Computer Algebra
Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

1-Algebraic-functions/1.2-Trinomial-products/1.2.3-General/47-1.2.3.3-d+e-x^n-^q-a+b-x^n+c-x^-2-n-^p

Nasser M. Abbasi

December 9, 2023 Compiled on December 9, 2023 at 6:00am

Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [96]. This is test number [47].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (96)	0.00 (0)
Mathematica	95.83 (92)	4.17 (4)
Maple	51.04 (49)	48.96 (47)
Fricas	51.04 (49)	48.96 (47)
Mupad	51.04 (49)	48.96 (47)
Sympy	43.75 (42)	56.25 (54)
Giac	38.54 (37)	61.46 (59)
Maxima	17.71 (17)	82.29 (79)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

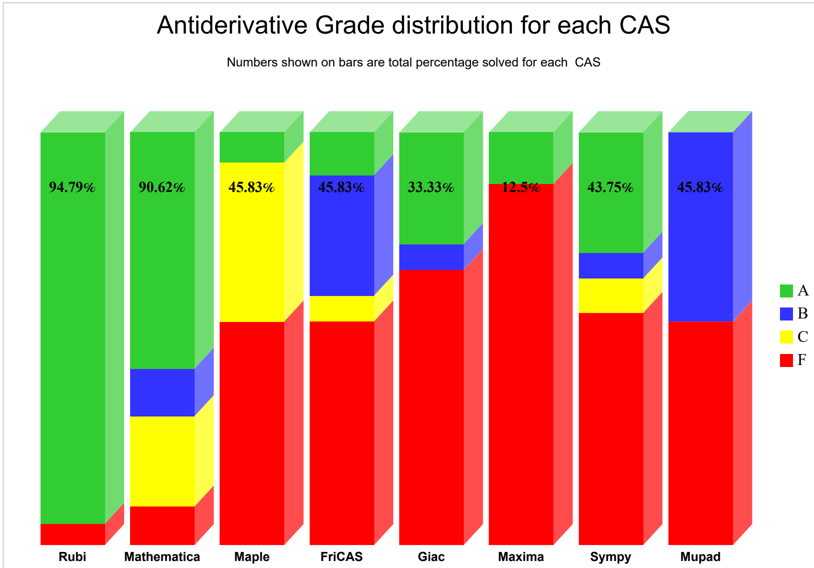
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

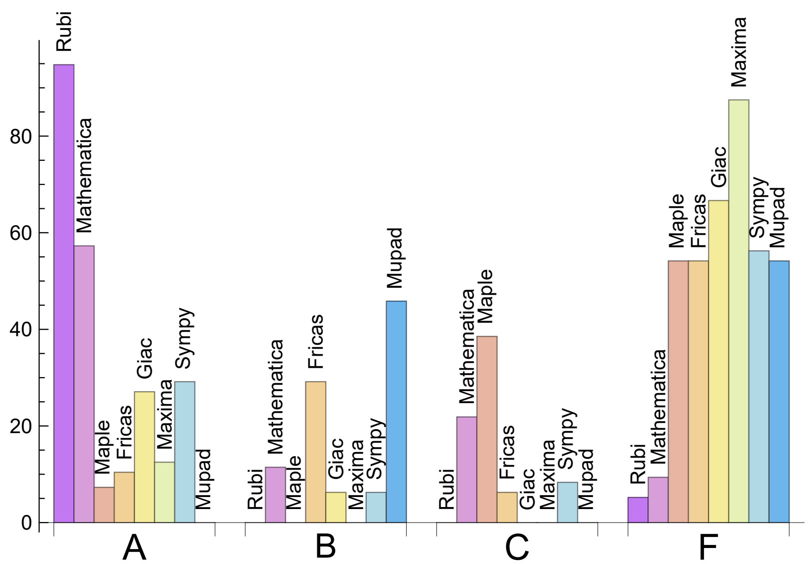
System	% A grade	% B grade	% C grade	% F grade
Rubi	94.792	0.000	0.000	5.208
Mathematica	57.292	11.458	21.875	9.375
Sympy	29.167	6.250	8.333	56.250
Giac	27.083	6.250	0.000	66.667
Maxima	12.500	0.000	0.000	87.500
Fricas	10.417	29.167	6.250	54.167
Maple	7.292	0.000	38.542	54.167
Mupad	0.000	45.833	0.000	54.167

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	4	100.00	0.00	0.00
Fricas	47	89.36	0.00	10.64
Maple	47	100.00	0.00	0.00
Mupad	47	0.00	100.00	0.00
Sympy	54	16.67	68.52	14.81
Giac	59	91.53	1.69	6.78
Maxima	79	98.73	0.00	1.27

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maple	0.19
Maxima	0.28
Giac	0.39
Fricas	0.48
Rubi	0.76
Mathematica	1.16
Mupad	7.00
Sympy	9.82

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maple	51.29	0.41	44.00	0.26
Maxima	128.29	1.10	72.00	1.08
Giac	331.35	1.75	147.00	0.94
Rubi	375.10	1.04	283.50	1.00
Sympy	435.00	2.78	110.50	0.48
Fricas	1192.80	3.44	349.00	2.21
Mathematica	2014.07	2.04	134.50	0.90
Mupad	2816.22	6.61	269.00	1.48

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

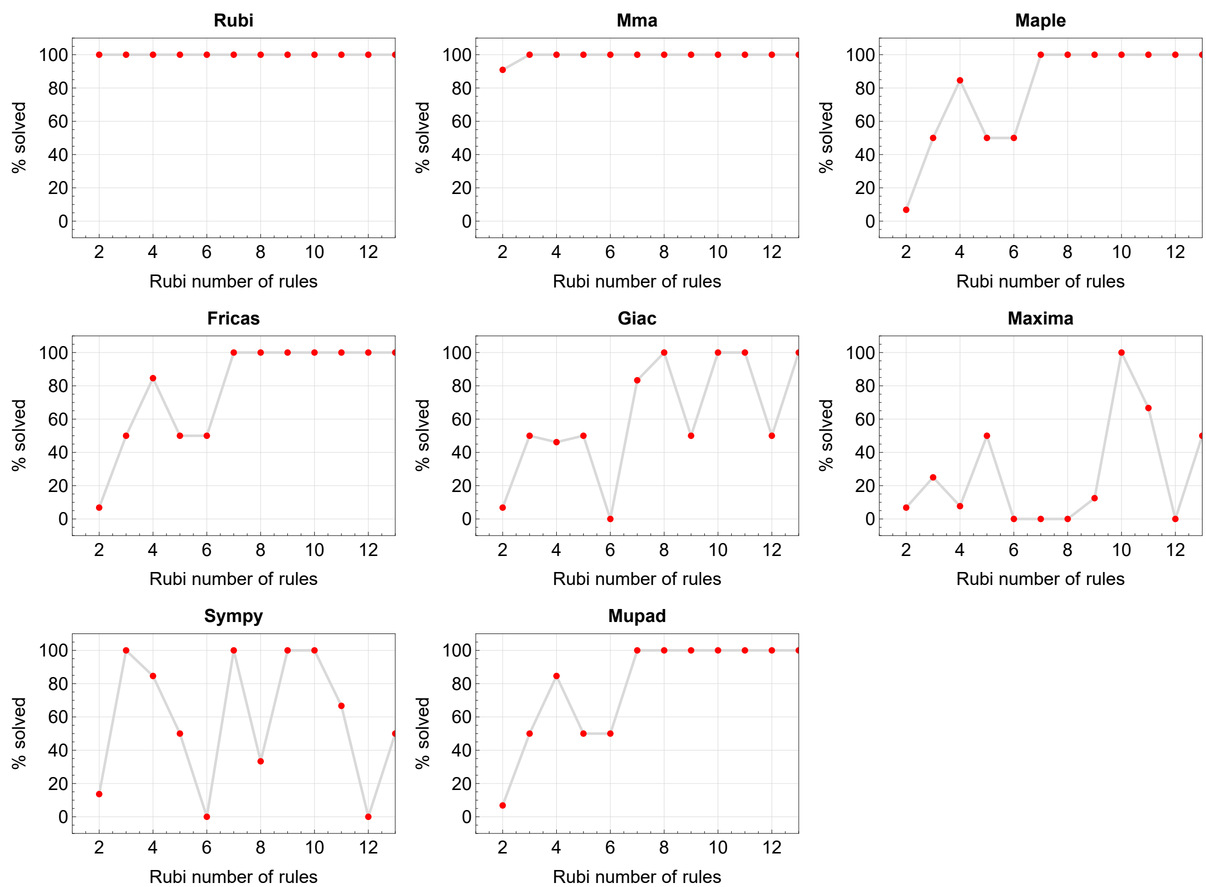


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

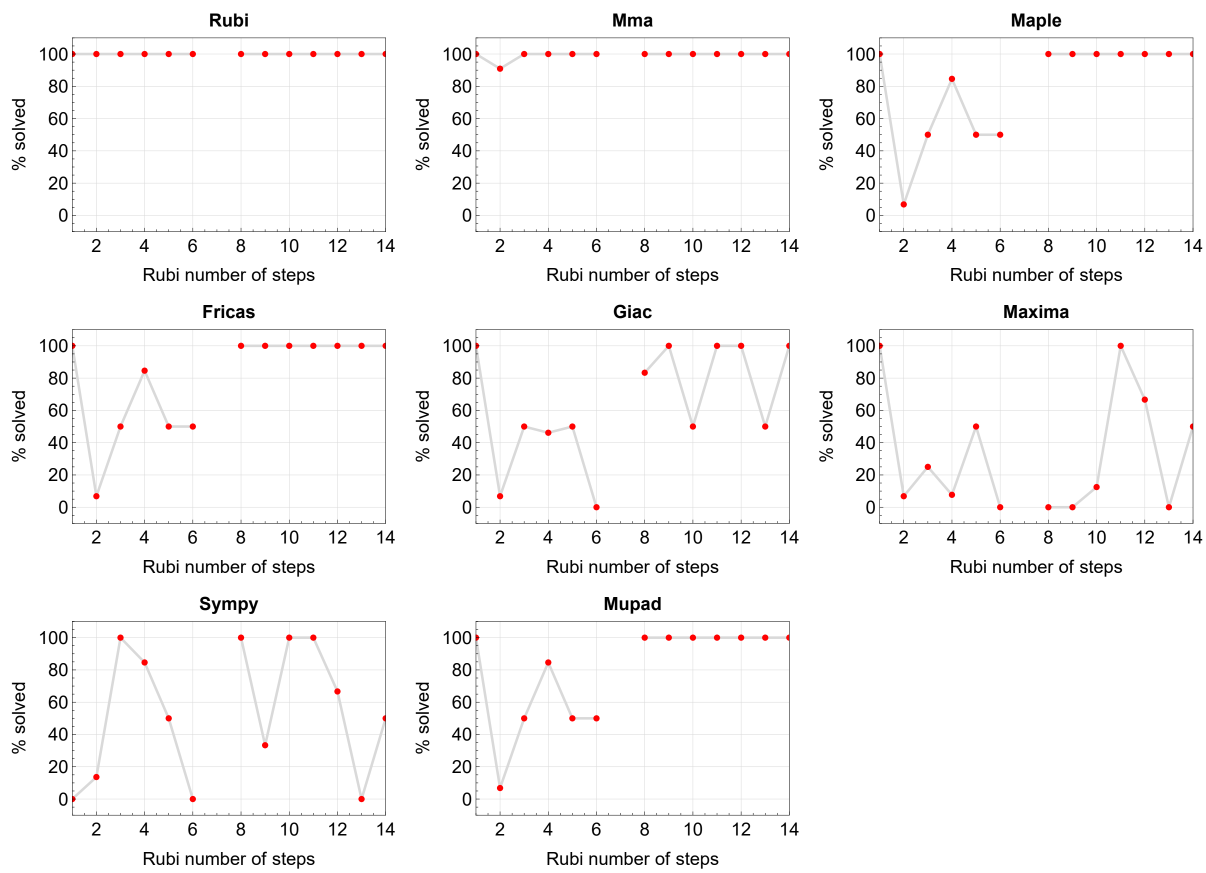


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

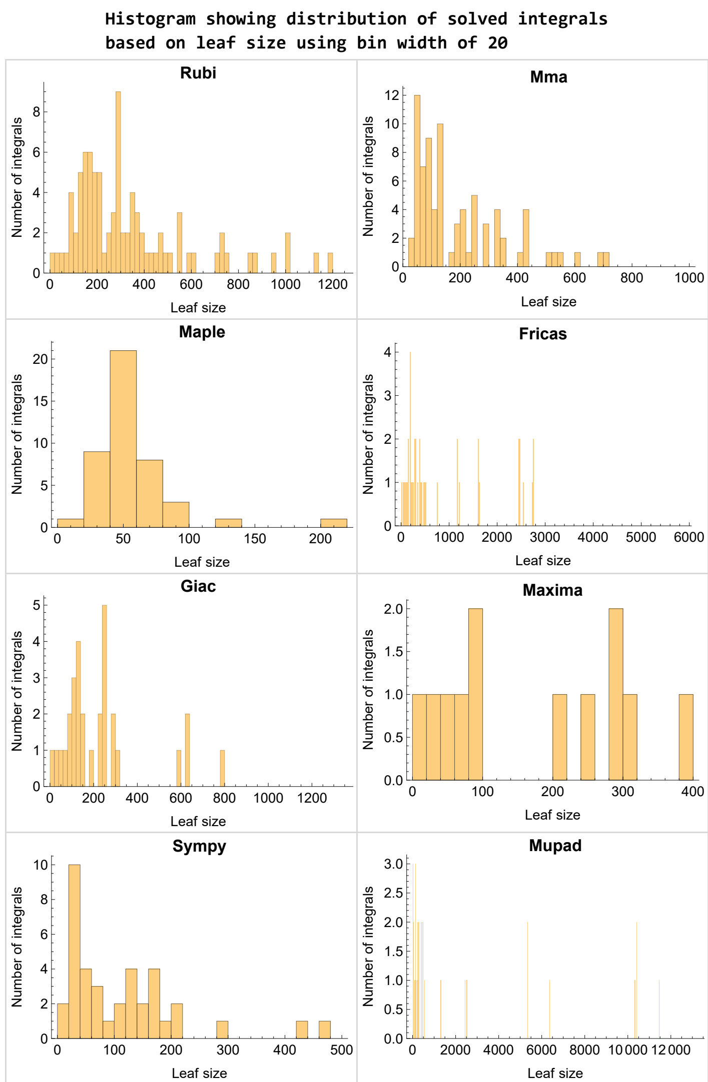


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

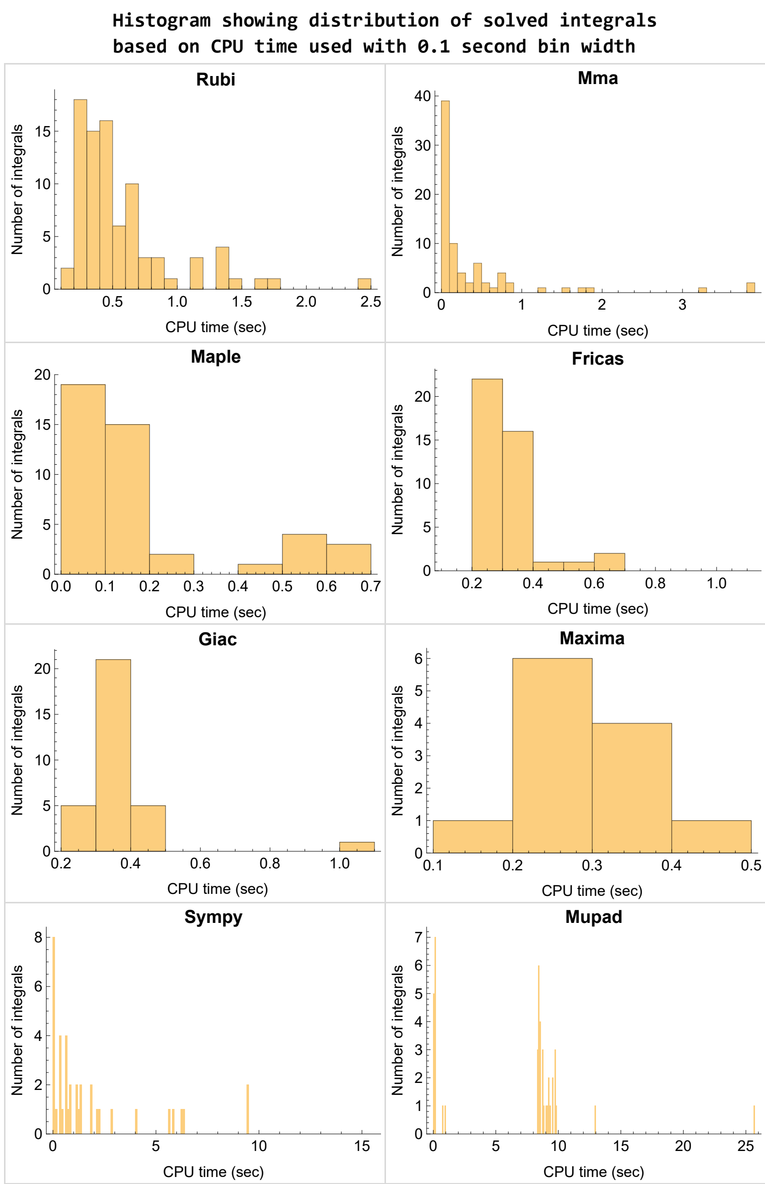


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

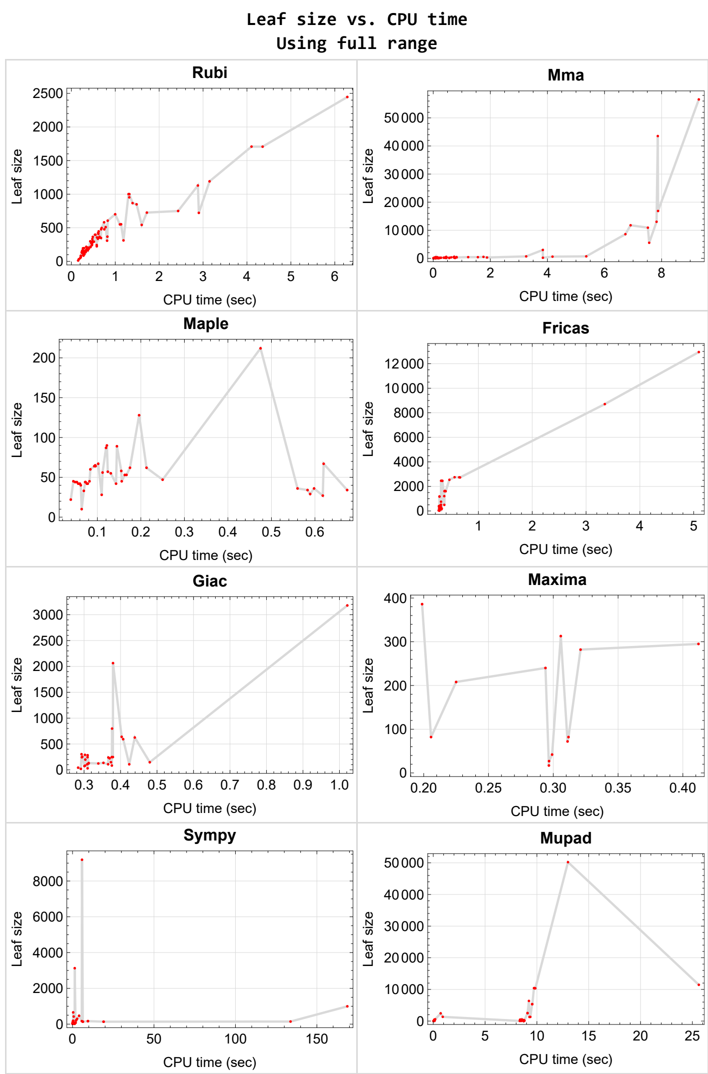


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{59, 90, 94, 95, 96\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica $\{79, 83, 84, 86, 89\}$

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

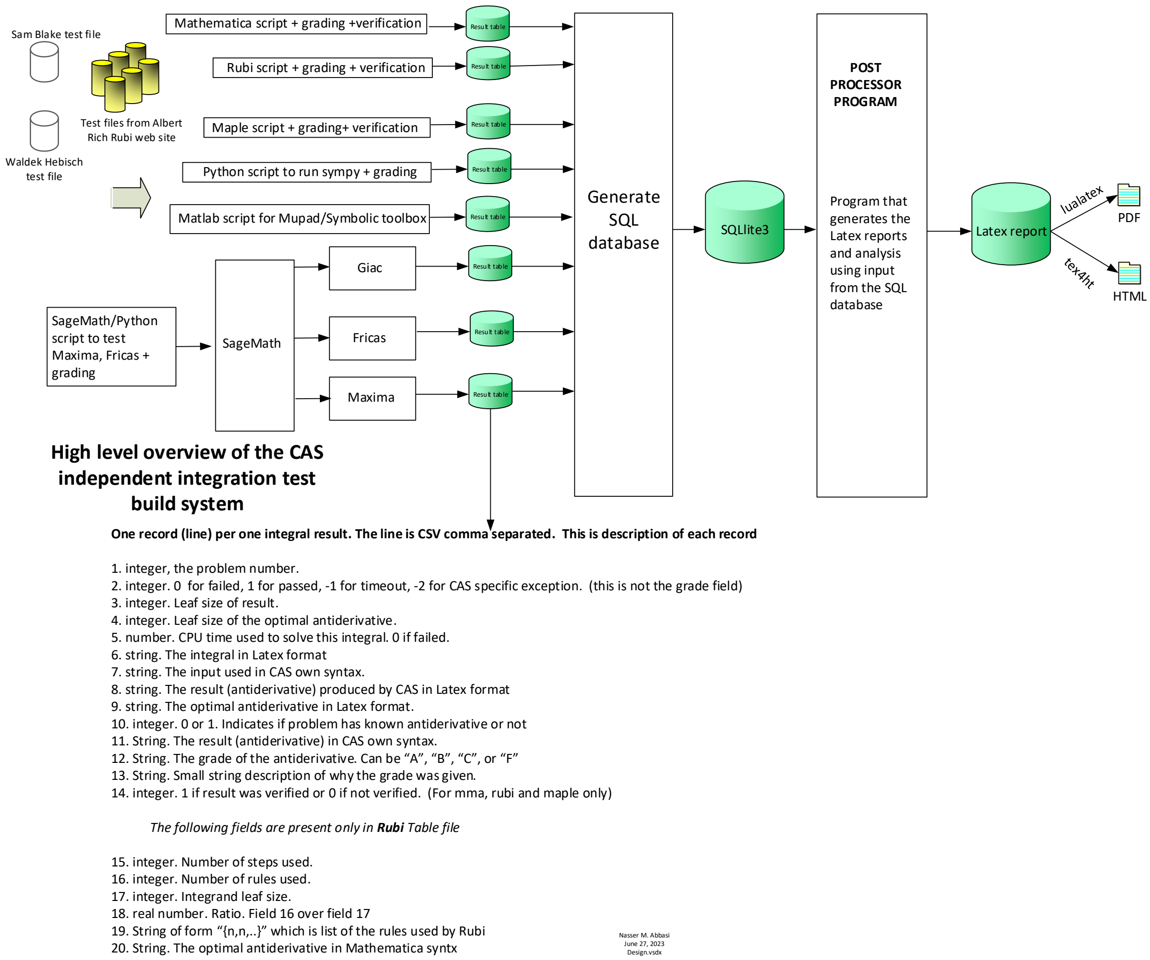
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 91, 92, 93 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 2, 3, 4, 11, 13, 15, 16, 19, 22, 24, 26, 27, 30, 34, 35, 36, 37, 38, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 60, 61, 62, 66, 67, 68, 69, 70, 71, 72, 73, 74, 77, 85, 87, 88, 91, 92, 93 }

B grade { 75, 76, 78, 79, 80, 81, 82, 83, 84, 86, 89 }

C grade { 5, 6, 7, 8, 9, 10, 12, 14, 17, 18, 20, 21, 23, 25, 28, 29, 31, 32, 33, 39, 41 }

F normal fail { 58, 63, 64, 65 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.3 Maple

A grade { 15, 26, 34, 35, 66, 67, 68 }

B grade { }

C grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 27, 28, 29, 30, 31, 32, 33, 36, 37, 38, 39, 40, 41 }

F normal fail { 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 60, 61, 62, 63, 64, 65, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 91, 92, 93 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 10, 14, 21, 23, 26, 31, 32, 33, 34, 35 }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 15, 16, 17, 18, 19, 20, 27, 28, 29, 30, 36, 37, 38, 39, 40, 41, 66, 67, 68 }

C grade { 11, 12, 13, 22, 24, 25 }

F normal fail { 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 60, 61, 62, 63, 64, 65, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 91, 92, 93 }

F(-1) timedout fail { }

F(-2) exception fail { 85, 86, 87, 88, 89 }

2.1.5 Maxima

A grade { 1, 2, 11, 15, 22, 26, 34, 36, 38, 66, 67, 68 }

B grade { }

C grade { }

F normal fail { 3, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14, 16, 17, 18, 19, 20, 21, 23, 24, 25, 27, 28, 29, 30, 31, 32, 33, 37, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 60, 61, 62, 63, 64, 65, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 91, 92, 93 }

F(-1) timedout fail { }

F(-2) exception fail { 35 }

2.1.6 Giac

A grade { 1, 2, 3, 10, 11, 12, 13, 14, 15, 16, 19, 21, 22, 23, 24, 25, 27, 30, 31, 32, 33, 34, 35, 36, 38, 40 }

B grade { 4, 26, 37, 66, 67, 68 }

C grade { }

F normal fail { 5, 6, 7, 8, 9, 17, 18, 20, 28, 29, 39, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 62, 63, 64, 65, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 93 }

F(-1) timedout fail { 41 }

F(-2) exception fail { 60, 61, 91, 92 }

2.1.7 Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 66, 67, 68 }

C grade { }

F normal fail { }

F(-1) timedout fail { 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 60, 61, 62, 63, 64, 65, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 91, 92, 93 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 1, 2, 5, 6, 7, 8, 9, 10, 11, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 24, 25, 27, 28, 29, 30, 31, 36, 38 }

B grade { 26, 34, 35, 66, 67, 68 }

C grade { 12, 23, 42, 43, 44, 47, 50, 62 }

F normal fail { 48, 49, 58, 70, 71, 85, 86, 87, 89 }

F(-1) timedout fail { 3, 4, 37, 39, 40, 41, 51, 52, 53, 54, 55, 56, 57, 59, 60, 61, 64, 65, 69, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 88, 90, 91, 92, 93, 94, 95, 96 }

F(-2) exception fail { 32, 33, 45, 46, 63, 72, 73, 74 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	305	352	334	34	282	1631	165	283	1331
N.S.	1	1.15	1.10	0.11	0.92	5.35	0.54	0.93	4.36
time (sec)	N/A	0.624	0.072	0.674	0.321	0.380	9.414	0.310	0.916

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	323	279	337	36	313	1613	168	303	1293
N.S.	1	0.86	1.04	0.11	0.97	4.99	0.52	0.94	4.00
time (sec)	N/A	0.451	0.070	0.560	0.306	0.369	9.495	0.293	9.281

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	754	849	534	34	0	2749	0	593	2510
N.S.	1	1.13	0.71	0.05	0.00	3.65	0.00	0.79	3.33
time (sec)	N/A	1.481	0.456	0.583	0.000	0.558	0.000	0.407	9.088

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	329	293	425	36	0	2741	0	625	2438
N.S.	1	0.89	1.29	0.11	0.00	8.33	0.00	1.90	7.41
time (sec)	N/A	0.496	0.134	0.598	0.000	0.643	0.000	0.439	0.715

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	791	1001	67	53	0	2461	136	0	10409
N.S.	1	1.27	0.08	0.07	0.00	3.11	0.17	0.00	13.16
time (sec)	N/A	1.301	0.035	0.167	0.000	0.322	19.014	0.000	9.717

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	791	1001	67	53	0	2461	136	0	10411
N.S.	1	1.27	0.08	0.07	0.00	3.11	0.17	0.00	13.16
time (sec)	N/A	1.306	0.026	0.163	0.000	0.306	6.282	0.000	9.776

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	349	365	69	57	0	2453	136	0	10337
N.S.	1	1.05	0.20	0.16	0.00	7.03	0.39	0.00	29.62
time (sec)	N/A	0.499	0.033	0.124	0.000	0.331	19.107	0.000	9.776

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	751	953	69	55	0	2453	136	0	10343
N.S.	1	1.27	0.09	0.07	0.00	3.27	0.18	0.00	13.77
time (sec)	N/A	1.347	0.029	0.131	0.000	0.327	6.382	0.000	9.850

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	411	497	55	42	0	1177	75	0	5341
N.S.	1	1.21	0.13	0.10	0.00	2.86	0.18	0.00	13.00
time (sec)	N/A	0.672	0.037	0.143	0.000	0.276	1.807	0.000	9.560

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	451	477	55	42	0	389	24	239	459
N.S.	1	1.06	0.12	0.09	0.00	0.86	0.05	0.53	1.02
time (sec)	N/A	0.732	0.016	0.076	0.000	0.267	0.864	0.367	0.102

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	90	64	22	72	61	73	72	33
N.S.	1	1.06	0.75	0.26	0.85	0.72	0.86	0.85	0.39
time (sec)	N/A	0.280	0.018	0.039	0.311	0.273	0.067	0.301	8.345

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	169	135	89	0	209	190	108	95
N.S.	1	1.21	0.96	0.64	0.00	1.49	1.36	0.77	0.68
time (sec)	N/A	0.355	0.144	0.145	0.000	0.272	0.341	0.366	0.087

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	347	397	258	27	0	189	19	247	311
N.S.	1	1.14	0.74	0.08	0.00	0.54	0.05	0.71	0.90
time (sec)	N/A	0.544	0.135	0.618	0.000	0.320	1.148	0.376	8.752

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	331	397	55	42	0	389	20	245	145
N.S.	1	1.20	0.17	0.13	0.00	1.18	0.06	0.74	0.44
time (sec)	N/A	0.542	0.017	0.078	0.000	0.266	1.277	0.310	0.137

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	32	31	28	27	43	26	29	21
N.S.	1	1.19	1.15	1.04	1.00	1.59	0.96	1.07	0.78
time (sec)	N/A	0.174	0.012	0.110	0.297	0.262	0.066	0.310	0.026

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	149	131	56	0	317	49	147	269
N.S.	1	1.14	1.00	0.43	0.00	2.42	0.37	1.12	2.05
time (sec)	N/A	0.273	0.063	0.112	0.000	0.281	0.661	0.374	0.117

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	53	40	0	349	24	0	399
N.S.	1	1.00	0.34	0.25	0.00	2.22	0.15	0.00	2.54
time (sec)	N/A	0.267	0.013	0.062	0.000	0.289	0.087	0.000	8.420

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	197	55	42	0	509	24	0	483
N.S.	1	1.15	0.32	0.25	0.00	2.98	0.14	0.00	2.82
time (sec)	N/A	0.287	0.015	0.060	0.000	0.304	0.086	0.000	8.465

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	127	111	58	0	221	49	123	233
N.S.	1	1.09	0.95	0.50	0.00	1.89	0.42	1.05	1.99
time (sec)	N/A	0.243	0.051	0.155	0.000	0.291	0.659	0.339	8.430

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	511	509	57	44	0	1177	76	0	5341
N.S.	1	1.00	0.11	0.09	0.00	2.30	0.15	0.00	10.45
time (sec)	N/A	0.773	0.035	0.049	0.000	0.277	1.868	0.000	9.534

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	411	477	57	44	0	421	26	223	447
N.S.	1	1.16	0.14	0.11	0.00	1.02	0.06	0.54	1.09
time (sec)	N/A	0.685	0.014	0.073	0.000	0.279	0.876	0.370	8.373

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	107	90	33	82	97	82	82	44
N.S.	1	1.10	0.93	0.34	0.85	1.00	0.85	0.85	0.45
time (sec)	N/A	0.296	0.043	0.069	0.312	0.284	0.070	0.303	0.046

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	164	129	87	0	195	148	108	109
N.S.	1	1.17	0.92	0.62	0.00	1.39	1.06	0.77	0.78
time (sec)	N/A	0.367	0.114	0.120	0.000	0.279	0.301	0.309	0.113

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	347	337	257	29	0	193	20	247	312
N.S.	1	0.97	0.74	0.08	0.00	0.56	0.06	0.71	0.90
time (sec)	N/A	0.604	0.098	0.589	0.000	0.303	1.175	0.379	8.547

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	355	347	57	44	0	417	26	253	208
N.S.	1	0.98	0.16	0.12	0.00	1.17	0.07	0.71	0.59
time (sec)	N/A	0.609	0.021	0.072	0.000	0.299	1.303	0.295	8.443

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	25	10	17	17	17	19	9
N.S.	1	1.00	1.92	0.77	1.31	1.31	1.31	1.46	0.69
time (sec)	N/A	0.153	0.005	0.064	0.297	0.274	0.062	0.291	0.014

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	155	129	64	0	285	51	147	269
N.S.	1	1.20	1.00	0.50	0.00	2.21	0.40	1.14	2.09
time (sec)	N/A	0.249	0.056	0.093	0.000	0.290	0.685	0.480	0.098

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	163	55	42	0	317	26	0	399
N.S.	1	0.99	0.33	0.25	0.00	1.92	0.16	0.00	2.42
time (sec)	N/A	0.259	0.013	0.059	0.000	0.273	0.086	0.000	0.108

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	195	57	44	0	477	26	0	483
N.S.	1	1.15	0.34	0.26	0.00	2.82	0.15	0.00	2.86
time (sec)	N/A	0.269	0.014	0.053	0.000	0.300	0.091	0.000	8.575

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	135	114	64	0	245	51	135	245
N.S.	1	1.08	0.91	0.51	0.00	1.96	0.41	1.08	1.96
time (sec)	N/A	0.234	0.039	0.096	0.000	0.276	0.672	0.353	8.513

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	189	71	47	0	153	163	107	133
N.S.	1	1.40	0.53	0.35	0.00	1.13	1.21	0.79	0.99
time (sec)	N/A	0.385	0.069	0.250	0.000	0.287	0.498	0.424	8.803

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F(-2)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	178	72	62	0	158	0	123	1
N.S.	1	1.09	0.44	0.38	0.00	0.96	0.00	0.75	0.01
time (sec)	N/A	0.347	0.040	0.213	0.000	0.286	0.000	0.313	8.700

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F(-2)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	188	89	62	0	181	0	131	1
N.S.	1	1.04	0.49	0.34	0.00	1.01	0.00	0.73	0.01
time (sec)	N/A	0.392	0.049	0.175	0.000	0.296	0.000	0.313	8.758

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	42	42	108	112	42	39
N.S.	1	1.00	1.00	0.86	0.86	2.20	2.29	0.86	0.80
time (sec)	N/A	0.201	0.019	0.056	0.299	0.286	0.136	0.284	8.496

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	86	90	0	291	423	83	127
N.S.	1	1.00	1.00	1.05	0.00	3.38	4.92	0.97	1.48
time (sec)	N/A	0.276	0.061	0.122	0.000	0.306	0.708	0.376	0.135

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	242	293	45	240	754	109	243	555
N.S.	1	0.96	1.16	0.18	0.95	2.98	0.43	0.96	2.19
time (sec)	N/A	0.467	0.073	0.045	0.294	0.305	0.348	0.294	0.186

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	212	251	65	0	2540	0	3179	6366
N.S.	1	1.02	1.21	0.31	0.00	12.21	0.00	15.28	30.61
time (sec)	N/A	0.463	0.119	0.095	0.000	0.461	0.000	1.022	9.231

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	311	348	346	45	295	1608	167	290	1308
N.S.	1	1.12	1.11	0.14	0.95	5.17	0.54	0.93	4.21
time (sec)	N/A	0.662	0.077	0.156	0.412	0.392	5.668	0.303	9.367

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	716	550	88	67	0	8707	0	0	11453
N.S.	1	0.77	0.12	0.09	0.00	12.16	0.00	0.00	16.00
time (sec)	N/A	1.109	0.036	0.620	0.000	3.349	0.000	0.000	25.624

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	753	867	551	45	0	2730	0	639	2520
N.S.	1	1.15	0.73	0.06	0.00	3.63	0.00	0.85	3.35
time (sec)	N/A	1.389	0.734	0.082	0.000	0.659	0.000	0.403	9.101

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	433	366	88	67	0	12946	0	0	50213
N.S.	1	0.85	0.20	0.15	0.00	29.90	0.00	0.00	115.97
time (sec)	N/A	0.656	0.060	0.102	0.000	5.095	0.000	0.000	13.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	141	141	127	0	0	0	466	0	0
N.S.	1	1.00	0.90	0.00	0.00	0.00	3.30	0.00	0.00
time (sec)	N/A	0.325	0.562	0.000	0.000	0.000	4.012	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	107	107	107	0	0	0	296	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	2.77	0.00	0.00
time (sec)	N/A	0.265	0.221	0.000	0.000	0.000	2.824	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	83	83	83	0	0	0	209	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	2.52	0.00	0.00
time (sec)	N/A	0.207	0.070	0.000	0.000	0.000	2.159	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	152	152	131	0	0	0	0	0	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.304	0.179	0.000	0.000	0.000	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	205	205	186	0	0	0	0	0	0
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.385	0.269	0.000	0.000	0.000	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	81	81	81	0	0	0	214	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	2.64	0.00	0.00
time (sec)	N/A	0.211	0.089	0.000	0.000	0.000	2.254	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	288	288	188	0	0	0	0	0	0
N.S.	1	1.00	0.65	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.448	0.384	0.000	0.000	0.000	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	203	203	136	0	0	0	0	0	0
N.S.	1	1.00	0.67	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.361	0.210	0.000	0.000	0.000	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	134	133	83	0	0	0	994	0	0
N.S.	1	0.99	0.62	0.00	0.00	0.00	7.42	0.00	0.00
time (sec)	N/A	0.270	0.078	0.000	0.000	0.000	168.683	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	333	333	227	0	0	0	0	0	0
N.S.	1	1.00	0.68	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.484	0.266	0.000	0.000	0.000	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	410	410	298	0	0	0	0	0	0
N.S.	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.630	0.469	0.000	0.000	0.000	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	424	424	188	0	0	0	0	0	0
N.S.	1	1.00	0.44	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.607	0.738	0.000	0.000	0.000	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	272	272	136	0	0	0	0	0	0
N.S.	1	1.00	0.50	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.432	0.445	0.000	0.000	0.000	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	184	194	83	0	0	0	0	0	0
N.S.	1	1.05	0.45	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.360	0.082	0.000	0.000	0.000	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	582	582	346	0	0	0	0	0	0
N.S.	1	1.00	0.59	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.746	0.424	0.000	0.000	0.000	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	701	701	426	0	0	0	0	0	0
N.S.	1	1.00	0.61	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.013	0.716	0.000	0.000	0.000	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	171	171	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.359	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	21	23	23	0	23	23
N.S.	1	1.00	1.10	1.00	1.10	1.10	0.00	1.10	1.10
time (sec)	N/A	0.153	0.301	0.160	0.278	0.297	0.000	0.370	8.931

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	299	299	213	0	0	0	0	0	0
N.S.	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.419	0.197	0.000	0.000	0.000	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	217	217	171	0	0	0	0	0	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.335	0.133	0.000	0.000	0.000	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	135	135	110	0	0	0	144	0	0
N.S.	1	1.00	0.81	0.00	0.00	0.00	1.07	0.00	0.00
time (sec)	N/A	0.262	0.072	0.000	0.000	0.000	133.828	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	167	167	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.334	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	261	261	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.465	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	357	357	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.593	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	57	60	82	137	656	198	59
N.S.	1	1.00	0.92	0.97	1.32	2.21	10.58	3.19	0.95
time (sec)	N/A	0.212	0.181	0.084	0.206	0.313	0.391	0.303	8.392

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	123	128	208	495	3128	798	131
N.S.	1	1.00	0.93	0.97	1.58	3.75	23.70	6.05	0.99
time (sec)	N/A	0.306	0.765	0.196	0.225	0.366	1.342	0.377	8.490

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	205	212	386	1209	9190	2064	227
N.S.	1	1.00	0.94	0.97	1.77	5.55	42.16	9.47	1.04
time (sec)	N/A	0.435	3.841	0.475	0.199	0.366	5.814	0.379	8.548

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	308	308	295	0	0	0	0	0	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.776	1.883	0.000	0.000	0.000	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	224	224	216	0	0	0	0	0	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.566	0.560	0.000	0.000	0.000	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	154	154	134	0	0	0	0	0	0
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.309	0.156	0.000	0.000	0.000	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	243	243	200	0	0	0	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.580	0.432	0.000	0.000	0.000	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	368	368	327	0	0	0	0	0	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.785	0.821	0.000	0.000	0.000	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	552	552	509	0	0	0	0	0	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.094	1.759	0.000	0.000	0.000	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	750	750	5537	0	0	0	0	0	0
N.S.	1	1.00	7.38	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.327	7.564	0.000	0.000	0.000	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	543	543	2980	0	0	0	0	0	0
N.S.	1	1.00	5.49	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.546	3.839	0.000	0.000	0.000	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	362	312	603	0	0	0	0	0	0
N.S.	1	0.86	1.67	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.172	4.178	0.000	0.000	0.000	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	726	726	11767	0	0	0	0	0	0
N.S.	1	1.00	16.21	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.658	6.913	0.000	0.000	0.000	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	1129	1129	16855	0	0	0	0	0	0
N.S.	1	1.00	14.93	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.840	7.875	0.000	0.000	0.000	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1707	1707	13018	0	0	0	0	0	0
N.S.	1	1.00	7.63	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	4.330	7.822	0.000	0.000	0.000	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1191	1191	10910	0	0	0	0	0	0
N.S.	1	1.00	9.16	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.115	7.518	0.000	0.000	0.000	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	713	722	8593	0	0	0	0	0	0
N.S.	1	1.01	12.05	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.868	6.731	0.000	0.000	0.000	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	1708	1708	43535	0	0	0	0	0	0
N.S.	1	1.00	25.49	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	4.173	7.866	0.000	0.000	0.000	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	2446	2446	56566	0	0	0	0	0	0
N.S.	1	1.00	23.13	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	6.314	9.303	0.000	0.000	0.000	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	292	292	424	0	0	0	0	0	0
N.S.	1	1.00	1.45	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.501	1.223	0.000	0.000	0.000	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	294	294	690	0	0	0	0	0	0
N.S.	1	1.00	2.35	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.467	3.255	0.000	0.000	0.000	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	292	292	245	0	0	0	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.461	0.366	0.000	0.000	0.000	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	298	298	414	0	0	0	0	0	0
N.S.	1	1.00	1.39	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.493	1.564	0.000	0.000	0.000	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	298	298	701	0	0	0	0	0	0
N.S.	1	1.00	2.35	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.473	5.360	0.000	0.000	0.000	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	26	28	28	0	28	28
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.00	1.08	1.08
time (sec)	N/A	0.156	1.326	0.115	0.261	0.307	0.000	1.205	10.626

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	606	606	438	0	0	0	0	0	0
N.S.	1	1.00	0.72	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.832	0.816	0.000	0.000	0.000	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	447	447	338	0	0	0	0	0	0
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.608	0.628	0.000	0.000	0.000	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	288	288	243	0	0	0	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.452	0.461	0.000	0.000	0.000	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	26	28	28	0	28	28
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.00	1.08	1.08
time (sec)	N/A	0.164	0.796	0.126	0.261	0.262	0.000	0.365	10.360

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	26	28	41	0	28	28
N.S.	1	1.00	1.08	1.00	1.08	1.58	0.00	1.08	1.08
time (sec)	N/A	0.162	0.624	0.079	0.271	0.276	0.000	0.372	11.181

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	26	28	54	0	28	28
N.S.	1	1.00	1.08	1.00	1.08	2.08	0.00	1.08	1.08
time (sec)	N/A	0.161	0.912	0.095	0.267	0.320	0.000	0.398	11.289

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [38] had the largest ratio of [.7647059999999997]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	12	11	1.15	17	0.647
2	A	11	10	0.86	18	0.556
3	A	12	11	1.13	17	0.647
4	A	14	13	0.89	18	0.722
5	A	10	9	1.27	26	0.346
6	A	10	9	1.27	26	0.346
7	A	4	4	1.05	27	0.148
8	A	10	9	1.27	27	0.333
9	A	8	7	1.21	18	0.389
10	A	10	9	1.06	18	0.500
11	A	10	9	1.06	18	0.500
12	A	8	7	1.21	16	0.438
13	A	8	7	1.14	13	0.538
14	A	8	7	1.20	18	0.389
15	A	5	5	1.19	18	0.278
16	A	4	4	1.14	18	0.222
17	A	4	4	1.00	18	0.222
18	A	4	4	1.15	18	0.222
19	A	4	4	1.09	18	0.222
20	A	10	9	1.00	20	0.450
21	A	10	9	1.16	20	0.450
22	A	11	10	1.10	20	0.500

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	10	9	1.17	18	0.500
24	A	8	7	0.97	15	0.467
25	A	9	8	0.98	20	0.400
26	A	4	4	1.00	20	0.200
27	A	4	4	1.20	20	0.200
28	A	4	4	0.99	20	0.200
29	A	4	4	1.15	20	0.200
30	A	4	4	1.08	20	0.200
31	A	8	7	1.40	25	0.280
32	A	9	8	1.09	26	0.308
33	A	9	8	1.04	33	0.242
34	A	3	3	1.00	17	0.176
35	A	3	3	1.00	22	0.136
36	A	12	11	0.96	17	0.647
37	A	4	4	1.02	22	0.182
38	A	14	13	1.12	17	0.765
39	A	13	12	0.77	22	0.545
40	A	13	12	1.15	17	0.706
41	A	6	6	0.85	22	0.273
42	A	2	2	1.00	21	0.095
43	A	2	2	1.00	21	0.095
44	A	3	3	1.00	19	0.158
45	A	2	2	1.00	21	0.095
46	A	2	2	1.00	21	0.095
47	A	3	3	1.00	20	0.150
48	A	2	2	1.00	21	0.095
49	A	2	2	1.00	21	0.095
50	A	4	4	0.99	19	0.211
51	A	2	2	1.00	21	0.095
52	A	2	2	1.00	21	0.095
53	A	2	2	1.00	21	0.095
54	A	2	2	1.00	21	0.095
55	A	5	5	1.05	19	0.263
56	A	2	2	1.00	21	0.095

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
57	A	2	2	1.00	21	0.095
58	A	2	2	1.00	23	0.087
59	N/A	1	0	1.00	21	0.000
60	A	2	2	1.00	21	0.095
61	A	2	2	1.00	21	0.095
62	A	2	2	1.00	19	0.105
63	A	2	2	1.00	21	0.095
64	A	2	2	1.00	21	0.095
65	A	2	2	1.00	21	0.095
66	A	2	2	1.00	22	0.091
67	A	2	2	1.00	24	0.083
68	A	2	2	1.00	24	0.083
69	A	2	2	1.00	26	0.077
70	A	2	2	1.00	26	0.077
71	A	2	2	1.00	24	0.083
72	A	2	2	1.00	26	0.077
73	A	2	2	1.00	26	0.077
74	A	2	2	1.00	26	0.077
75	A	2	2	1.00	26	0.077
76	A	2	2	1.00	26	0.077
77	A	4	4	0.86	24	0.167
78	A	2	2	1.00	26	0.077
79	A	2	2	1.00	26	0.077
80	A	2	2	1.00	26	0.077
81	A	2	2	1.00	26	0.077
82	A	6	6	1.01	24	0.250
83	A	2	2	1.00	26	0.077
84	A	2	2	1.00	26	0.077
85	A	2	2	1.00	26	0.077
86	A	2	2	1.00	26	0.077
87	A	2	2	1.00	26	0.077
88	A	2	2	1.00	26	0.077
89	A	2	2	1.00	26	0.077

Continued on next page

2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
90	N/A	1	0	1.00	26	0.000
91	A	2	2	1.00	26	0.077
92	A	2	2	1.00	26	0.077
93	A	2	2	1.00	24	0.083
94	N/A	1	0	1.00	26	0.000
95	N/A	1	0	1.00	26	0.000
96	N/A	1	0	1.00	26	0.000

2.3. Detailed conclusion table specific for Rubi results

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \frac{d+ex^3}{a+cx^6}\,dx$

3.1.1 Optimal result

Integrand size = 17, antiderivative size = 305

$$\int \frac{d+ex^3}{a+cx^6}\,dx = \frac{d\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt[6]{c}} - \frac{\left(\sqrt{c}d+\sqrt{3}\sqrt{a}e\right)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{6a^{5/6}c^{2/3}} + \frac{\left(\sqrt{c}d-\sqrt{3}\sqrt{a}e\right)\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{6a^{5/6}c^{2/3}} - \frac{e\log\left(\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{6\sqrt[3]{a}c^{2/3}} - \frac{\left(\sqrt{3}\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{12a^{5/6}c^{2/3}} + \frac{\left(\sqrt{3}\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{12a^{5/6}c^{2/3}}$$

output
```
1/3*d*arctan(c^(1/6)*x/a^(1/6))/a^(5/6)/c^(1/6)-1/6*e*ln(a^(1/3)+c^(1/3)*x
^2)/a^(1/3)/c^(2/3)+1/6*arctan(2*c^(1/6)*x/a^(1/6)+3^(1/2))*(-e*3^(1/2)*a^
(1/2)+d*c^(1/2))/a^(5/6)/c^(2/3)+1/6*arctan(2*c^(1/6)*x/a^(1/6)-3^(1/2))*(
e*3^(1/2)*a^(1/2)+d*c^(1/2))/a^(5/6)/c^(2/3)-1/12*ln(a^(1/3)+c^(1/3)*x^2-a
^(1/6)*c^(1/6)*x*3^(1/2))*(-e*a^(1/2)+d*3^(1/2)*c^(1/2))/a^(5/6)/c^(2/3)+1
/12*ln(a^(1/3)+c^(1/3)*x^2+a^(1/6)*c^(1/6)*x*3^(1/2))*(e*a^(1/2)+d*3^(1/2)
*c^(1/2))/a^(5/6)/c^(2/3)
```

3.1.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.10

$$\int \frac{d+ex^3}{a+cx^6}\,dx = \frac{d\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt[6]{c}} + \frac{\left(\sqrt[6]{a}\sqrt{c}d+\sqrt{3}a^{2/3}e\right)\arctan\left(\frac{-\sqrt{3}\sqrt[6]{a}+2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{6ac^{2/3}} + \frac{\left(\sqrt[6]{a}\sqrt{c}d-\sqrt{3}a^{2/3}e\right)\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}+2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{6ac^{2/3}} - \frac{e\log\left(\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{6\sqrt[3]{a}c^{2/3}} - \frac{\left(\sqrt{3}\sqrt[6]{a}\sqrt{c}d-a^{2/3}e\right)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{12ac^{2/3}} - \frac{\left(-\sqrt{3}\sqrt[6]{a}\sqrt{c}d-a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{12ac^{2/3}}$$

input
```
Integrate[(d + e*x^3)/(a + c*x^6),x]
```

output
```
(d*ArcTan[(c^(1/6)*x)/a^(1/6)])/(3*a^(5/6)*c^(1/6)) + ((a^(1/6)*Sqrt[c]*d
+ Sqrt[3]*a^(2/3)*e)*ArcTan[(-(Sqrt[3]*a^(1/6)) + 2*c^(1/6)*x)/a^(1/6)])/(
6*a*c^(2/3)) + ((a^(1/6)*Sqrt[c]*d - Sqrt[3]*a^(2/3)*e)*ArcTan[(Sqrt[3]*a^
(1/6) + 2*c^(1/6)*x)/a^(1/6)])/(6*a*c^(2/3)) - (e*Log[a^(1/3) + c^(1/3)*x^
2])/(6*a^(1/3)*c^(2/3)) - ((Sqrt[3]*a^(1/6)*Sqrt[c]*d - a^(2/3)*e)*Log[a^(
1/3) - Sqrt[3]*a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(12*a*c^(2/3)) - ((-(Sqrt
[3]*a^(1/6)*Sqrt[c]*d) - a^(2/3)*e)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*c^(1/6)*
x + c^(1/3)*x^2])/(12*a*c^(2/3))
```

3.1.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.15, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.647$, Rules used = {1746, 27, 452, 218, 240, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^3}{a+cx^6}\,dx$$

$$\Big\downarrow \; 1746$$

$$\frac{\int \frac{\sqrt[3]{c}d-\sqrt[3]{a}ex}{\sqrt[3]{c}x^2+\sqrt[3]{a}}dx}{3a^{2/3}\sqrt[3]{c}}+\frac{\int \frac{2\sqrt[3]{c}d-\sqrt[3]{a}\left(\frac{\sqrt{3}\sqrt{c}d}{\sqrt{a}}-e\right)x}{\sqrt[3]{a}\left(\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1\right)}dx}{6a^{2/3}\sqrt[3]{c}}+\frac{\int \frac{2\sqrt[3]{c}d+\sqrt[3]{a}\left(\frac{\sqrt{3}\sqrt{c}d}{\sqrt{a}}+e\right)x}{\sqrt[3]{a}\left(\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1\right)}dx}{6a^{2/3}\sqrt[3]{c}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt[3]{c}d-\sqrt[3]{a}ex}{\sqrt[3]{c}x^2+\sqrt[3]{a}}dx}{3a^{2/3}\sqrt[3]{c}}+\frac{\int \frac{2\sqrt[3]{c}d-\sqrt[3]{a}\left(\frac{\sqrt{3}\sqrt{c}d}{\sqrt{a}}-e\right)x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a\sqrt[3]{c}}+\frac{\int \frac{2\sqrt[3]{c}d+\sqrt[3]{a}\left(\frac{\sqrt{3}\sqrt{c}d}{\sqrt{a}}+e\right)x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a\sqrt[3]{c}}$$

$$\Big\downarrow \text{452}$$

$$\frac{\sqrt[3]{c}d\int \frac{1}{\sqrt[3]{c}x^2+\sqrt[3]{a}}dx-\sqrt[3]{a}e\int \frac{x}{\sqrt[3]{c}x^2+\sqrt[3]{a}}dx}{3a^{2/3}\sqrt[3]{c}}+\frac{\int \frac{2\sqrt[3]{c}d-\sqrt[3]{a}\left(\frac{\sqrt{3}\sqrt{c}d}{\sqrt{a}}-e\right)x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a\sqrt[3]{c}}+\frac{\int \frac{2\sqrt[3]{c}d+\sqrt[3]{a}\left(\frac{\sqrt{3}\sqrt{c}d}{\sqrt{a}}+e\right)x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a\sqrt[3]{c}}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\sqrt[6]{c}d\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{\sqrt[6]{a}}-\sqrt[3]{a}e\int \frac{x}{\sqrt[3]{c}x^2+\sqrt[3]{a}}dx}{3a^{2/3}\sqrt[3]{c}}+\frac{\int \frac{2\sqrt[3]{c}d-\sqrt[3]{a}\left(\frac{\sqrt{3}\sqrt{c}d}{\sqrt{a}}-e\right)x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a\sqrt[3]{c}}+\frac{\int \frac{2\sqrt[3]{c}d+\sqrt[3]{a}\left(\frac{\sqrt{3}\sqrt{c}d}{\sqrt{a}}+e\right)x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a\sqrt[3]{c}}$$

$$\Big\downarrow \text{240}$$

$$\frac{\int \frac{2\sqrt[3]{c}d-\sqrt[3]{a}\left(\frac{\sqrt{3}\sqrt{c}d}{\sqrt{a}}-e\right)x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a\sqrt[3]{c}}+\frac{\int \frac{2\sqrt[3]{c}d+\sqrt[3]{a}\left(\frac{\sqrt{3}\sqrt{c}d}{\sqrt{a}}+e\right)x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a\sqrt[3]{c}}+\frac{\frac{\sqrt[6]{c}d\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{\sqrt[6]{a}}-\frac{\sqrt[3]{a}e\log\left(\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{2\sqrt[3]{c}}}{3a^{2/3}\sqrt[3]{c}}$$

$$\Big\downarrow \text{1142}$$

3.1. $\int \frac{d+ex^3}{a+cx^6}\,dx$

$$\frac{\frac{\left(\sqrt{3}\sqrt{a}e+\sqrt{c}d\right)\int \frac{1}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{2\sqrt[6]{c}}-\frac{a^{2/3}\left(\frac{\sqrt{3}\sqrt{c}d}{\sqrt{a}}-e\right)\int -\frac{\sqrt[6]{c}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{c}x\right)}{\sqrt[3]{a}\left(\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1\right)}dx}{2\sqrt[3]{c}}}{6a\sqrt[3]{c}}+\frac{\frac{\left(\sqrt{c}d-\sqrt{3}\sqrt{a}e\right)\int \frac{1}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{2\sqrt[6]{c}}+\frac{\sqrt[6]{a}\left(\sqrt{a}e+\sqrt{3}\sqrt{c}d\right)\int \frac{\sqrt[6]{c}\left(2\sqrt[6]{c}x+\sqrt{3}\sqrt[6]{a}\right)}{\sqrt[3]{a}\left(\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1\right)}dx}{2\sqrt[3]{c}}}{6a\sqrt[3]{c}}+\frac{\frac{\sqrt[6]{c}d\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{\sqrt[6]{a}}-\frac{\sqrt[3]{a}e\log\left(\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{2\sqrt[3]{c}}}{3a^{2/3}\sqrt[3]{c}}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{a^{2/3}\left(\frac{\sqrt{3}\sqrt{c}d}{\sqrt{a}}-e\right)\int \frac{\sqrt[6]{c}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{c}x\right)}{\sqrt[3]{a}\left(\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1\right)}dx}{2\sqrt[3]{c}}+\frac{\left(\sqrt{3}\sqrt{a}e+\sqrt{c}d\right)\int \frac{1}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{2\sqrt[6]{c}}}{6a\sqrt[3]{c}}+\frac{\frac{\left(\sqrt{c}d-\sqrt{3}\sqrt{a}e\right)\int \frac{1}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{2\sqrt[6]{c}}+\frac{\sqrt[6]{a}\left(\sqrt{a}e+\sqrt{3}\sqrt{c}d\right)\int \frac{\sqrt[6]{c}\left(2\sqrt[6]{c}x+\sqrt{3}\sqrt[6]{a}\right)}{\sqrt[3]{a}\left(\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1\right)}dx}{2\sqrt[3]{c}}}{6a\sqrt[3]{c}}+\frac{\frac{\sqrt[6]{c}d\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{\sqrt[6]{a}}-\frac{\sqrt[3]{a}e\log\left(\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{2\sqrt[3]{c}}}{3a^{2/3}\sqrt[3]{c}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\left(\sqrt{3}\sqrt{a}e+\sqrt{c}d\right)\int \frac{1}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{2\sqrt[6]{c}}+\frac{\sqrt[3]{a}\left(\frac{\sqrt{3}\sqrt{c}d}{\sqrt{a}}-e\right)\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{c}x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{2\sqrt[6]{c}}}{6a\sqrt[3]{c}}+\frac{\frac{\left(\sqrt{c}d-\sqrt{3}\sqrt{a}e\right)\int \frac{1}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{2\sqrt[6]{c}}+\frac{\left(\sqrt{a}e+\sqrt{3}\sqrt{c}d\right)\int \frac{2\sqrt[6]{c}x+\sqrt{3}\sqrt[6]{a}}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{2\sqrt[6]{a}\sqrt[6]{c}}}{6a\sqrt[3]{c}}+\frac{\frac{\sqrt[6]{c}d\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{\sqrt[6]{a}}-\frac{\sqrt[3]{a}e\log\left(\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{2\sqrt[3]{c}}}{3a^{2/3}\sqrt[3]{c}}$$

3.1. $\int \frac{d+ex^3}{a+cx^6}\,dx$

$$\Big\downarrow \; 1082$$

$$\frac{\frac{\sqrt[3]{a}\left(\frac{\sqrt{3}\sqrt{c}d}{\sqrt{a}}-e\right)\int\frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{c}x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{2\sqrt[6]{c}}+\frac{\sqrt[6]{a}\left(\sqrt{3}\sqrt{a}e+\sqrt{c}d\right)\int\frac{1}{-\left(1-\frac{2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)^2-\frac{1}{3}}d\left(1-\frac{2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)}{\sqrt{3}\sqrt[3]{c}}}{6a\sqrt[3]{c}}+\frac{\frac{\left(\sqrt{a}e+\sqrt{3}\sqrt{c}d\right)\int\frac{2\sqrt[6]{c}x+\sqrt{3}\sqrt[6]{a}}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{2\sqrt[6]{a}\sqrt[6]{c}}-\frac{\sqrt[6]{a}\left(\sqrt{c}d-\sqrt{3}\sqrt{a}e\right)\int\frac{1}{-\left(\frac{2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}+1\right)^2-\frac{1}{3}}d\left(\frac{2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}+1\right)}{\sqrt{3}\sqrt[3]{c}}}{6a\sqrt[3]{c}}+\frac{\frac{\sqrt[6]{c}d\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{\sqrt[6]{a}}-\frac{\sqrt[3]{a}e\log\left(\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{2\sqrt[3]{c}}}{3a^{2/3}\sqrt[3]{c}}$$

$$\Big\downarrow \; 217$$

$$\frac{\frac{\sqrt[3]{a}\left(\frac{\sqrt{3}\sqrt{c}d}{\sqrt{a}}-e\right)\int\frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{c}x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{2\sqrt[6]{c}}-\frac{\sqrt[6]{a}\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)\right)\left(\sqrt{3}\sqrt{a}e+\sqrt{c}d\right)}{\sqrt[3]{c}}}{6a\sqrt[3]{c}}+\frac{\frac{\left(\sqrt{a}e+\sqrt{3}\sqrt{c}d\right)\int\frac{2\sqrt[6]{c}x+\sqrt{3}\sqrt[6]{a}}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{2\sqrt[6]{a}\sqrt[6]{c}}+\frac{\sqrt[6]{a}\arctan\left(\sqrt{3}\left(\frac{2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}+1\right)\right)\left(\sqrt{c}d-\sqrt{3}\sqrt{a}e\right)}{\sqrt[3]{c}}}{6a\sqrt[3]{c}}+\frac{\frac{\sqrt[6]{c}d\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{\sqrt[6]{a}}-\frac{\sqrt[3]{a}e\log\left(\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{2\sqrt[3]{c}}}{3a^{2/3}\sqrt[3]{c}}$$

$$\Big\downarrow \; 1103$$

$$\frac{\frac{\sqrt[6]{c}d\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{\sqrt[6]{a}}-\frac{\sqrt[3]{a}e\log\left(\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{2\sqrt[3]{c}}}{3a^{2/3}\sqrt[3]{c}}+\frac{-\frac{a^{2/3}\left(\frac{\sqrt{3}\sqrt{c}d}{\sqrt{a}}-e\right)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{2\sqrt[3]{c}}-\frac{\sqrt[6]{a}\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)\right)\left(\sqrt{3}\sqrt{a}e+\sqrt{c}d\right)}{\sqrt[3]{c}}}{6a\sqrt[3]{c}}+\frac{\frac{\sqrt[6]{a}\arctan\left(\sqrt{3}\left(\frac{2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}+1\right)\right)\left(\sqrt{c}d-\sqrt{3}\sqrt{a}e\right)}{\sqrt[3]{c}}+\frac{\sqrt[6]{a}\left(\sqrt{a}e+\sqrt{3}\sqrt{c}d\right)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{2\sqrt[3]{c}}}{6a\sqrt[3]{c}}$$

3.1. $\int \frac{d+ex^3}{a+cx^6}\,dx$

input
```
Int[(d + e*x^3)/(a + c*x^6),x]
```

output
```
((c^(1/6)*d*ArcTan[(c^(1/6)*x)/a^(1/6)])/a^(1/6) - (a^(1/3)*e*Log[a^(1/3)
+ c^(1/3)*x^2])/(2*c^(1/3)))/(3*a^(2/3)*c^(1/3)) + (-((a^(1/6)*(Sqrt[c]*d
+ Sqrt[3]*Sqrt[a]*e)*ArcTan[Sqrt[3]*(1 - (2*c^(1/6)*x)/(Sqrt[3]*a^(1/6)))]
)/c^(1/3)) - (a^(2/3)*((Sqrt[3]*Sqrt[c]*d)/Sqrt[a] - e)*Log[a^(1/3) - Sqrt
[3]*a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(2*c^(1/3)))/(6*a*c^(1/3)) + ((a^(1/
6)*(Sqrt[c]*d - Sqrt[3]*Sqrt[a]*e)*ArcTan[Sqrt[3]*(1 + (2*c^(1/6)*x)/(Sqrt
[3]*a^(1/6)))])/c^(1/3) + (a^(1/6)*(Sqrt[3]*Sqrt[c]*d + Sqrt[a]*e)*Log[a^(
1/3) + Sqrt[3]*a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(2*c^(1/3)))/(6*a*c^(1/3)
)
```

3.1.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1746
```
Int[((d_) + (e_.)*(x_)^3)/((a_) + (c_.)*(x_)^6), x_Symbol] :> With[{q = Rt[
c/a, 6]}, Simp[1/(3*a*q^2)   Int[(q^2*d - e*x)/(1 + q^2*x^2), x], x] + (Sim
p[1/(6*a*q^2)   Int[(2*q^2*d - (Sqrt[3]*q^3*d - e)*x)/(1 - Sqrt[3]*q*x + q^
2*x^2), x], x] + Simp[1/(6*a*q^2)   Int[(2*q^2*d + (Sqrt[3]*q^3*d + e)*x)/(
1 + Sqrt[3]*q*x + q^2*x^2), x], x])] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2
 + a*e^2, 0] && PosQ[c/a]
```

3.1.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.67 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.11

method	result
risch	$\frac{\sum_{\_R=\operatorname{RootOf}\left(\_Z^{6}c+a\right)}\frac{\left(\_R^{3}e+d\right)\ln\left(x-\_R\right)}{\_R^{5}}}{6c}$
default	$\frac{\ln\left(x^{2}-\sqrt{3}\,\left(\frac{a}{c}\right)^{\frac{1}{6}}x+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)\left(\frac{a}{c}\right)^{\frac{2}{3}}e}{12a}-\frac{\ln\left(x^{2}-\sqrt{3}\,\left(\frac{a}{c}\right)^{\frac{1}{6}}x+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)\sqrt{3}\,\left(\frac{a}{c}\right)^{\frac{1}{6}}d}{12a}+\frac{\left(\frac{a}{c}\right)^{\frac{2}{3}}\arctan\left(\frac{2x}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}-\sqrt{3}\right)\sqrt{3}\,e}{6a}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{6}}\operatorname{arcta}}{}$

input
```
int((e*x^3+d)/(c*x^6+a),x,method=_RETURNVERBOSE)
```

output
```
1/6/c*sum((_R^3*e+d)/_R^5*ln(x-_R),_R=RootOf(_Z^6*c+a))
```

3.1.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1631 vs. 2(207) = 414.

Time = 0.38 (sec) , antiderivative size = 1631, normalized size of antiderivative = 5.35

$$\int \frac{d+ex^3}{a+cx^6}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^3+d)/(c*x^6+a),x, algorithm="fricas")
```

output
```
-1/12*(sqrt(-3) + 1)*((a^2*c^2*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*
e^4)/(a^5*c^3)) + 3*c*d^2*e - a*e^3)/(a^2*c^2))^(1/3)*log(-(c^2*d^5 - 2*a*
c*d^3*e^2 - 3*a^2*d*e^4)*x + 1/2*(a*c^2*d^4 - 3*a^2*c*d^2*e^2 + sqrt(-3)*(
a*c^2*d^4 - 3*a^2*c*d^2*e^2) + (sqrt(-3)*a^4*c^2*e + a^4*c^2*e)*sqrt(-(c^2
*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)))*((a^2*c^2*sqrt(-(c^2*d^6
 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) + 3*c*d^2*e - a*e^3)/(a^2*c^2
))^(1/3)) + 1/12*(sqrt(-3) - 1)*((a^2*c^2*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 +
 9*a^2*d^2*e^4)/(a^5*c^3)) + 3*c*d^2*e - a*e^3)/(a^2*c^2))^(1/3)*log(-(c^2
*d^5 - 2*a*c*d^3*e^2 - 3*a^2*d*e^4)*x + 1/2*(a*c^2*d^4 - 3*a^2*c*d^2*e^2 -
 sqrt(-3)*(a*c^2*d^4 - 3*a^2*c*d^2*e^2) - (sqrt(-3)*a^4*c^2*e - a^4*c^2*e)
*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)))*((a^2*c^2*sqr
t(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) + 3*c*d^2*e - a*e^
3)/(a^2*c^2))^(1/3)) - 1/12*(sqrt(-3) + 1)*(-(a^2*c^2*sqrt(-(c^2*d^6 - 6*a
*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) - 3*c*d^2*e + a*e^3)/(a^2*c^2))^(1/
3)*log(-(c^2*d^5 - 2*a*c*d^3*e^2 - 3*a^2*d*e^4)*x + 1/2*(a*c^2*d^4 - 3*a^2
*c*d^2*e^2 + sqrt(-3)*(a*c^2*d^4 - 3*a^2*c*d^2*e^2) - (sqrt(-3)*a^4*c^2*e
+ a^4*c^2*e)*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)))*(
-(a^2*c^2*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) - 3*c
*d^2*e + a*e^3)/(a^2*c^2))^(1/3)) + 1/12*(sqrt(-3) - 1)*(-(a^2*c^2*sqrt(-(
c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) - 3*c*d^2*e + a*e^3...
```

3.1.6 Sympy [A] (verification not implemented)

Time = 9.41 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.54

$$\int \frac{d+ex^3}{a+cx^6}\,dx$$
$$= \text{RootSum}\Big(46656t^6a^5c^4 + t^3\cdot\left(432a^4c^2e^3 - 1296a^3c^3d^2e\right) + a^3e^6 + 3a^2cd^2e^4 + 3ac^2d^4e^2 + c^3d^6, \Big(t \mapsto t\,\text{lo}$$

input
```
integrate((e*x**3+d)/(c*x**6+a),x)
```

output
```
RootSum(46656*_t**6*a**5*c**4 + _t**3*(432*a**4*c**2*e**3 - 1296*a**3*c**3
*d**2*e) + a**3*e**6 + 3*a**2*c*d**2*e**4 + 3*a*c**2*d**4*e**2 + c**3*d**6
, Lambda(_t, _t*log(x + (-1296*_t**4*a**4*c**2*e - 6*_t*a**3*e**4 + 36*_t*
a**2*c*d**2*e**2 - 6*_t*a*c**2*d**4)/(3*a**2*d*e**4 + 2*a*c*d**3*e**2 - c*
*2*d**5))))
```

3.1.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 282, normalized size of antiderivative = 0.92

$$\int \frac{d+ex^3}{a+cx^6}\,dx = -\frac{e\log\left(c^{\frac{1}{3}}x^2+a^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}c^{\frac{2}{3}}} + \frac{d\arctan\left(\frac{c^{\frac{1}{3}}x}{\sqrt{a^{\frac{1}{3}}c^{\frac{1}{3}}}}\right)}{3\,a^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}c^{\frac{1}{3}}}} + \frac{\left(\sqrt{3}a^{\frac{1}{6}}\sqrt{c}d+a^{\frac{2}{3}}e\right)\log\left(c^{\frac{1}{3}}x^2+\sqrt{3}a^{\frac{1}{6}}c^{\frac{1}{6}}x+a^{\frac{1}{3}}\right)}{12\,ac^{\frac{2}{3}}} - \frac{\left(\sqrt{3}a^{\frac{1}{6}}\sqrt{c}d-a^{\frac{2}{3}}e\right)\log\left(c^{\frac{1}{3}}x^2-\sqrt{3}a^{\frac{1}{6}}c^{\frac{1}{6}}x+a^{\frac{1}{3}}\right)}{12\,ac^{\frac{2}{3}}} - \frac{\left(\sqrt{3}a^{\frac{5}{6}}c^{\frac{1}{6}}e-a^{\frac{1}{3}}c^{\frac{2}{3}}d\right)\arctan\left(\frac{2\,c^{\frac{1}{3}}x+\sqrt{3}a^{\frac{1}{6}}c^{\frac{1}{6}}}{\sqrt{a^{\frac{1}{3}}c^{\frac{1}{3}}}}\right)}{6\,ac^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}c^{\frac{1}{3}}}} + \frac{\left(\sqrt{3}a^{\frac{5}{6}}c^{\frac{1}{6}}e+a^{\frac{1}{3}}c^{\frac{2}{3}}d\right)\arctan\left(\frac{2\,c^{\frac{1}{3}}x-\sqrt{3}a^{\frac{1}{6}}c^{\frac{1}{6}}}{\sqrt{a^{\frac{1}{3}}c^{\frac{1}{3}}}}\right)}{6\,ac^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}c^{\frac{1}{3}}}}$$

input
```
integrate((e*x^3+d)/(c*x^6+a),x, algorithm="maxima")
```

output
```
-1/6*e*log(c^(1/3)*x^2 + a^(1/3))/(a^(1/3)*c^(2/3)) + 1/3*d*arctan(c^(1/3)
*x/sqrt(a^(1/3)*c^(1/3)))/(a^(2/3)*sqrt(a^(1/3)*c^(1/3))) + 1/12*(sqrt(3)*
a^(1/6)*sqrt(c)*d + a^(2/3)*e)*log(c^(1/3)*x^2 + sqrt(3)*a^(1/6)*c^(1/6)*x
 + a^(1/3))/(a*c^(2/3)) - 1/12*(sqrt(3)*a^(1/6)*sqrt(c)*d - a^(2/3)*e)*log
(c^(1/3)*x^2 - sqrt(3)*a^(1/6)*c^(1/6)*x + a^(1/3))/(a*c^(2/3)) - 1/6*(sqr
t(3)*a^(5/6)*c^(1/6)*e - a^(1/3)*c^(2/3)*d)*arctan((2*c^(1/3)*x + sqrt(3)*
a^(1/6)*c^(1/6))/sqrt(a^(1/3)*c^(1/3)))/(a*c^(2/3)*sqrt(a^(1/3)*c^(1/3)))
+ 1/6*(sqrt(3)*a^(5/6)*c^(1/6)*e + a^(1/3)*c^(2/3)*d)*arctan((2*c^(1/3)*x
- sqrt(3)*a^(1/6)*c^(1/6))/sqrt(a^(1/3)*c^(1/3)))/(a*c^(2/3)*sqrt(a^(1/3)*
c^(1/3)))
```

3.1.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.93

$$\begin{aligned}\int \frac{d+ex^3}{a+cx^6}\,dx = &-\frac{e|c|\log\left(x^2+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{6\,(ac^5)^{\frac{1}{3}}}+\frac{(ac^5)^{\frac{1}{6}}\,d\arctan\left(\frac{x}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{3\,ac}\\ &+\frac{\left((ac^5)^{\frac{1}{6}}\,c^3d-\sqrt{3}(ac^5)^{\frac{2}{3}}\,e\right)\arctan\left(\frac{2\,x+\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6\,ac^4}\\ &+\frac{\left((ac^5)^{\frac{1}{6}}\,c^3d+\sqrt{3}(ac^5)^{\frac{2}{3}}\,e\right)\arctan\left(\frac{2\,x-\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6\,ac^4}\\ &+\frac{\left(\sqrt{3}(ac^5)^{\frac{1}{6}}\,c^3d+(ac^5)^{\frac{2}{3}}\,e\right)\log\left(x^2+\sqrt{3}x\left(\frac{a}{c}\right)^{\frac{1}{6}}+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12\,ac^4}\\ &-\frac{\left(\sqrt{3}(ac^5)^{\frac{1}{6}}\,c^3d-(ac^5)^{\frac{2}{3}}\,e\right)\log\left(x^2-\sqrt{3}x\left(\frac{a}{c}\right)^{\frac{1}{6}}+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12\,ac^4}\end{aligned}$$

input
```
integrate((e*x^3+d)/(c*x^6+a),x, algorithm="giac")
```

output
```
-1/6*e*abs(c)*log(x^2 + (a/c)^(1/3))/(a*c^5)^(1/3) + 1/3*(a*c^5)^(1/6)*d*a
rctan(x/(a/c)^(1/6))/(a*c) + 1/6*((a*c^5)^(1/6)*c^3*d - sqrt(3)*(a*c^5)^(2
/3)*e)*arctan((2*x + sqrt(3)*(a/c)^(1/6))/(a/c)^(1/6))/(a*c^4) + 1/6*((a*c
^5)^(1/6)*c^3*d + sqrt(3)*(a*c^5)^(2/3)*e)*arctan((2*x - sqrt(3)*(a/c)^(1/
6))/(a/c)^(1/6))/(a*c^4) + 1/12*(sqrt(3)*(a*c^5)^(1/6)*c^3*d + (a*c^5)^(2/
3)*e)*log(x^2 + sqrt(3)*x*(a/c)^(1/6) + (a/c)^(1/3))/(a*c^4) - 1/12*(sqrt(
3)*(a*c^5)^(1/6)*c^3*d - (a*c^5)^(2/3)*e)*log(x^2 - sqrt(3)*x*(a/c)^(1/6)
+ (a/c)^(1/3))/(a*c^4)
```

3.1.9 Mupad [B] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 1331, normalized size of antiderivative = 4.36

$$\int \frac{d+ex^3}{a+cx^6}\,dx = \ln\left(a^3\,c^3\,\left(-\frac{a^4\,c^2\,e^3+c\,d^3\,\sqrt{-a^5\,c^5}-3\,a^3\,c^3\,d^2\,e-3\,a\,d\,e^2\,\sqrt{-a^5\,c^5}}{a^5\,c^4}\right)^{1/3}\right.$$
$$\left.+e\,x\,\sqrt{-a^5\,c^5}+a^2\,c^3\,d\,x\right)\left(-\frac{a^4\,c^2\,e^3+c\,d^3\,\sqrt{-a^5\,c^5}-3\,a^3\,c^3\,d^2\,e-3\,a\,d\,e^2\,\sqrt{-a^5\,c^5}}{216\,a^5\,c^4}\right)^{1/3}$$
$$+\ln\left(a^3\,c^3\,\left(-\frac{a^4\,c^2\,e^3-c\,d^3\,\sqrt{-a^5\,c^5}-3\,a^3\,c^3\,d^2\,e+3\,a\,d\,e^2\,\sqrt{-a^5\,c^5}}{a^5\,c^4}\right)^{1/3}-e\,x\,\sqrt{-a^5\,c^5}+a^2\,c^3\,d\,x\right)\left(-\right.$$

input
```
int((d + e*x^3)/(a + c*x^6),x)
```

output
```
log(a^3*c^3*(-(a^4*c^2*e^3 + c*d^3*(-a^5*c^5)^(1/2) - 3*a^3*c^3*d^2*e - 3*
a*d*e^2*(-a^5*c^5)^(1/2))/(a^5*c^4))^(1/3) + e*x*(-a^5*c^5)^(1/2) + a^2*c^
3*d*x)*(-(a^4*c^2*e^3 + c*d^3*(-a^5*c^5)^(1/2) - 3*a^3*c^3*d^2*e - 3*a*d*e
^2*(-a^5*c^5)^(1/2))/(216*a^5*c^4))^(1/3) + log(a^3*c^3*(-(a^4*c^2*e^3 - c
*d^3*(-a^5*c^5)^(1/2) - 3*a^3*c^3*d^2*e + 3*a*d*e^2*(-a^5*c^5)^(1/2))/(a^5
*c^4))^(1/3) - e*x*(-a^5*c^5)^(1/2) + a^2*c^3*d*x)*(-(a^4*c^2*e^3 - c*d^3*
(-a^5*c^5)^(1/2) - 3*a^3*c^3*d^2*e + 3*a*d*e^2*(-a^5*c^5)^(1/2))/(216*a^5*
c^4))^(1/3) - log(a^3*c^3*(-(a^4*c^2*e^3 + c*d^3*(-a^5*c^5)^(1/2) - 3*a^3*
c^3*d^2*e - 3*a*d*e^2*(-a^5*c^5)^(1/2))/(a^5*c^4))^(1/3) - 2*e*x*(-a^5*c^5
)^(1/2) + 3^(1/2)*a^3*c^3*(-(a^4*c^2*e^3 + c*d^3*(-a^5*c^5)^(1/2) - 3*a^3*
c^3*d^2*e - 3*a*d*e^2*(-a^5*c^5)^(1/2))/(a^5*c^4))^(1/3)*1i - 2*a^2*c^3*d*
x)*((3^(1/2)*1i)/2 + 1/2)*(-(a^4*c^2*e^3 + c*d^3*(-a^5*c^5)^(1/2) - 3*a^3*
c^3*d^2*e - 3*a*d*e^2*(-a^5*c^5)^(1/2))/(216*a^5*c^4))^(1/3) + log(e*x*(-a
^5*c^5)^(1/2) - (a^3*c^3*(-(a^4*c^2*e^3 + c*d^3*(-a^5*c^5)^(1/2) - 3*a^3*c
^3*d^2*e - 3*a*d*e^2*(-a^5*c^5)^(1/2))/(a^5*c^4))^(1/3))/2 + (3^(1/2)*a^3*
c^3*(-(a^4*c^2*e^3 + c*d^3*(-a^5*c^5)^(1/2) - 3*a^3*c^3*d^2*e - 3*a*d*e^2*
(-a^5*c^5)^(1/2))/(a^5*c^4))^(1/3)*1i)/2 + a^2*c^3*d*x)*((3^(1/2)*1i)/2 -
1/2)*(-(a^4*c^2*e^3 + c*d^3*(-a^5*c^5)^(1/2) - 3*a^3*c^3*d^2*e - 3*a*d*e^2
*(-a^5*c^5)^(1/2))/(216*a^5*c^4))^(1/3) + log(a^3*c^3*(-(a^4*c^2*e^3 - c*d
^3*(-a^5*c^5)^(1/2) - 3*a^3*c^3*d^2*e + 3*a*d*e^2*(-a^5*c^5)^(1/2))/(a^...
```

3.2 $\int \frac{d+ex^3}{a-cx^6}\,dx$

3.2.1 Optimal result

Integrand size = 18, antiderivative size = 323

$$\int \frac{d+ex^3}{a-cx^6}\,dx = -\frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\arctan\left(\frac{\sqrt[6]{a}-2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)}{2\sqrt{3}a^{5/6}\sqrt[6]{c}} + \frac{(\sqrt{c}d+\sqrt{a}e)\arctan\left(\frac{\sqrt[6]{a}+2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)}{2\sqrt{3}a^{5/6}c^{2/3}} - \frac{(\sqrt{c}d+\sqrt{a}e)\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{6a^{5/6}c^{2/3}} + \frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\log\left(\sqrt[6]{a}+\sqrt[6]{c}x\right)}{6a^{5/6}\sqrt[6]{c}} - \frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\log\left(\sqrt[3]{a}-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{12a^{5/6}\sqrt[6]{c}} + \frac{(\sqrt{c}d+\sqrt{a}e)\log\left(\sqrt[3]{a}+\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{12a^{5/6}c^{2/3}}$$

output
```
1/6*ln(a^(1/6)+c^(1/6)*x)*(d-e*a^(1/2)/c^(1/2))/a^(5/6)/c^(1/6)-1/12*ln(a^
(1/3)-a^(1/6)*c^(1/6)*x+c^(1/3)*x^2)*(d-e*a^(1/2)/c^(1/2))/a^(5/6)/c^(1/6)
-1/6*arctan(1/3*(a^(1/6)-2*c^(1/6)*x)/a^(1/6)*3^(1/2))*(d-e*a^(1/2)/c^(1/2
))/a^(5/6)/c^(1/6)*3^(1/2)-1/6*ln(a^(1/6)-c^(1/6)*x)*(e*a^(1/2)+d*c^(1/2))
/a^(5/6)/c^(2/3)+1/12*ln(a^(1/3)+a^(1/6)*c^(1/6)*x+c^(1/3)*x^2)*(e*a^(1/2)
+d*c^(1/2))/a^(5/6)/c^(2/3)+1/6*arctan(1/3*(a^(1/6)+2*c^(1/6)*x)/a^(1/6)*3
^(1/2))*(e*a^(1/2)+d*c^(1/2))/a^(5/6)/c^(2/3)*3^(1/2)
```

3.2.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.04

$$\int \frac{d+ex^3}{a-cx^6}\,dx$$
$$= \frac{-2\sqrt{3}\left(\sqrt{c}d-\sqrt{a}e\right)\arctan\left(\frac{1-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}}{\sqrt{3}}\right)+2\sqrt{3}\left(\sqrt{c}d+\sqrt{a}e\right)\arctan\left(\frac{1+\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}}{\sqrt{3}}\right)-2\sqrt{c}d\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{}$$

input
```
Integrate[(d + e*x^3)/(a - c*x^6),x]
```

output
```
(-2*Sqrt[3]*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[(1 - (2*c^(1/6)*x)/a^(1/6))/Sqr
t[3]] + 2*Sqrt[3]*(Sqrt[c]*d + Sqrt[a]*e)*ArcTan[(1 + (2*c^(1/6)*x)/a^(1/6
))/Sqrt[3]] - 2*Sqrt[c]*d*Log[a^(1/6) - c^(1/6)*x] - 2*Sqrt[a]*e*Log[a^(1/
6) - c^(1/6)*x] + 2*Sqrt[c]*d*Log[a^(1/6) + c^(1/6)*x] - 2*Sqrt[a]*e*Log[a
^(1/6) + c^(1/6)*x] - Sqrt[c]*d*Log[a^(1/3) - a^(1/6)*c^(1/6)*x + c^(1/3)*
x^2] + Sqrt[a]*e*Log[a^(1/3) - a^(1/6)*c^(1/6)*x + c^(1/3)*x^2] + Sqrt[c]*
d*Log[a^(1/3) + a^(1/6)*c^(1/6)*x + c^(1/3)*x^2] + Sqrt[a]*e*Log[a^(1/3) +
 a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(12*a^(5/6)*c^(2/3))
```

3.2.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.86, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {1747, 750, 16, 27, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^3}{a-cx^6}\,dx$$
$$\Big\downarrow \text{1747}$$
$$\frac{1}{2}\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\int \frac{1}{a-\sqrt{a}\sqrt{c}x^3}dx+\frac{1}{2}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{1}{\sqrt{a}\sqrt{c}x^3+a}dx$$
$$\Big\downarrow \text{750}$$

$$\frac{1}{2}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\left(\frac{\int \frac{\sqrt[6]{a}\left(2\sqrt[6]{a}-\sqrt[6]{c}x\right)}{\sqrt[3]{a}\sqrt[3]{c}x^2-\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)+\frac{1}{2}\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\left(\frac{\int \frac{\sqrt[6]{a}\left(\sqrt[6]{c}x+2\sqrt[6]{a}\right)}{\sqrt[3]{a}\sqrt[3]{c}x^2+\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{a}-\sqrt[6]{a}\sqrt[6]{c}x}dx}{3a^{2/3}}\right)$$

$$\Big\downarrow \; 16$$

$$\frac{1}{2}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\left(\frac{\int \frac{\sqrt[6]{a}\left(2\sqrt[6]{a}-\sqrt[6]{c}x\right)}{\sqrt[3]{a}\sqrt[3]{c}x^2-\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[6]{a}+\sqrt[6]{c}x\right)}{3a^{5/6}\sqrt[6]{c}}\right)+\frac{1}{2}\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\left(\frac{\int \frac{\sqrt[6]{a}\left(\sqrt[6]{c}x+2\sqrt[6]{a}\right)}{\sqrt[3]{a}\sqrt[3]{c}x^2+\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx}{3a^{2/3}}-\frac{\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{3a^{5/6}\sqrt[6]{c}}\right)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{2}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\left(\frac{\int \frac{2\sqrt[6]{a}-\sqrt[6]{c}x}{\sqrt[3]{a}\sqrt[3]{c}x^2-\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx}{3\sqrt{a}}+\frac{\log\left(\sqrt[6]{a}+\sqrt[6]{c}x\right)}{3a^{5/6}\sqrt[6]{c}}\right)+\frac{1}{2}\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\left(\frac{\int \frac{\sqrt[6]{c}x+2\sqrt[6]{a}}{\sqrt[3]{a}\sqrt[3]{c}x^2+\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx}{3\sqrt{a}}-\frac{\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{3a^{5/6}\sqrt[6]{c}}\right)$$

$$\Big\downarrow \; 1142$$

$$\frac{1}{2}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\left(\frac{\frac{3}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{a}\sqrt[3]{c}x^2-\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx-\frac{\int -\frac{\sqrt[3]{a}\sqrt[6]{c}\left(\sqrt[6]{a}-2\sqrt[6]{c}x\right)}{\sqrt[3]{a}\sqrt[3]{c}x^2-\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx}{2\sqrt[3]{a}\sqrt[6]{c}}}{3\sqrt{a}}+\frac{\log\left(\sqrt[6]{a}+\sqrt[6]{c}x\right)}{3a^{5/6}\sqrt[6]{c}}\right)+\frac{1}{2}\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\left(\frac{\frac{3}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{a}\sqrt[3]{c}x^2+\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx+\frac{\int \frac{\sqrt[3]{a}\sqrt[6]{c}\left(2\sqrt[6]{c}x+\sqrt[6]{a}\right)}{\sqrt[3]{a}\sqrt[3]{c}x^2+\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx}{2\sqrt[3]{a}\sqrt[6]{c}}}{3\sqrt{a}}-\frac{\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{3a^{5/6}\sqrt[6]{c}}\right)$$

3.2. $\int \frac{d+ex^3}{a-cx^6}\,dx$

$$\Big\downarrow \textcolor{blue}{25}$$

$$\frac{1}{2}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\left(\frac{\frac{3}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{a}\sqrt[3]{c}x^2-\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx+\frac{\int \frac{\sqrt[6]{c}\left(\sqrt[6]{a}-2\sqrt[6]{c}x\right)}{\sqrt[3]{c}x^2-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{2\sqrt[3]{a}\sqrt[6]{c}}}{3\sqrt{a}}+\frac{\log\left(\sqrt[6]{a}+\sqrt[6]{c}x\right)}{3a^{5/6}\sqrt[6]{c}}\right)+\frac{1}{2}\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\left(\frac{\frac{3}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{a}\sqrt[3]{c}x^2+\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx+\frac{\int \frac{\sqrt[3]{a}\sqrt[6]{c}\left(2\sqrt[6]{c}x+\sqrt[6]{a}\right)}{\sqrt[3]{a}\sqrt[3]{c}x^2+\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx}{2\sqrt[3]{a}\sqrt[6]{c}}}{3\sqrt{a}}-\frac{\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{3a^{5/6}\sqrt[6]{c}}\right)$$

$$\Big\downarrow \textcolor{blue}{27}$$

$$\frac{1}{2}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\left(\frac{\frac{3}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{a}\sqrt[3]{c}x^2-\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx+\frac{\int \frac{\sqrt[6]{a}-2\sqrt[6]{c}x}{\sqrt[3]{c}x^2-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{2\sqrt[3]{a}}}{3\sqrt{a}}+\frac{\log\left(\sqrt[6]{a}+\sqrt[6]{c}x\right)}{3a^{5/6}\sqrt[6]{c}}\right)+\frac{1}{2}\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\left(\frac{\frac{3}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{a}\sqrt[3]{c}x^2+\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx+\frac{1}{2}\int \frac{2\sqrt[6]{c}x+\sqrt[6]{a}}{\sqrt[3]{a}\sqrt[3]{c}x^2+\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx}{3\sqrt{a}}-\frac{\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{3a^{5/6}\sqrt[6]{c}}\right)$$

$$\Big\downarrow \textcolor{blue}{1082}$$

$$\frac{1}{2}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\left(\frac{\frac{\int \frac{\sqrt[6]{a}-2\sqrt[6]{c}x}{\sqrt[3]{c}x^2-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{2\sqrt[3]{a}}+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{\sqrt[3]{a}\sqrt[6]{c}}}{3\sqrt{a}}+\frac{\log\left(\sqrt[6]{a}+\sqrt[6]{c}x\right)}{3a^{5/6}\sqrt[6]{c}}\right)+\frac{1}{2}\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\left(\frac{\frac{1}{2}\int \frac{2\sqrt[6]{c}x+\sqrt[6]{a}}{\sqrt[3]{a}\sqrt[3]{c}x^2+\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx-\frac{3\int \frac{1}{-\left(\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}+1\right)^2-3}d\left(\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}+1\right)}{\sqrt[3]{a}\sqrt[6]{c}}}{3\sqrt{a}}-\frac{\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{3a^{5/6}\sqrt[6]{c}}\right)$$

$$\Big\downarrow \textcolor{blue}{217}$$

3.2. $\int \frac{d+ex^3}{a-cx^6}\,dx$

$$\frac{1}{2}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\left(\frac{\frac{\int \frac{\sqrt[6]{a}-2\sqrt[6]{c}x}{\sqrt[3]{c}x^2-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{2\sqrt[3]{a}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}\sqrt[6]{c}}}{3\sqrt{a}}+\frac{\log\left(\sqrt[6]{a}+\sqrt[6]{c}x\right)}{3a^{5/6}\sqrt[6]{c}}\right)+$$

$$\frac{1}{2}\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\left(\frac{\frac{1}{2}\int \frac{2\sqrt[6]{c}x+\sqrt[6]{a}}{\sqrt[3]{a}\sqrt[3]{c}x^2+\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx+\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{a}\sqrt[6]{c}}}{3\sqrt{a}}-\frac{\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{3a^{5/6}\sqrt[6]{c}}\right)$$

$$\Big\downarrow \; 1103$$

$$\frac{1}{2}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\left(\frac{\log\left(\sqrt[6]{a}+\sqrt[6]{c}x\right)}{3a^{5/6}\sqrt[6]{c}}+\frac{-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}\sqrt[6]{c}}-\frac{\log\left(-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{2\sqrt[3]{a}\sqrt[6]{c}}}{3\sqrt{a}}\right)+$$

$$\frac{1}{2}\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\left(\frac{\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{a}\sqrt[6]{c}}+\frac{\log\left(\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{2\sqrt[3]{a}\sqrt[6]{c}}}{3\sqrt{a}}-\frac{\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{3a^{5/6}\sqrt[6]{c}}\right)$$

input
```
Int[(d + e*x^3)/(a - c*x^6),x]
```

3.2. $\int \frac{d+ex^3}{a-cx^6}\,dx$

output
```
((d - (Sqrt[a]*e)/Sqrt[c])*(Log[a^(1/6) + c^(1/6)*x]/(3*a^(5/6)*c^(1/6)) +
 (-((Sqrt[3]*ArcTan[(1 - (2*c^(1/6)*x)/a^(1/6))/Sqrt[3]])/(a^(1/3)*c^(1/6)
)) - Log[a^(1/3) - a^(1/6)*c^(1/6)*x + c^(1/3)*x^2]/(2*a^(1/3)*c^(1/6)))/(
3*Sqrt[a])))/2 + ((d + (Sqrt[a]*e)/Sqrt[c])*(-1/3*Log[a^(1/6) - c^(1/6)*x]
/(a^(5/6)*c^(1/6)) + ((Sqrt[3]*ArcTan[(1 + (2*c^(1/6)*x)/a^(1/6))/Sqrt[3]]
)/(a^(1/3)*c^(1/6)) + Log[a^(1/3) + a^(1/6)*c^(1/6)*x + c^(1/3)*x^2]/(2*a^
(1/3)*c^(1/6)))/(3*Sqrt[a])))/2
```

3.2.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1747
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> With[{
q = Rt[-a/c, 2]}, Simp[(d + e*q)/2   Int[1/(a + c*q*x^n), x], x] + Simp[(d
- e*q)/2   Int[1/(a - c*q*x^n), x], x]] /; FreeQ[{a, c, d, e, n}, x] && EqQ
[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && NegQ[a*c] && IntegerQ[n]
```

3.2.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.56 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.11

method	result
risch	$-\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^6 c-a\right)} \frac{\left(\_R^3 e+d\right)\ln\left(x-\_R\right)}{\_R^5}}{6c}$
default	$-\frac{\ln\left(-x+\left(\frac{a}{c}\right)^{\frac{1}{6}}\right)e}{6c\left(\frac{a}{c}\right)^{\frac{1}{3}}}-\frac{\ln\left(-x+\left(\frac{a}{c}\right)^{\frac{1}{6}}\right)d}{6c\left(\frac{a}{c}\right)^{\frac{5}{6}}}+\frac{e\left(\frac{a}{c}\right)^{\frac{2}{3}}\ln\left(x^2+\left(\frac{a}{c}\right)^{\frac{1}{6}}x+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12a}+\frac{e\left(\frac{a}{c}\right)^{\frac{2}{3}}\sqrt{3}\arctan\left(\frac{2x\sqrt{3}}{3\left(\frac{a}{c}\right)^{\frac{1}{6}}}+\frac{\sqrt{3}}{3}\right)}{6a}+\frac{d\left(\frac{a}{c}\right)^{\frac{1}{6}}\ln(\cdots}{}$

input
```
int((e*x^3+d)/(-c*x^6+a),x,method=_RETURNVERBOSE)
```

output
```
-1/6/c*sum((_R^3*e+d)/_R^5*ln(x-_R),_R=RootOf(_Z^6*c-a))
```

3.2.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1613 vs. 2(223) = 446.

Time = 0.37 (sec) , antiderivative size = 1613, normalized size of antiderivative = 4.99

$$\int \frac{d+ex^3}{a-cx^6}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^3+d)/(-c*x^6+a),x, algorithm="fricas")
```

output
```
-1/12*(sqrt(-3) + 1)*(-(a^2*c^2*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*
e^4)/(a^5*c^3)) + 3*c*d^2*e + a*e^3)/(a^2*c^2))^(1/3)*log(-(c^2*d^5 + 2*a*
c*d^3*e^2 - 3*a^2*d*e^4)*x + 1/2*(a*c^2*d^4 + 3*a^2*c*d^2*e^2 + sqrt(-3)*(
a*c^2*d^4 + 3*a^2*c*d^2*e^2) - (sqrt(-3)*a^4*c^2*e + a^4*c^2*e)*sqrt((c^2*
d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)))*(-(a^2*c^2*sqrt((c^2*d^6
+ 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) + 3*c*d^2*e + a*e^3)/(a^2*c^2)
)^(1/3)) + 1/12*(sqrt(-3) - 1)*(-(a^2*c^2*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 +
9*a^2*d^2*e^4)/(a^5*c^3)) + 3*c*d^2*e + a*e^3)/(a^2*c^2))^(1/3)*log(-(c^2*
d^5 + 2*a*c*d^3*e^2 - 3*a^2*d*e^4)*x + 1/2*(a*c^2*d^4 + 3*a^2*c*d^2*e^2 -
sqrt(-3)*(a*c^2*d^4 + 3*a^2*c*d^2*e^2) + (sqrt(-3)*a^4*c^2*e - a^4*c^2*e)*
sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)))*(-(a^2*c^2*sqrt
((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) + 3*c*d^2*e + a*e^3)
/(a^2*c^2))^(1/3)) - 1/12*(sqrt(-3) + 1)*((a^2*c^2*sqrt((c^2*d^6 + 6*a*c*d
^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) - 3*c*d^2*e - a*e^3)/(a^2*c^2))^(1/3)*l
og(-(c^2*d^5 + 2*a*c*d^3*e^2 - 3*a^2*d*e^4)*x + 1/2*(a*c^2*d^4 + 3*a^2*c*d
^2*e^2 + sqrt(-3)*(a*c^2*d^4 + 3*a^2*c*d^2*e^2) + (sqrt(-3)*a^4*c^2*e + a^
4*c^2*e)*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)))*((a^2*
c^2*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) - 3*c*d^2*e
- a*e^3)/(a^2*c^2))^(1/3)) + 1/12*(sqrt(-3) - 1)*((a^2*c^2*sqrt((c^2*d^6 +
 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) - 3*c*d^2*e - a*e^3)/(a^2*c^...
```

3.2.6 Sympy [A] (verification not implemented)

Time = 9.50 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.52

$$\int \frac{d+ex^3}{a-cx^6}\,dx =$$

$$-\operatorname{RootSum}\left(46656t^6a^5c^4+t^3\left(-432a^4c^2e^3-1296a^3c^3d^2e\right)+a^3e^6-3a^2cd^2e^4+3ac^2d^4e^2-c^3d^6,\left(t\mapsto t\right.\right.$$

input
```
integrate((e*x**3+d)/(-c*x**6+a),x)
```

output
```
-RootSum(46656*_t**6*a**5*c**4 + _t**3*(-432*a**4*c**2*e**3 - 1296*a**3*c*
*3*d**2*e) + a**3*e**6 - 3*a**2*c*d**2*e**4 + 3*a*c**2*d**4*e**2 - c**3*d*
*6, Lambda(_t, _t*log(x + (-1296*_t**4*a**4*c**2*e + 6*_t*a**3*e**4 + 36*_
t*a**2*c*d**2*e**2 + 6*_t*a*c**2*d**4)/(3*a**2*d*e**4 - 2*a*c*d**3*e**2 -
c**2*d**5))))
```

3.2.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 313, normalized size of antiderivative = 0.97

$$\int \frac{d+ex^3}{a-cx^6}\,dx = \frac{\sqrt{3}(\sqrt{c}d+\sqrt{a}e)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}\right)}{3\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}}\right)}{6\sqrt{ac}\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{2}{3}}} + \frac{\sqrt{3}(\sqrt{c}d-\sqrt{a}e)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}\right)}{3\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}}\right)}{6\sqrt{ac}\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{2}{3}}} + \frac{(\sqrt{c}d+\sqrt{a}e)\log\left(x^2+x\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}+\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{2}{3}}\right)}{12\sqrt{ac}\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{2}{3}}} - \frac{(\sqrt{c}d-\sqrt{a}e)\log\left(x^2-x\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}+\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{2}{3}}\right)}{12\sqrt{ac}\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{2}{3}}} + \frac{(\sqrt{c}d-\sqrt{a}e)\log\left(x+\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}\right)}{6\sqrt{ac}\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{2}{3}}} - \frac{(\sqrt{c}d+\sqrt{a}e)\log\left(x-\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}\right)}{6\sqrt{ac}\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{2}{3}}}$$

input
```
integrate((e*x^3+d)/(-c*x^6+a),x, algorithm="maxima")
```

output
```
1/6*sqrt(3)*(sqrt(c)*d + sqrt(a)*e)*arctan(1/3*sqrt(3)*(2*x + (sqrt(a)/sqr
t(c))^(1/3))/(sqrt(a)/sqrt(c))^(1/3))/(sqrt(a)*c*(sqrt(a)/sqrt(c))^(2/3))
+ 1/6*sqrt(3)*(sqrt(c)*d - sqrt(a)*e)*arctan(1/3*sqrt(3)*(2*x - (sqrt(a)/s
qrt(c))^(1/3))/(sqrt(a)/sqrt(c))^(1/3))/(sqrt(a)*c*(sqrt(a)/sqrt(c))^(2/3)
) + 1/12*(sqrt(c)*d + sqrt(a)*e)*log(x^2 + x*(sqrt(a)/sqrt(c))^(1/3) + (sq
rt(a)/sqrt(c))^(2/3))/(sqrt(a)*c*(sqrt(a)/sqrt(c))^(2/3)) - 1/12*(sqrt(c)*
d - sqrt(a)*e)*log(x^2 - x*(sqrt(a)/sqrt(c))^(1/3) + (sqrt(a)/sqrt(c))^(2/
3))/(sqrt(a)*c*(sqrt(a)/sqrt(c))^(2/3)) + 1/6*(sqrt(c)*d - sqrt(a)*e)*log(
x + (sqrt(a)/sqrt(c))^(1/3))/(sqrt(a)*c*(sqrt(a)/sqrt(c))^(2/3)) - 1/6*(sq
rt(c)*d + sqrt(a)*e)*log(x - (sqrt(a)/sqrt(c))^(1/3))/(sqrt(a)*c*(sqrt(a)/
sqrt(c))^(2/3))
```

3.2.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.94

$$\int \frac{d+ex^3}{a-cx^6}\,dx = \frac{e|c|\log\left(x^2+\left(-\frac{a}{c}\right)^{\frac{1}{3}}\right)}{6\left(-ac^5\right)^{\frac{1}{3}}} + \frac{\left(-ac^5\right)^{\frac{1}{6}} d \arctan\left(\frac{x}{\left(-\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{3\,ac} + \frac{\left(\left(-ac^5\right)^{\frac{1}{6}} c^3 d - \sqrt{3}\left(-ac^5\right)^{\frac{2}{3}} e\right)\arctan\left(\frac{2\,x+\sqrt{3}\left(-\frac{a}{c}\right)^{\frac{1}{6}}}{\left(-\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6\,ac^4} + \frac{\left(\left(-ac^5\right)^{\frac{1}{6}} c^3 d + \sqrt{3}\left(-ac^5\right)^{\frac{2}{3}} e\right)\arctan\left(\frac{2\,x-\sqrt{3}\left(-\frac{a}{c}\right)^{\frac{1}{6}}}{\left(-\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6\,ac^4} + \frac{\left(\sqrt{3}\left(-ac^5\right)^{\frac{1}{6}} c^3 d + \left(-ac^5\right)^{\frac{2}{3}} e\right)\log\left(x^2+\sqrt{3}x\left(-\frac{a}{c}\right)^{\frac{1}{6}}+\left(-\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12\,ac^4} - \frac{\left(\sqrt{3}\left(-ac^5\right)^{\frac{1}{6}} c^3 d - \left(-ac^5\right)^{\frac{2}{3}} e\right)\log\left(x^2-\sqrt{3}x\left(-\frac{a}{c}\right)^{\frac{1}{6}}+\left(-\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12\,ac^4}$$

input
```
integrate((e*x^3+d)/(-c*x^6+a),x, algorithm="giac")
```

output
```
1/6*e*abs(c)*log(x^2 + (-a/c)^(1/3))/(-a*c^5)^(1/3) + 1/3*(-a*c^5)^(1/6)*d
*arctan(x/(-a/c)^(1/6))/(a*c) + 1/6*((-a*c^5)^(1/6)*c^3*d - sqrt(3)*(-a*c^
5)^(2/3)*e)*arctan((2*x + sqrt(3)*(-a/c)^(1/6))/(-a/c)^(1/6))/(a*c^4) + 1/
6*((-a*c^5)^(1/6)*c^3*d + sqrt(3)*(-a*c^5)^(2/3)*e)*arctan((2*x - sqrt(3)*
(-a/c)^(1/6))/(-a/c)^(1/6))/(a*c^4) + 1/12*(sqrt(3)*(-a*c^5)^(1/6)*c^3*d +
 (-a*c^5)^(2/3)*e)*log(x^2 + sqrt(3)*x*(-a/c)^(1/6) + (-a/c)^(1/3))/(a*c^4
) - 1/12*(sqrt(3)*(-a*c^5)^(1/6)*c^3*d - (-a*c^5)^(2/3)*e)*log(x^2 - sqrt(
3)*x*(-a/c)^(1/6) + (-a/c)^(1/3))/(a*c^4)
```

3.2.9 Mupad [B] (verification not implemented)

Time = 9.28 (sec) , antiderivative size = 1293, normalized size of antiderivative = 4.00

$$\int \frac{d+ex^3}{a-cx^6}\,dx = \ln\left(a^3\,c^3\,\left(-\frac{a^4\,c^2\,e^3+c\,d^3\,\sqrt{a^5\,c^5}+3\,a^3\,c^3\,d^2\,e+3\,a\,d\,e^2\,\sqrt{a^5\,c^5}}{a^5\,c^4}\right)^{1/3}+e\,x\,\sqrt{a^5\,c^5}+a^2\,c^3\,d\,x\right)\left(-\frac{a^4\,c^2\,e^3+c\,d^3\,\sqrt{a^5\,c^5}+3\,a^3\,c^3\,d^2\,e+3\,a\,d\,e^2\,\sqrt{a^5\,c^5}}{216\,a^5\,c^4}\right)^{1/3}+\ln\left(a^3\,c^3\,\left(-\frac{a^4\,c^2\,e^3-c\,d^3\,\sqrt{a^5\,c^5}+3\,a^3\,c^3\,d^2\,e-3\,a\,d\,e^2\,\sqrt{a^5\,c^5}}{a^5\,c^4}\right)^{1/3}-e\,x\,\sqrt{a^5\,c^5}+a^2\,c^3\,d\,x\right)\left(-\frac{a^4\,c^2}{}\right.$$

input
```
int((d + e*x^3)/(a - c*x^6),x)
```

output
```
log(a^3*c^3*(-(a^4*c^2*e^3 + c*d^3*(a^5*c^5)^(1/2) + 3*a^3*c^3*d^2*e + 3*a
*d*e^2*(a^5*c^5)^(1/2))/(a^5*c^4))^(1/3) + e*x*(a^5*c^5)^(1/2) + a^2*c^3*d
*x)*(-(a^4*c^2*e^3 + c*d^3*(a^5*c^5)^(1/2) + 3*a^3*c^3*d^2*e + 3*a*d*e^2*(
a^5*c^5)^(1/2))/(216*a^5*c^4))^(1/3) + log(a^3*c^3*(-(a^4*c^2*e^3 - c*d^3*
(a^5*c^5)^(1/2) + 3*a^3*c^3*d^2*e - 3*a*d*e^2*(a^5*c^5)^(1/2))/(a^5*c^4))^
(1/3) - e*x*(a^5*c^5)^(1/2) + a^2*c^3*d*x)*(-(a^4*c^2*e^3 - c*d^3*(a^5*c^5
)^(1/2) + 3*a^3*c^3*d^2*e - 3*a*d*e^2*(a^5*c^5)^(1/2))/(216*a^5*c^4))^(1/3
) - log(a^3*c^3*(-(a^4*c^2*e^3 + c*d^3*(a^5*c^5)^(1/2) + 3*a^3*c^3*d^2*e +
 3*a*d*e^2*(a^5*c^5)^(1/2))/(a^5*c^4))^(1/3) - 2*e*x*(a^5*c^5)^(1/2) + 3^(
1/2)*a^3*c^3*(-(a^4*c^2*e^3 + c*d^3*(a^5*c^5)^(1/2) + 3*a^3*c^3*d^2*e + 3*
a*d*e^2*(a^5*c^5)^(1/2))/(a^5*c^4))^(1/3)*1i - 2*a^2*c^3*d*x)*((3^(1/2)*1i
)/2 + 1/2)*(-(a^4*c^2*e^3 + c*d^3*(a^5*c^5)^(1/2) + 3*a^3*c^3*d^2*e + 3*a*
d*e^2*(a^5*c^5)^(1/2))/(216*a^5*c^4))^(1/3) + log(e*x*(a^5*c^5)^(1/2) - (a
^3*c^3*(-(a^4*c^2*e^3 + c*d^3*(a^5*c^5)^(1/2) + 3*a^3*c^3*d^2*e + 3*a*d*e^
2*(a^5*c^5)^(1/2))/(a^5*c^4))^(1/3))/2 + (3^(1/2)*a^3*c^3*(-(a^4*c^2*e^3 +
 c*d^3*(a^5*c^5)^(1/2) + 3*a^3*c^3*d^2*e + 3*a*d*e^2*(a^5*c^5)^(1/2))/(a^5
*c^4))^(1/3)*1i)/2 + a^2*c^3*d*x)*((3^(1/2)*1i)/2 - 1/2)*(-(a^4*c^2*e^3 +
c*d^3*(a^5*c^5)^(1/2) + 3*a^3*c^3*d^2*e + 3*a*d*e^2*(a^5*c^5)^(1/2))/(216*
a^5*c^4))^(1/3) + log(a^3*c^3*(-(a^4*c^2*e^3 - c*d^3*(a^5*c^5)^(1/2) + 3*a
^3*c^3*d^2*e - 3*a*d*e^2*(a^5*c^5)^(1/2))/(a^5*c^4))^(1/3) + 2*e*x*(a^5...
```

3.3 $\int \frac{d+ex^4}{a+cx^8}\,dx$

3.3.1 Optimal result

Integrand size = 17, antiderivative size = 754

$$
\begin{aligned}
\int \frac{d+ex^4}{a+cx^8}\,dx = & -\frac{\sqrt{2-\sqrt{2}}\left(\left(1+\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)\arctan\left(\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}-2\sqrt[8]{c}x}{\sqrt{2+\sqrt{2}}\sqrt[8]{a}}\right)}{8a^{7/8}c^{5/8}} \\
& +\frac{\sqrt{2+\sqrt{2}}\left(\left(1-\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)\arctan\left(\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}-2\sqrt[8]{c}x}{\sqrt{2-\sqrt{2}}\sqrt[8]{a}}\right)}{8a^{7/8}c^{5/8}} \\
& +\frac{\sqrt{2-\sqrt{2}}\left(\left(1+\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)\arctan\left(\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}+2\sqrt[8]{c}x}{\sqrt{2+\sqrt{2}}\sqrt[8]{a}}\right)}{8a^{7/8}c^{5/8}} \\
& -\frac{\sqrt{2+\sqrt{2}}\left(\left(1-\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)\arctan\left(\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}+2\sqrt[8]{c}x}{\sqrt{2-\sqrt{2}}\sqrt[8]{a}}\right)}{8a^{7/8}c^{5/8}} \\
& +\frac{\left(\left(1-\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt[4]{a}-\sqrt{2-\sqrt{2}}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{c}x^2\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}a^{7/8}c^{5/8}} \\
& -\frac{\left(\left(1-\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt[4]{a}+\sqrt{2-\sqrt{2}}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{c}x^2\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}a^{7/8}c^{5/8}} \\
& -\frac{\left(\left(1+\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt[4]{a}-\sqrt{2+\sqrt{2}}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{c}x^2\right)}{8\sqrt{2\left(2+\sqrt{2}\right)}a^{7/8}c^{5/8}} \\
& +\frac{\left(d+\sqrt{2}d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\log\left(\sqrt[4]{a}+\sqrt{2+\sqrt{2}}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{c}x^2\right)}{8\sqrt{2\left(2+\sqrt{2}\right)}a^{7/8}\sqrt[8]{c}}
\end{aligned}
$$

output
```
-1/8*arctan((-2*c^(1/8)*x+a^(1/8)*(2-2^(1/2))^(1/2))/a^(1/8)/(2+2^(1/2))^(
1/2))*(-e*a^(1/2)+d*(1+2^(1/2))*c^(1/2))*(2-2^(1/2))^(1/2)/a^(7/8)/c^(5/8)
+1/8*arctan((2*c^(1/8)*x+a^(1/8)*(2-2^(1/2))^(1/2))/a^(1/8)/(2+2^(1/2))^(1
/2))*(-e*a^(1/2)+d*(1+2^(1/2))*c^(1/2))*(2-2^(1/2))^(1/2)/a^(7/8)/c^(5/8)+
1/4*arctan((-2*c^(1/8)*x+a^(1/8)*(2+2^(1/2))^(1/2))/a^(1/8)/(2-2^(1/2))^(1
/2))*(-e*a^(1/2)+d*(1-2^(1/2))*c^(1/2))/a^(7/8)/c^(5/8)/(4-2*2^(1/2))^(1/2
)-1/4*arctan((2*c^(1/8)*x+a^(1/8)*(2+2^(1/2))^(1/2))/a^(1/8)/(2-2^(1/2))^(
1/2))*(-e*a^(1/2)+d*(1-2^(1/2))*c^(1/2))/a^(7/8)/c^(5/8)/(4-2*2^(1/2))^(1/
2)+1/8*ln(a^(1/4)+c^(1/4)*x^2-a^(1/8)*c^(1/8)*x*(2-2^(1/2))^(1/2))*(-e*a^(
1/2)+d*(1-2^(1/2))*c^(1/2))/a^(7/8)/c^(5/8)/(4-2*2^(1/2))^(1/2)-1/8*ln(a^(
1/4)+c^(1/4)*x^2+a^(1/8)*c^(1/8)*x*(2-2^(1/2))^(1/2))*(-e*a^(1/2)+d*(1-2^(
1/2))*c^(1/2))/a^(7/8)/c^(5/8)/(4-2*2^(1/2))^(1/2)+1/8*ln(a^(1/4)+c^(1/4)*
x^2+a^(1/8)*c^(1/8)*x*(2+2^(1/2))^(1/2))*(d+d*2^(1/2)-e*a^(1/2)/c^(1/2))/a
^(7/8)/c^(1/8)/(4+2*2^(1/2))^(1/2)-1/8*ln(a^(1/4)+c^(1/4)*x^2-a^(1/8)*c^(1
/8)*x*(2+2^(1/2))^(1/2))*(-e*a^(1/2)+d*(1+2^(1/2))*c^(1/2))/a^(7/8)/c^(5/8
)/(4+2*2^(1/2))^(1/2)
```

3.3.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 534, normalized size of antiderivative = 0.71

$$\int \frac{d+ex^4}{a+cx^8}\,dx$$
$$= \frac{-2\sqrt[8]{a}\arctan\left(\cot\left(\frac{\pi}{8}\right)-\frac{\sqrt[8]{c}x\csc\left(\frac{\pi}{8}\right)}{\sqrt[8]{a}}\right)\left(\sqrt{a}e\cos\left(\frac{\pi}{8}\right)+\sqrt{c}d\sin\left(\frac{\pi}{8}\right)\right)+2\sqrt[8]{a}\arctan\left(\cot\left(\frac{\pi}{8}\right)+\frac{\sqrt[8]{c}x\csc\left(\frac{\pi}{8}\right)}{\sqrt[8]{a}}\right)}{}$$

input
```
Integrate[(d + e*x^4)/(a + c*x^8),x]
```

output
```
(-2*a^(1/8)*ArcTan[Cot[Pi/8] - (c^(1/8)*x*Csc[Pi/8])/a^(1/8)]*(Sqrt[a]*e*C
os[Pi/8] + Sqrt[c]*d*Sin[Pi/8]) + 2*a^(1/8)*ArcTan[Cot[Pi/8] + (c^(1/8)*x*
Csc[Pi/8])/a^(1/8)]*(Sqrt[a]*e*Cos[Pi/8] + Sqrt[c]*d*Sin[Pi/8]) - a^(1/8)*
Log[a^(1/4) + c^(1/4)*x^2 - 2*a^(1/8)*c^(1/8)*x*Sin[Pi/8]]*(Sqrt[a]*e*Cos[
Pi/8] + Sqrt[c]*d*Sin[Pi/8]) + a^(1/8)*Log[a^(1/4) + c^(1/4)*x^2 + 2*a^(1/
8)*c^(1/8)*x*Sin[Pi/8]]*(Sqrt[a]*e*Cos[Pi/8] + Sqrt[c]*d*Sin[Pi/8]) + a^(1
/8)*Log[a^(1/4) + c^(1/4)*x^2 - 2*a^(1/8)*c^(1/8)*x*Cos[Pi/8]]*(-(Sqrt[c]*
d*Cos[Pi/8]) + Sqrt[a]*e*Sin[Pi/8]) - a^(1/8)*Log[a^(1/4) + c^(1/4)*x^2 +
2*a^(1/8)*c^(1/8)*x*Cos[Pi/8]]*(-(Sqrt[c]*d*Cos[Pi/8]) + Sqrt[a]*e*Sin[Pi/
8]) + 2*ArcTan[(c^(1/8)*x*Sec[Pi/8])/a^(1/8) - Tan[Pi/8]]*(a^(1/8)*Sqrt[c]
*d*Cos[Pi/8] - a^(5/8)*e*Sin[Pi/8]) + 2*ArcTan[(c^(1/8)*x*Sec[Pi/8])/a^(1/
8) + Tan[Pi/8]]*(a^(1/8)*Sqrt[c]*d*Cos[Pi/8] - a^(5/8)*e*Sin[Pi/8]))/(8*a*
c^(5/8))
```

3.3.3 Rubi [A] (verified)

Time = 1.48 (sec) , antiderivative size = 849, normalized size of antiderivative = 1.13, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.647$, Rules used = {1745, 27, 1483, 27, 27, 1142, 25, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^4}{a+cx^8} \, dx$$

$$\Big\downarrow \ 1745$$

$$\frac{\int \frac{\sqrt{2}\sqrt[4]{a}d-\sqrt[4]{c}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)x^2}{\sqrt[4]{c}\left(x^4-\frac{\sqrt{2}\sqrt[4]{a}x^2}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}a^{3/4}\sqrt[4]{c}}+\frac{\int \frac{\sqrt[4]{c}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)x^2+\sqrt{2}\sqrt[4]{a}d}{\sqrt[4]{c}\left(x^4+\frac{\sqrt{2}\sqrt[4]{a}x^2}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}a^{3/4}\sqrt[4]{c}}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{\sqrt{2}\sqrt[4]{a}d-\sqrt[4]{c}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)x^2}{x^4-\frac{\sqrt{2}\sqrt[4]{a}x^2}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{2}a^{3/4}\sqrt{c}}+\frac{\int \frac{\sqrt[4]{c}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)x^2+\sqrt{2}\sqrt[4]{a}d}{x^4+\frac{\sqrt{2}\sqrt[4]{a}x^2}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{2}a^{3/4}\sqrt{c}}$$

$$\Big\downarrow \ 1483$$

$$\frac{\frac{c^{3/8}\int \frac{\sqrt[4]{a}\left(\sqrt{2\left(2-\sqrt{2}\right)}\sqrt[8]{a}d+\sqrt[8]{c}\left(-\sqrt{2}d+d-\frac{\sqrt{a}e}{\sqrt{c}}\right)x\right)}{\sqrt[8]{c}\left(x^2-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}}+\frac{\sqrt[4]{a}}{\sqrt[4]{c}}\right)}dx}{2\sqrt{2-\sqrt{2}}a^{3/8}}+\frac{c^{3/8}\int \frac{\sqrt[4]{a}\left(\sqrt{2\left(2-\sqrt{2}\right)}\sqrt[8]{a}d-\sqrt[8]{c}\left(\left(1-\sqrt{2}\right)d-\frac{\sqrt{a}e}{\sqrt{c}}\right)x\right)}{\sqrt[8]{c}\left(x^2+\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}}+\frac{\sqrt[4]{a}}{\sqrt[4]{c}}\right)}dx}{2\sqrt{2-\sqrt{2}}a^{3/8}}}{2\sqrt{2}a^{3/4}\sqrt{c}}+\frac{\frac{c^{3/8}\int \frac{\sqrt[4]{a}\left(\sqrt{2\left(2+\sqrt{2}\right)}\sqrt[8]{a}d-\sqrt[8]{c}\left(\sqrt{2}d+d-\frac{\sqrt{a}e}{\sqrt{c}}\right)x\right)}{\sqrt[8]{c}\left(x^2-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}}+\frac{\sqrt[4]{a}}{\sqrt[4]{c}}\right)}dx}{2\sqrt{2+\sqrt{2}}a^{3/8}}+\frac{c^{3/8}\int \frac{\sqrt[4]{a}\left(\sqrt{2\left(2+\sqrt{2}\right)}\sqrt[8]{a}d+\sqrt[8]{c}\left(\sqrt{2}d+d-\frac{\sqrt{a}e}{\sqrt{c}}\right)x\right)}{\sqrt[8]{c}\left(x^2+\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}}+\frac{\sqrt[4]{a}}{\sqrt[4]{c}}\right)}dx}{2\sqrt{2+\sqrt{2}}a^{3/8}}}{2\sqrt{2}a^{3/4}\sqrt{c}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\sqrt[4]{c}\int \frac{\sqrt{2\left(2-\sqrt{2}\right)}\sqrt[8]{a}c^{3/8}d-\left(\left(1-\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)x}{c^{3/8}\left(x^2+\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}}+\frac{\sqrt[4]{a}}{\sqrt[4]{c}}\right)}dx}{2\sqrt{2-\sqrt{2}}\sqrt[8]{a}}+\frac{\sqrt[4]{c}\int \frac{\sqrt{2\left(2-\sqrt{2}\right)}\sqrt[8]{a}d+\sqrt[8]{c}\left(-\sqrt{2}d+d-\frac{\sqrt{a}e}{\sqrt{c}}\right)x}{x^2-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}}+\frac{\sqrt[4]{a}}{\sqrt[4]{c}}}dx}{2\sqrt{2-\sqrt{2}}\sqrt[8]{a}}}{2\sqrt{2}a^{3/4}\sqrt{c}}+\frac{\frac{\sqrt[4]{c}\int \frac{\sqrt{2\left(2+\sqrt{2}\right)}\sqrt[8]{a}d-\sqrt[8]{c}\left(\sqrt{2}d+d-\frac{\sqrt{a}e}{\sqrt{c}}\right)x}{x^2-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}}+\frac{\sqrt[4]{a}}{\sqrt[4]{c}}}dx}{2\sqrt{2+\sqrt{2}}\sqrt[8]{a}}+\frac{\sqrt[4]{c}\int \frac{\sqrt{2\left(2+\sqrt{2}\right)}\sqrt[8]{a}d+\sqrt[8]{c}\left(\sqrt{2}d+d-\frac{\sqrt{a}e}{\sqrt{c}}\right)x}{x^2+\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}}+\frac{\sqrt[4]{a}}{\sqrt[4]{c}}}dx}{2\sqrt{2+\sqrt{2}}\sqrt[8]{a}}}{2\sqrt{2}a^{3/4}\sqrt{c}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{\sqrt{2\left(2-\sqrt{2}\right)}\sqrt[8]{a}c^{3/8}d-\left(\left(1-\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)x}{x^2+\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}}+\frac{\sqrt[4]{a}}{\sqrt[4]{c}}}dx}{2\sqrt{2-\sqrt{2}}\sqrt[8]{a}\sqrt[8]{c}}+\frac{\sqrt[4]{c}\int \frac{\sqrt{2\left(2-\sqrt{2}\right)}\sqrt[8]{a}d+\sqrt[8]{c}\left(-\sqrt{2}d+d-\frac{\sqrt{a}e}{\sqrt{c}}\right)x}{x^2-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}}+\frac{\sqrt[4]{a}}{\sqrt[4]{c}}}dx}{2\sqrt{2-\sqrt{2}}\sqrt[8]{a}}}{2\sqrt{2}a^{3/4}\sqrt{c}}+\frac{\frac{\sqrt[4]{c}\int \frac{\sqrt{2\left(2+\sqrt{2}\right)}\sqrt[8]{a}d-\sqrt[8]{c}\left(\sqrt{2}d+d-\frac{\sqrt{a}e}{\sqrt{c}}\right)x}{x^2-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}}+\frac{\sqrt[4]{a}}{\sqrt[4]{c}}}dx}{2\sqrt{2+\sqrt{2}}\sqrt[8]{a}}+\frac{\sqrt[4]{c}\int \frac{\sqrt{2\left(2+\sqrt{2}\right)}\sqrt[8]{a}d+\sqrt[8]{c}\left(\sqrt{2}d+d-\frac{\sqrt{a}e}{\sqrt{c}}\right)x}{x^2+\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}}+\frac{\sqrt[4]{a}}{\sqrt[4]{c}}}dx}{2\sqrt{2+\sqrt{2}}\sqrt[8]{a}}}{2\sqrt{2}a^{3/4}\sqrt{c}}$$

$$\Big\downarrow \; 1142$$

3.3. $\int \frac{d+ex^4}{a+cx^8}\,dx$

$$\frac{\frac{\sqrt[4]{c}\left(\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}\left(\left(1+\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)\int \frac{1}{x^2-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}}+\frac{\sqrt[4]{a}}{\sqrt[4]{c}}}dx}{2\sqrt{c}}+\frac{1}{2}\sqrt[8]{c}\left(-\sqrt{2}d+d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\int -\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}-2\sqrt[8]{c}x}{\sqrt[8]{c}\left(x^2-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}}+\frac{\sqrt[4]{a}}{\sqrt[4]{c}}\right)}dx\right)}{2\sqrt{2-\sqrt{2}}\sqrt[8]{a}}+\cdots}{2\sqrt{2}a^{3/4}\sqrt{c}}$$

$$\frac{\frac{\sqrt[4]{c}\left(-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}\left(\left(1-\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)\int \frac{1}{x^2-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}}+\frac{\sqrt[4]{a}}{\sqrt[4]{c}}}dx}{2\sqrt{c}}-\frac{1}{2}\sqrt[8]{c}\left(\sqrt{2}d+d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\int -\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}-2\sqrt[8]{c}x}{\sqrt[8]{c}\left(x^2-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}}+\frac{\sqrt[4]{a}}{\sqrt[4]{c}}\right)}dx\right)}{2\sqrt{2+\sqrt{2}}\sqrt[8]{a}}+\cdots}{2\sqrt{2}a^{3/4}\sqrt{c}}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\sqrt[4]{c}\left(\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}\left(\left(1+\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)\int \frac{1}{x^2-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}}+\frac{\sqrt[4]{a}}{\sqrt[4]{c}}}dx}{2\sqrt{c}}-\frac{1}{2}\sqrt[8]{c}\left(-\sqrt{2}d+d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}-2\sqrt[8]{c}x}{\sqrt[8]{c}\left(x^2-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}}+\frac{\sqrt[4]{a}}{\sqrt[4]{c}}\right)}dx\right)}{2\sqrt{2-\sqrt{2}}\sqrt[8]{a}}+\cdots}{2\sqrt{2}a^{3/4}\sqrt{c}}$$

$$\frac{\frac{\sqrt[4]{c}\left(\frac{1}{2}\sqrt[8]{c}\left(\sqrt{2}d+d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}-2\sqrt[8]{c}x}{\sqrt[8]{c}\left(x^2-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}}+\frac{\sqrt[4]{a}}{\sqrt[4]{c}}\right)}dx-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}\left(\left(1-\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)\int \frac{1}{x^2-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}}+\frac{\sqrt[4]{a}}{\sqrt[4]{c}}}dx}{2\sqrt{c}}\right)}{2\sqrt{2+\sqrt{2}}\sqrt[8]{a}}+\frac{\sqrt[4]{c}\left(\frac{1}{2}\sqrt[8]{c}\cdots\right)}{\cdots}}{2\sqrt{2}a^{3/4}\sqrt{c}}$$

$$\Big\downarrow \ 27$$

3.3. $\int \frac{d+ex^4}{a+cx^8}\,dx$

$$\frac{\frac{\sqrt[4]{c}\left(\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}\left(\left(1+\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)\int \frac{1}{x^2-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}}+\frac{\sqrt[4]{a}}{\sqrt[4]{c}}}dx}{2\sqrt{c}}-\frac{1}{2}\left(-\sqrt{2}d+d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}-2\sqrt[8]{c}x}{x^2-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}}+\frac{\sqrt[4]{a}}{\sqrt[4]{c}}}dx\right)}{2\sqrt{2-\sqrt{2}}\sqrt[8]{a}}+\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}\left(\left(1+\sqrt{2}\right)\cdots\right.}{\cdots}}{2\sqrt{2}a^{3/4}\sqrt{c}}$$

$$\frac{\frac{\sqrt[4]{c}\left(\frac{1}{2}\left(\sqrt{2}d+d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}-2\sqrt[8]{c}x}{x^2-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}}+\frac{\sqrt[4]{a}}{\sqrt[4]{c}}}dx-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}\left(\left(1-\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)\int \frac{1}{x^2-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}}+\frac{\sqrt[4]{a}}{\sqrt[4]{c}}}dx}{2\sqrt{c}}\right)}{2\sqrt{2+\sqrt{2}}\sqrt[8]{a}}+\frac{\sqrt[4]{c}\left(\frac{1}{2}\left(\sqrt{2}d+d-\frac{\sqrt{a}e}{\sqrt{c}}\cdots\right.\right.}{\cdots}}{2\sqrt{2}a^{3/4}\sqrt{c}}$$

↓ 1083

$$\frac{\frac{\sqrt[4]{c}\left(-\frac{1}{2}\left(-\sqrt{2}d+d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}-2\sqrt[8]{c}x}{x^2-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}}+\frac{\sqrt[4]{a}}{\sqrt[4]{c}}}dx-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}\left(\left(1+\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)\int \frac{1}{-\left(2x-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}}{\sqrt[8]{c}}\right)^2-\frac{\left(2+\sqrt{2}\right)\sqrt[4]{a}}{\sqrt[4]{c}}}d\left(2x-\frac{\sqrt{2-\sqrt{2}}\cdots}{\sqrt[8]{c}}\cdots\right.}{\sqrt{c}}\cdots\right.}{2\sqrt{2-\sqrt{2}}\sqrt[8]{a}}\cdots}{2\cdots}$$

$$\frac{\frac{\sqrt[4]{c}\left(\frac{1}{2}\left(\sqrt{2}d+d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}-2\sqrt[8]{c}x}{x^2-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}}+\frac{\sqrt[4]{a}}{\sqrt[4]{c}}}dx+\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}\left(\left(1-\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)\int \frac{1}{-\left(2x-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}}{\sqrt[8]{c}}\right)^2-\frac{\left(2-\sqrt{2}\right)\sqrt[4]{a}}{\sqrt[4]{c}}}d\left(2x-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}}{\sqrt[8]{c}}\cdots\right.}{\sqrt{c}}\cdots\right.}{2\sqrt{2+\sqrt{2}}\sqrt[8]{a}}\cdots}{2\cdots}$$

↓ 217

3.3. $\int \frac{d+ex^4}{a+cx^8}\,dx$

$$\frac{\frac{\sqrt[4]{c}\left(\frac{\sqrt{\frac{2-\sqrt{2}}{2+\sqrt{2}}}\left(\left(1+\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)\arctan\left(\frac{\sqrt[8]{c}\left(2x-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}}{\sqrt[8]{c}}\right)}{\sqrt{2+\sqrt{2}}\sqrt[8]{a}}\right)}{c^{3/8}}-\frac{1}{2}\left(-\sqrt{2}d+d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\int\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}-2\sqrt[8]{c}x}{x^2-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}}+\frac{\sqrt[4]{a}}{\sqrt[4]{c}}}dx\right)}{2\sqrt{2-\sqrt{2}}\sqrt[8]{a}}+\frac{\sqrt{\frac{2-\sqrt{2}}{2+\sqrt{2}}}\left(\left(1+\sqrt{2}\right)\sqrt{\cdots}\right.}{\cdots}}{2\sqrt{2}a^{3/4}\sqrt{c}}$$

$$\frac{\frac{\sqrt[4]{c}\left(\frac{1}{2}\left(\sqrt{2}d+d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\int\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}-2\sqrt[8]{c}x}{x^2-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}}+\frac{\sqrt[4]{a}}{\sqrt[4]{c}}}dx-\frac{\sqrt{\frac{2+\sqrt{2}}{2-\sqrt{2}}}\left(\left(1-\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)\arctan\left(\frac{\sqrt[8]{c}\left(2x-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}}{\sqrt[8]{c}}\right)}{\sqrt{2-\sqrt{2}}\sqrt[8]{a}}\right)}{c^{3/8}}\right)}{2\sqrt{2+\sqrt{2}}\sqrt[8]{a}}+\frac{\sqrt[4]{c}\left(\frac{1}{2}\left(\sqrt{2}d+d-\frac{\sqrt{a}e}{\sqrt{c}}\cdots\right.\right.}{\cdots}}{2\sqrt{2}a^{3/4}\sqrt{c}}$$

$$\Big\downarrow \; 1103$$

$$\frac{\frac{\sqrt[4]{c}\left(\frac{\sqrt{\frac{2-\sqrt{2}}{2+\sqrt{2}}}\left(\left(1+\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)\arctan\left(\frac{\sqrt[8]{c}\left(2x-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}}{\sqrt[8]{c}}\right)}{\sqrt{2+\sqrt{2}}\sqrt[8]{a}}\right)}{c^{3/8}}+\frac{1}{2}\sqrt[8]{c}\left(-\sqrt{2}d+d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\log\left(\sqrt[4]{c}x^2-\sqrt{2-\sqrt{2}}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{a}\right)\right)}{2\sqrt{2-\sqrt{2}}\sqrt[8]{a}}+\frac{\sqrt{\frac{2-\cdots}{2+\cdots}}\cdots}{\cdots}}{2\sqrt{2}a^{3/4}\sqrt{c}}$$

$$\frac{\frac{\sqrt[4]{c}\left(-\frac{\sqrt{\frac{2+\sqrt{2}}{2-\sqrt{2}}}\left(\left(1-\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)\arctan\left(\frac{\sqrt[8]{c}\left(2x-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}}{\sqrt[8]{c}}\right)}{\sqrt{2-\sqrt{2}}\sqrt[8]{a}}\right)}{c^{3/8}}-\frac{1}{2}\sqrt[8]{c}\left(\sqrt{2}d+d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\log\left(\sqrt[4]{c}x^2-\sqrt{2+\sqrt{2}}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{a}\right)\right)}{2\sqrt{2+\sqrt{2}}\sqrt[8]{a}}+\frac{\sqrt[4]{c}\left(\cdots\right.}{\cdots}}{2\sqrt{2}a^{3/4}\sqrt{c}}$$

input
```
Int[(d + e*x^4)/(a + c*x^8),x]
```

3.3. $\int \frac{d+ex^4}{a+cx^8}\,dx$

output
```
((c^(1/4)*((Sqrt[(2 - Sqrt[2])/(2 + Sqrt[2])]*((1 + Sqrt[2])*Sqrt[c]*d - S
qrt[a]*e)*ArcTan[(c^(1/8)*(-((Sqrt[2 - Sqrt[2]]*a^(1/8))/c^(1/8)) + 2*x))/
(Sqrt[2 + Sqrt[2]]*a^(1/8))])/c^(3/8) + (c^(1/8)*(d - Sqrt[2]*d - (Sqrt[a]
*e)/Sqrt[c])*Log[a^(1/4) - Sqrt[2 - Sqrt[2]]*a^(1/8)*c^(1/8)*x + c^(1/4)*x
^2])/2))/(2*Sqrt[2 - Sqrt[2]]*a^(1/8)) + (Sqrt[(2 - Sqrt[2])/(2 + Sqrt[2])
]*((1 + Sqrt[2])*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[(c^(1/8)*((Sqrt[2 - Sqrt[2]
]*a^(1/8))/c^(1/8) + 2*x))/(Sqrt[2 + Sqrt[2]]*a^(1/8))] - (((1 - Sqrt[2])*
Sqrt[c]*d - Sqrt[a]*e)*Log[a^(1/4) + Sqrt[2 - Sqrt[2]]*a^(1/8)*c^(1/8)*x +
 c^(1/4)*x^2])/2)/(2*Sqrt[2 - Sqrt[2]]*a^(1/8)*c^(1/8)))/(2*Sqrt[2]*a^(3/4
)*Sqrt[c]) + ((c^(1/4)*(-((Sqrt[(2 + Sqrt[2])/(2 - Sqrt[2])]*((1 - Sqrt[2]
)*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[(c^(1/8)*(-((Sqrt[2 + Sqrt[2]]*a^(1/8))/c^
(1/8)) + 2*x))/(Sqrt[2 - Sqrt[2]]*a^(1/8))])/c^(3/8)) - (c^(1/8)*(d + Sqrt
[2]*d - (Sqrt[a]*e)/Sqrt[c])*Log[a^(1/4) - Sqrt[2 + Sqrt[2]]*a^(1/8)*c^(1/
8)*x + c^(1/4)*x^2])/2))/(2*Sqrt[2 + Sqrt[2]]*a^(1/8)) + (c^(1/4)*(-((Sqrt
[(2 + Sqrt[2])/(2 - Sqrt[2])]*((1 - Sqrt[2])*Sqrt[c]*d - Sqrt[a]*e)*ArcTan
[(c^(1/8)*((Sqrt[2 + Sqrt[2]]*a^(1/8))/c^(1/8) + 2*x))/(Sqrt[2 - Sqrt[2]]*
a^(1/8))])/c^(3/8)) + (c^(1/8)*(d + Sqrt[2]*d - (Sqrt[a]*e)/Sqrt[c])*Log[a
^(1/4) + Sqrt[2 + Sqrt[2]]*a^(1/8)*c^(1/8)*x + c^(1/4)*x^2])/2))/(2*Sqrt[2
 + Sqrt[2]]*a^(1/8)))/(2*Sqrt[2]*a^(3/4)*Sqrt[c])
```

3.3.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 1745
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> With[{
q = Rt[a/c, 4]}, Simp[1/(2*Sqrt[2]*c*q^3)   Int[(Sqrt[2]*d*q - (d - e*q^2)*
x^(n/2))/(q^2 - Sqrt[2]*q*x^(n/2) + x^n), x], x] + Simp[1/(2*Sqrt[2]*c*q^3)
   Int[(Sqrt[2]*d*q + (d - e*q^2)*x^(n/2))/(q^2 + Sqrt[2]*q*x^(n/2) + x^n),
 x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0]
 && NeQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && PosQ[a*c]
```

3.3.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.58 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.05

method	result	size
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+a\right)} \frac{\left(\_R^4 e+d\right) \ln\left(x-\_R\right)}{\_R^7}}{8c}$	34
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+a\right)} \frac{\left(\_R^4 e+d\right) \ln\left(x-\_R\right)}{\_R^7}}{8c}$	34

input
```
int((e*x^4+d)/(c*x^8+a),x,method=_RETURNVERBOSE)
```

output
```
1/8/c*sum((_R^4*e+d)/_R^7*ln(x-_R),_R=RootOf(_Z^8*c+a))
```

3.3.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2749 vs. 2(514) = 1028.

Time = 0.56 (sec) , antiderivative size = 2749, normalized size of antiderivative = 3.65

$$\int \frac{d+ex^4}{a+cx^8}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^4+d)/(c*x^8+a),x, algorithm="fricas")
```

output
```
1/8*sqrt(-sqrt(-(a^3*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^
4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 4*c*d^3*e - 4*a*d*e^3)/(a
^3*c^2)))*log((c^3*d^6 - 5*a*c^2*d^4*e^2 - 5*a^2*c*d^2*e^4 + a^3*e^6)*x +
(a^5*c^3*e*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3
*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + a*c^3*d^5 - 6*a^2*c^2*d^3*e^2 + a^3*c*d
*e^4)*sqrt(-sqrt(-(a^3*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*
d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 4*c*d^3*e - 4*a*d*e^3)/
(a^3*c^2)))) - 1/8*sqrt(-sqrt(-(a^3*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2
+ 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 4*c*d^3*e
- 4*a*d*e^3)/(a^3*c^2)))*log((c^3*d^6 - 5*a*c^2*d^4*e^2 - 5*a^2*c*d^2*e^4
+ a^3*e^6)*x - (a^5*c^3*e*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d
^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + a*c^3*d^5 - 6*a^2*c^2*d^
3*e^2 + a^3*c*d*e^4)*sqrt(-sqrt(-(a^3*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^
2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 4*c*d^3*
e - 4*a*d*e^3)/(a^3*c^2)))) - 1/8*sqrt(-sqrt((a^3*c^2*sqrt(-(c^4*d^8 - 12*
a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)
) - 4*c*d^3*e + 4*a*d*e^3)/(a^3*c^2)))*log((c^3*d^6 - 5*a*c^2*d^4*e^2 - 5*
a^2*c*d^2*e^4 + a^3*e^6)*x + (a^5*c^3*e*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2
+ 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - a*c^3*d^5
+ 6*a^2*c^2*d^3*e^2 - a^3*c*d*e^4)*sqrt(-sqrt((a^3*c^2*sqrt(-(c^4*d^8 -...
```

3.3.6 Sympy [F(-1)]

Timed out.

$$\int \frac{d + ex^4}{a + cx^8} \, dx = \text{Timed out}$$

input
```
integrate((e*x**4+d)/(c*x**8+a),x)
```

output
```
Timed out
```

3.3.7 Maxima [F]

$$\int \frac{d + ex^4}{a + cx^8} \, dx = \int \frac{ex^4 + d}{cx^8 + a} \, dx$$

input
```
integrate((e*x^4+d)/(c*x^8+a),x, algorithm="maxima")
```

output
```
integrate((e*x^4 + d)/(c*x^8 + a), x)
```

3.3.8 Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 593, normalized size of antiderivative = 0.79

$$\int \frac{d+ex^4}{a+cx^8}\,dx$$
$$= -\frac{\left(e\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{5}{8}} - d\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}\right)\arctan\left(\frac{2x+\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{8a}$$
$$- \frac{\left(e\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{5}{8}} - d\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}\right)\arctan\left(\frac{2x-\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{8a}$$
$$+ \frac{\left(e\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{5}{8}} + d\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}\right)\arctan\left(\frac{2x+\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{8a}$$
$$+ \frac{\left(e\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{5}{8}} + d\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}\right)\arctan\left(\frac{2x-\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{8a}$$
$$- \frac{\left(e\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{5}{8}} - d\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}\right)\log\left(x^2+x\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{16a}$$
$$+ \frac{\left(e\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{5}{8}} - d\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}\right)\log\left(x^2-x\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{16a}$$
$$+ \frac{\left(e\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{5}{8}} + d\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}\right)\log\left(x^2+x\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{16a}$$
$$- \frac{\left(e\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{5}{8}} + d\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}\right)\log\left(x^2-x\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{16a}$$

input
```
integrate((e*x^4+d)/(c*x^8+a),x, algorithm="giac")
```

output
```
-1/8*(e*sqrt(-sqrt(2) + 2)*(a/c)^(5/8) - d*sqrt(sqrt(2) + 2)*(a/c)^(1/8))*
arctan((2*x + sqrt(-sqrt(2) + 2)*(a/c)^(1/8))/(sqrt(sqrt(2) + 2)*(a/c)^(1/
8)))/a - 1/8*(e*sqrt(-sqrt(2) + 2)*(a/c)^(5/8) - d*sqrt(sqrt(2) + 2)*(a/c)
^(1/8))*arctan((2*x - sqrt(-sqrt(2) + 2)*(a/c)^(1/8))/(sqrt(sqrt(2) + 2)*(
a/c)^(1/8)))/a + 1/8*(e*sqrt(sqrt(2) + 2)*(a/c)^(5/8) + d*sqrt(-sqrt(2) +
2)*(a/c)^(1/8))*arctan((2*x + sqrt(sqrt(2) + 2)*(a/c)^(1/8))/(sqrt(-sqrt(2
) + 2)*(a/c)^(1/8)))/a + 1/8*(e*sqrt(sqrt(2) + 2)*(a/c)^(5/8) + d*sqrt(-sq
rt(2) + 2)*(a/c)^(1/8))*arctan((2*x - sqrt(sqrt(2) + 2)*(a/c)^(1/8))/(sqrt
(-sqrt(2) + 2)*(a/c)^(1/8)))/a - 1/16*(e*sqrt(-sqrt(2) + 2)*(a/c)^(5/8) -
d*sqrt(sqrt(2) + 2)*(a/c)^(1/8))*log(x^2 + x*sqrt(sqrt(2) + 2)*(a/c)^(1/8)
 + (a/c)^(1/4))/a + 1/16*(e*sqrt(-sqrt(2) + 2)*(a/c)^(5/8) - d*sqrt(sqrt(2
) + 2)*(a/c)^(1/8))*log(x^2 - x*sqrt(sqrt(2) + 2)*(a/c)^(1/8) + (a/c)^(1/4
))/a + 1/16*(e*sqrt(sqrt(2) + 2)*(a/c)^(5/8) + d*sqrt(-sqrt(2) + 2)*(a/c)^
(1/8))*log(x^2 + x*sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + (a/c)^(1/4))/a - 1/16*
(e*sqrt(sqrt(2) + 2)*(a/c)^(5/8) + d*sqrt(-sqrt(2) + 2)*(a/c)^(1/8))*log(x
^2 - x*sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + (a/c)^(1/4))/a
```

3.3.9 Mupad [B] (verification not implemented)

Time = 9.09 (sec) , antiderivative size = 2510, normalized size of antiderivative = 3.33

$$\int \frac{d+ex^4}{a+cx^8}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^4)/(a + c*x^8),x)
```

output
```
(atan((c^3*d^6*x - a^3*e^6*x + a*c^2*d^4*e^2*x - a^2*c*d^2*e^4*x + (2*d*e*
x*(a^2*e^4*(-a^7*c^5)^(1/2) + c^2*d^4*(-a^7*c^5)^(1/2) - 4*a^4*c^4*d^3*e +
 4*a^5*c^3*d*e^3 - 6*a*c*d^2*e^2*(-a^7*c^5)^(1/2)))/(a^3*c^2))/(a*c^3*d^5*
((a^2*e^4*(-a^7*c^5)^(1/2) + c^2*d^4*(-a^7*c^5)^(1/2) - 4*a^4*c^4*d^3*e +
4*a^5*c^3*d*e^3 - 6*a*c*d^2*e^2*(-a^7*c^5)^(1/2))/(a^7*c^5))^(1/4) + a^5*c
^3*e*((a^2*e^4*(-a^7*c^5)^(1/2) + c^2*d^4*(-a^7*c^5)^(1/2) - 4*a^4*c^4*d^3
*e + 4*a^5*c^3*d*e^3 - 6*a*c*d^2*e^2*(-a^7*c^5)^(1/2))/(a^7*c^5))^(5/4) -
2*a^2*c^2*d^3*e^2*((a^2*e^4*(-a^7*c^5)^(1/2) + c^2*d^4*(-a^7*c^5)^(1/2) -
4*a^4*c^4*d^3*e + 4*a^5*c^3*d*e^3 - 6*a*c*d^2*e^2*(-a^7*c^5)^(1/2))/(a^7*c
^5))^(1/4) - 3*a^3*c*d*e^4*((a^2*e^4*(-a^7*c^5)^(1/2) + c^2*d^4*(-a^7*c^5)
^(1/2) - 4*a^4*c^4*d^3*e + 4*a^5*c^3*d*e^3 - 6*a*c*d^2*e^2*(-a^7*c^5)^(1/2
))/(a^7*c^5))^(1/4)))*((a^2*e^4*(-a^7*c^5)^(1/2) + c^2*d^4*(-a^7*c^5)^(1/2
) - 4*a^4*c^4*d^3*e + 4*a^5*c^3*d*e^3 - 6*a*c*d^2*e^2*(-a^7*c^5)^(1/2))/(a
^7*c^5))^(1/4))/4 - (atan((a^3*e^6*x - c^3*d^6*x - a*c^2*d^4*e^2*x + a^2*c
*d^2*e^4*x + (2*d*e*x*(a^2*e^4*(-a^7*c^5)^(1/2) + c^2*d^4*(-a^7*c^5)^(1/2)
 + 4*a^4*c^4*d^3*e - 4*a^5*c^3*d*e^3 - 6*a*c*d^2*e^2*(-a^7*c^5)^(1/2)))/(a
^3*c^2))/(a*c^3*d^5*(-(a^2*e^4*(-a^7*c^5)^(1/2) + c^2*d^4*(-a^7*c^5)^(1/2)
 + 4*a^4*c^4*d^3*e - 4*a^5*c^3*d*e^3 - 6*a*c*d^2*e^2*(-a^7*c^5)^(1/2))/(a^
7*c^5))^(1/4) + a^5*c^3*e*(-(a^2*e^4*(-a^7*c^5)^(1/2) + c^2*d^4*(-a^7*c^5)
^(1/2) + 4*a^4*c^4*d^3*e - 4*a^5*c^3*d*e^3 - 6*a*c*d^2*e^2*(-a^7*c^5)^(...
```

3.3. $\int \frac{d+ex^4}{a+cx^8}\,dx$

3.4 $\int \frac{d+ex^4}{a-cx^8}\,dx$

3.4.1 Optimal result

Integrand size = 18, antiderivative size = 329

$$\begin{aligned}\int \frac{d+ex^4}{a-cx^8}\,dx &= \frac{\left(\sqrt{c}d+\sqrt{a}e\right)\arctan\left(\frac{\sqrt[8]{c}x}{\sqrt[8]{a}}\right)}{4a^{7/8}c^{5/8}}-\frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}x}{\sqrt[8]{a}}\right)}{4\sqrt{2}a^{7/8}\sqrt[8]{c}}\\
&+\frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[8]{c}x}{\sqrt[8]{a}}\right)}{4\sqrt{2}a^{7/8}\sqrt[8]{c}}+\frac{\left(\sqrt{c}d+\sqrt{a}e\right)\operatorname{arctanh}\left(\frac{\sqrt[8]{c}x}{\sqrt[8]{a}}\right)}{4a^{7/8}c^{5/8}}\\
&-\frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\log\left(\sqrt[4]{a}-\sqrt{2}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{c}x^2\right)}{8\sqrt{2}a^{7/8}\sqrt[8]{c}}\\
&+\frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\log\left(\sqrt[4]{a}+\sqrt{2}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{c}x^2\right)}{8\sqrt{2}a^{7/8}\sqrt[8]{c}}\end{aligned}$$

output
```
1/8*arctan(-1+c^(1/8)*x*2^(1/2)/a^(1/8))*(d-e*a^(1/2)/c^(1/2))/a^(7/8)/c^(
1/8)*2^(1/2)+1/8*arctan(1+c^(1/8)*x*2^(1/2)/a^(1/8))*(d-e*a^(1/2)/c^(1/2))
/a^(7/8)/c^(1/8)*2^(1/2)-1/16*ln(a^(1/4)+c^(1/4)*x^2-a^(1/8)*c^(1/8)*x*2^(
1/2))*(d-e*a^(1/2)/c^(1/2))/a^(7/8)/c^(1/8)*2^(1/2)+1/16*ln(a^(1/4)+c^(1/4
)*x^2+a^(1/8)*c^(1/8)*x*2^(1/2))*(d-e*a^(1/2)/c^(1/2))/a^(7/8)/c^(1/8)*2^(
1/2)+1/4*arctan(c^(1/8)*x/a^(1/8))*(e*a^(1/2)+d*c^(1/2))/a^(7/8)/c^(5/8)+1
/4*arctanh(c^(1/8)*x/a^(1/8))*(e*a^(1/2)+d*c^(1/2))/a^(7/8)/c^(5/8)
```

3.4.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.29

$$\int \frac{d+ex^4}{a-cx^8}\,dx = \frac{\left(\sqrt[8]{a}\sqrt{c}d+a^{5/8}e\right)\arctan\left(\frac{\sqrt[8]{c}x}{\sqrt[8]{a}}\right)}{4ac^{5/8}} - \frac{\left(-\sqrt[8]{a}\sqrt{c}d+a^{5/8}e\right)\arctan\left(\frac{-\sqrt{2}\sqrt[8]{a}+2\sqrt[8]{c}x}{\sqrt{2}\sqrt[8]{a}}\right)}{4\sqrt{2}ac^{5/8}} - \frac{\left(-\sqrt[8]{a}\sqrt{c}d+a^{5/8}e\right)\arctan\left(\frac{\sqrt{2}\sqrt[8]{a}+2\sqrt[8]{c}x}{\sqrt{2}\sqrt[8]{a}}\right)}{4\sqrt{2}ac^{5/8}} - \frac{\left(\sqrt[8]{a}\sqrt{c}d+a^{5/8}e\right)\log\left(\sqrt[8]{a}-\sqrt[8]{c}x\right)}{8ac^{5/8}} - \frac{\left(-\sqrt[8]{a}\sqrt{c}d-a^{5/8}e\right)\log\left(\sqrt[8]{a}+\sqrt[8]{c}x\right)}{8ac^{5/8}} + \frac{\left(-\sqrt[8]{a}\sqrt{c}d+a^{5/8}e\right)\log\left(\sqrt[4]{a}-\sqrt{2}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{c}x^2\right)}{8\sqrt{2}ac^{5/8}} - \frac{\left(-\sqrt[8]{a}\sqrt{c}d+a^{5/8}e\right)\log\left(\sqrt[4]{a}+\sqrt{2}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{c}x^2\right)}{8\sqrt{2}ac^{5/8}}$$

input
```
Integrate[(d + e*x^4)/(a - c*x^8),x]
```

output
```
((a^(1/8)*Sqrt[c]*d + a^(5/8)*e)*ArcTan[(c^(1/8)*x)/a^(1/8)])/(4*a*c^(5/8)
) - ((-(a^(1/8)*Sqrt[c]*d) + a^(5/8)*e)*ArcTan[(-(Sqrt[2]*a^(1/8)) + 2*c^(
1/8)*x)/(Sqrt[2]*a^(1/8))])/(4*Sqrt[2]*a*c^(5/8)) - ((-(a^(1/8)*Sqrt[c]*d)
 + a^(5/8)*e)*ArcTan[(Sqrt[2]*a^(1/8) + 2*c^(1/8)*x)/(Sqrt[2]*a^(1/8))])/(
4*Sqrt[2]*a*c^(5/8)) - ((a^(1/8)*Sqrt[c]*d + a^(5/8)*e)*Log[a^(1/8) - c^(1
/8)*x])/(8*a*c^(5/8)) - ((-(a^(1/8)*Sqrt[c]*d) - a^(5/8)*e)*Log[a^(1/8) +
c^(1/8)*x])/(8*a*c^(5/8)) + ((-(a^(1/8)*Sqrt[c]*d) + a^(5/8)*e)*Log[a^(1/4
) - Sqrt[2]*a^(1/8)*c^(1/8)*x + c^(1/4)*x^2])/(8*Sqrt[2]*a*c^(5/8)) - ((-(
a^(1/8)*Sqrt[c]*d) + a^(5/8)*e)*Log[a^(1/4) + Sqrt[2]*a^(1/8)*c^(1/8)*x +
c^(1/4)*x^2])/(8*Sqrt[2]*a*c^(5/8))
```

3.4.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 293, normalized size of antiderivative = 0.89, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.722, Rules used = {1747, 755, 27, 756, 218, 221, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^4}{a-cx^8}dx$$

$$\Big\downarrow \text{1747}$$

$$\frac{1}{2}\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\int \frac{1}{a-\sqrt{a}\sqrt{c}x^4}dx+\frac{1}{2}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{1}{\sqrt{a}\sqrt{c}x^4+a}dx$$

$$\Big\downarrow \text{755}$$

$$\frac{1}{2}\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\int \frac{1}{a-\sqrt{a}\sqrt{c}x^4}dx+\frac{1}{2}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\left(\frac{\int \frac{\sqrt[4]{a}-\sqrt[4]{c}x^2}{\sqrt{a}\left(\sqrt{c}x^4+\sqrt{a}\right)}dx}{2\sqrt[4]{a}}+\frac{\int \frac{\sqrt[4]{c}x^2+\sqrt[4]{a}}{\sqrt{a}\left(\sqrt{c}x^4+\sqrt{a}\right)}dx}{2\sqrt[4]{a}}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\left(\frac{\int \frac{\sqrt[4]{a}-\sqrt[4]{c}x^2}{\sqrt{c}x^4+\sqrt{a}}dx}{2a^{3/4}}+\frac{\int \frac{\sqrt[4]{c}x^2+\sqrt[4]{a}}{\sqrt{c}x^4+\sqrt{a}}dx}{2a^{3/4}}\right)+\frac{1}{2}\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\int \frac{1}{a-\sqrt{a}\sqrt{c}x^4}dx$$

$$\Big\downarrow \text{756}$$

$$\frac{1}{2}\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\left(\frac{\int \frac{1}{\sqrt[4]{a}-\sqrt[4]{c}x^2}dx}{2a^{3/4}}+\frac{\int \frac{1}{\sqrt[4]{c}x^2+\sqrt[4]{a}}dx}{2a^{3/4}}\right)+\frac{1}{2}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\left(\frac{\int \frac{\sqrt[4]{a}-\sqrt[4]{c}x^2}{\sqrt{c}x^4+\sqrt{a}}dx}{2a^{3/4}}+\frac{\int \frac{\sqrt[4]{c}x^2+\sqrt[4]{a}}{\sqrt{c}x^4+\sqrt{a}}dx}{2a^{3/4}}\right)$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{2}\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\left(\frac{\int \frac{1}{\sqrt[4]{a}-\sqrt[4]{c}x^2}dx}{2a^{3/4}}+\frac{\arctan\left(\frac{\sqrt[8]{c}x}{\sqrt[8]{a}}\right)}{2a^{7/8}\sqrt[8]{c}}\right)+\frac{1}{2}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\left(\frac{\int \frac{\sqrt[4]{a}-\sqrt[4]{c}x^2}{\sqrt{c}x^4+\sqrt{a}}dx}{2a^{3/4}}+\frac{\int \frac{\sqrt[4]{c}x^2+\sqrt[4]{a}}{\sqrt{c}x^4+\sqrt{a}}dx}{2a^{3/4}}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\left(\frac{\int \frac{\sqrt[4]{a}-\sqrt[4]{c}x^2}{\sqrt{c}x^4+\sqrt{a}}dx}{2a^{3/4}}+\frac{\int \frac{\sqrt[4]{c}x^2+\sqrt[4]{a}}{\sqrt{c}x^4+\sqrt{a}}dx}{2a^{3/4}}\right)+\frac{1}{2}\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\left(\frac{\arctan\left(\frac{\sqrt[8]{c}x}{\sqrt[8]{a}}\right)}{2a^{7/8}\sqrt[8]{c}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}x}{\sqrt[8]{a}}\right)}{2a^{7/8}\sqrt[8]{c}}\right)$$

$$\Big\downarrow \text{ 1476}$$

$$\frac{1}{2}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[8]{a}x}{\sqrt[8]{c}}+\frac{\sqrt[4]{a}}{\sqrt[4]{c}}}dx}{2\sqrt[4]{c}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[8]{a}x}{\sqrt[8]{c}}+\frac{\sqrt[4]{a}}{\sqrt[4]{c}}}dx}{2\sqrt[4]{c}}}{2a^{3/4}}+\frac{\int \frac{\sqrt[4]{a}-\sqrt[4]{c}x^2}{\sqrt{c}x^4+\sqrt{a}}dx}{2a^{3/4}}\right)+\frac{1}{2}\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\left(\frac{\arctan\left(\frac{\sqrt[8]{c}x}{\sqrt[8]{a}}\right)}{2a^{7/8}\sqrt[8]{c}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}x}{\sqrt[8]{a}}\right)}{2a^{7/8}\sqrt[8]{c}}\right)$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{1}{2}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\left(\frac{\int \frac{\sqrt[4]{a}-\sqrt[4]{c}x^2}{\sqrt{c}x^4+\sqrt{a}}dx}{2a^{3/4}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[8]{c}x}{\sqrt[8]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[8]{c}x}{\sqrt[8]{a}}\right)}{\sqrt{2}\sqrt[8]{a}\sqrt[8]{c}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[8]{c}x}{\sqrt[8]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[8]{c}x}{\sqrt[8]{a}}+1\right)}{\sqrt{2}\sqrt[8]{a}\sqrt[8]{c}}}{2a^{3/4}}\right)+\frac{1}{2}\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\left(\frac{\arctan\left(\frac{\sqrt[8]{c}x}{\sqrt[8]{a}}\right)}{2a^{7/8}\sqrt[8]{c}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}x}{\sqrt[8]{a}}\right)}{2a^{7/8}\sqrt[8]{c}}\right)$$

$$\Big\downarrow \text{ 217}$$

$$\frac{1}{2}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\left(\frac{\int \frac{\sqrt[4]{a}-\sqrt[4]{c}x^2}{\sqrt{c}x^4+\sqrt{a}}dx}{2a^{3/4}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}x}{\sqrt[8]{a}}+1\right)}{\sqrt{2}\sqrt[8]{a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}x}{\sqrt[8]{a}}\right)}{\sqrt{2}\sqrt[8]{a}\sqrt[8]{c}}}{2a^{3/4}}\right)+\frac{1}{2}\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\left(\frac{\arctan\left(\frac{\sqrt[8]{c}x}{\sqrt[8]{a}}\right)}{2a^{7/8}\sqrt[8]{c}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}x}{\sqrt[8]{a}}\right)}{2a^{7/8}\sqrt[8]{c}}\right)$$

3.4. $\int \frac{d+ex^4}{a-cx^8}\,dx$

$$\Big\downarrow \ 1479$$

$$\frac{1}{2}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[8]{a}-2\sqrt[8]{c}x}{\sqrt[8]{c}\left(x^2-\frac{\sqrt{2}\sqrt[8]{a}x}{\sqrt[8]{c}}+\frac{\sqrt[4]{a}}{\sqrt[4]{c}}\right)}dx}{2\sqrt{2}\sqrt[8]{a}\sqrt[8]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[8]{c}x+\sqrt[8]{a}\right)}{\sqrt[8]{c}\left(x^2+\frac{\sqrt{2}\sqrt[8]{a}x}{\sqrt[8]{c}}+\frac{\sqrt[4]{a}}{\sqrt[4]{c}}\right)}dx}{2\sqrt{2}\sqrt[8]{a}\sqrt[8]{c}}}{2a^{3/4}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}x}{\sqrt[8]{a}}+1\right)}{\sqrt{2}\sqrt[8]{a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}x}{\sqrt[8]{a}}\right)}{\sqrt{2}\sqrt[8]{a}\sqrt[8]{c}}}{2a^{3/4}}\right)+\frac{1}{2}\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\left(\frac{\arctan\left(\frac{\sqrt[8]{c}x}{\sqrt[8]{a}}\right)}{2a^{7/8}\sqrt[8]{c}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}x}{\sqrt[8]{a}}\right)}{2a^{7/8}\sqrt[8]{c}}\right)$$

$$\Big\downarrow \ 25$$

$$\frac{1}{2}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[8]{a}-2\sqrt[8]{c}x}{\sqrt[8]{c}\left(x^2-\frac{\sqrt{2}\sqrt[8]{a}x}{\sqrt[8]{c}}+\frac{\sqrt[4]{a}}{\sqrt[4]{c}}\right)}dx}{2\sqrt{2}\sqrt[8]{a}\sqrt[8]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[8]{c}x+\sqrt[8]{a}\right)}{\sqrt[8]{c}\left(x^2+\frac{\sqrt{2}\sqrt[8]{a}x}{\sqrt[8]{c}}+\frac{\sqrt[4]{a}}{\sqrt[4]{c}}\right)}dx}{2\sqrt{2}\sqrt[8]{a}\sqrt[8]{c}}}{2a^{3/4}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}x}{\sqrt[8]{a}}+1\right)}{\sqrt{2}\sqrt[8]{a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}x}{\sqrt[8]{a}}\right)}{\sqrt{2}\sqrt[8]{a}\sqrt[8]{c}}}{2a^{3/4}}\right)+\frac{1}{2}\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\left(\frac{\arctan\left(\frac{\sqrt[8]{c}x}{\sqrt[8]{a}}\right)}{2a^{7/8}\sqrt[8]{c}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}x}{\sqrt[8]{a}}\right)}{2a^{7/8}\sqrt[8]{c}}\right)$$

$$\Big\downarrow \ 27$$

$$\frac{1}{2}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[8]{a}-2\sqrt[8]{c}x}{x^2-\frac{\sqrt{2}\sqrt[8]{a}x}{\sqrt[8]{c}}+\frac{\sqrt[4]{a}}{\sqrt[4]{c}}}dx}{2\sqrt{2}\sqrt[8]{a}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\sqrt[8]{c}x+\sqrt[8]{a}}{x^2+\frac{\sqrt{2}\sqrt[8]{a}x}{\sqrt[8]{c}}+\frac{\sqrt[4]{a}}{\sqrt[4]{c}}}dx}{2\sqrt[8]{a}\sqrt[4]{c}}}{2a^{3/4}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}x}{\sqrt[8]{a}}+1\right)}{\sqrt{2}\sqrt[8]{a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}x}{\sqrt[8]{a}}\right)}{\sqrt{2}\sqrt[8]{a}\sqrt[8]{c}}}{2a^{3/4}}\right)+\frac{1}{2}\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\left(\frac{\arctan\left(\frac{\sqrt[8]{c}x}{\sqrt[8]{a}}\right)}{2a^{7/8}\sqrt[8]{c}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}x}{\sqrt[8]{a}}\right)}{2a^{7/8}\sqrt[8]{c}}\right)$$

$$\Big\downarrow \ 1103$$

3.4. $\int \frac{d+ex^4}{a-cx^8}\,dx$

$$\frac{1}{2}\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\left(\frac{\arctan\left(\frac{\sqrt[8]{c}x}{\sqrt[8]{a}}\right)}{2a^{7/8}\sqrt[8]{c}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}x}{\sqrt[8]{a}}\right)}{2a^{7/8}\sqrt[8]{c}}\right)+$$

$$\frac{1}{2}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}x}{\sqrt[8]{a}}+1\right)}{\sqrt{2}\sqrt[8]{a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}x}{\sqrt[8]{a}}\right)}{\sqrt{2}\sqrt[8]{a}\sqrt[8]{c}}}{2a^{3/4}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{a}+\sqrt[4]{c}x^2\right)}{2\sqrt{2}\sqrt[8]{a}\sqrt[8]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{a}+\sqrt[4]{c}x\right.}{2\sqrt{2}\sqrt[8]{a}\sqrt[8]{c}}}{2a^{3/4}}\right.$$

input
```
Int[(d + e*x^4)/(a - c*x^8),x]
```

output
```
((d + (Sqrt[a]*e)/Sqrt[c])*(ArcTan[(c^(1/8)*x)/a^(1/8)]/(2*a^(7/8)*c^(1/8)
) + ArcTanh[(c^(1/8)*x)/a^(1/8)]/(2*a^(7/8)*c^(1/8))))/2 + ((d - (Sqrt[a]*
e)/Sqrt[c])*((-(ArcTan[1 - (Sqrt[2]*c^(1/8)*x)/a^(1/8)]/(Sqrt[2]*a^(1/8)*c
^(1/8))) + ArcTan[1 + (Sqrt[2]*c^(1/8)*x)/a^(1/8)]/(Sqrt[2]*a^(1/8)*c^(1/8
)))/(2*a^(3/4)) + (-1/2*Log[a^(1/4) - Sqrt[2]*a^(1/8)*c^(1/8)*x + c^(1/4)*
x^2]/(Sqrt[2]*a^(1/8)*c^(1/8)) + Log[a^(1/4) + Sqrt[2]*a^(1/8)*c^(1/8)*x +
 c^(1/4)*x^2]/(2*Sqrt[2]*a^(1/8)*c^(1/8)))/(2*a^(3/4))))/2
```

3.4.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1747
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> With[{
q = Rt[-a/c, 2]}, Simp[(d + e*q)/2   Int[1/(a + c*q*x^n), x], x] + Simp[(d
- e*q)/2   Int[1/(a - c*q*x^n), x], x]] /; FreeQ[{a, c, d, e, n}, x] && EqQ
[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && NegQ[a*c] && IntegerQ[n]
```

3.4. $\int \frac{d+ex^4}{a-cx^8} \, dx$

3.4.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.60 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.11

method	result	size
default	$-\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{8}-a\right)}\frac{\left(\_R^{4}e+d\right)\ln\left(x-\_R\right)}{\_R^{7}}}{8c}$	36
risch	$-\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{8}-a\right)}\frac{\left(\_R^{4}e+d\right)\ln\left(x-\_R\right)}{\_R^{7}}}{8c}$	36

input
```
int((e*x^4+d)/(-c*x^8+a),x,method=_RETURNVERBOSE)
```

output
```
-1/8/c*sum((_R^4*e+d)/_R^7*ln(x-_R),_R=RootOf(_Z^8*c-a))
```

3.4.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2741 vs. 2(220) = 440.

Time = 0.64 (sec) , antiderivative size = 2741, normalized size of antiderivative = 8.33

$$\int \frac{d+ex^4}{a-cx^8}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^4+d)/(-c*x^8+a),x, algorithm="fricas")
```

output
```
1/8*sqrt(-sqrt((a^3*c^2*sqrt((c^4*d^8 + 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*
e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 4*c*d^3*e + 4*a*d*e^3)/(a^3
*c^2)))*log(-(c^3*d^6 + 5*a*c^2*d^4*e^2 - 5*a^2*c*d^2*e^4 - a^3*e^6)*x + (
a^5*c^3*e*sqrt((c^4*d^8 + 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*a^3*c
*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - a*c^3*d^5 - 6*a^2*c^2*d^3*e^2 - a^3*c*d*e
^4)*sqrt(-sqrt((a^3*c^2*sqrt((c^4*d^8 + 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*
e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 4*c*d^3*e + 4*a*d*e^3)/(a^3
*c^2)))) - 1/8*sqrt(-sqrt((a^3*c^2*sqrt((c^4*d^8 + 12*a*c^3*d^6*e^2 + 38*a
^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 4*c*d^3*e + 4*a*
d*e^3)/(a^3*c^2)))*log(-(c^3*d^6 + 5*a*c^2*d^4*e^2 - 5*a^2*c*d^2*e^4 - a^3
*e^6)*x - (a^5*c^3*e*sqrt((c^4*d^8 + 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4
 + 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - a*c^3*d^5 - 6*a^2*c^2*d^3*e^2
- a^3*c*d*e^4)*sqrt(-sqrt((a^3*c^2*sqrt((c^4*d^8 + 12*a*c^3*d^6*e^2 + 38*a
^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 4*c*d^3*e + 4*a*
d*e^3)/(a^3*c^2)))) - 1/8*sqrt(-sqrt(-(a^3*c^2*sqrt((c^4*d^8 + 12*a*c^3*d^
6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - 4*c*
d^3*e - 4*a*d*e^3)/(a^3*c^2)))*log(-(c^3*d^6 + 5*a*c^2*d^4*e^2 - 5*a^2*c*d
^2*e^4 - a^3*e^6)*x + (a^5*c^3*e*sqrt((c^4*d^8 + 12*a*c^3*d^6*e^2 + 38*a^2
*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + a*c^3*d^5 + 6*a^2*
c^2*d^3*e^2 + a^3*c*d*e^4)*sqrt(-sqrt(-(a^3*c^2*sqrt((c^4*d^8 + 12*a*c^...
```

3.4.6 Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^4}{a-cx^8}\,dx = \text{Timed out}$$

input
```
integrate((e*x**4+d)/(-c*x**8+a),x)
```

output
```
Timed out
```

3.4.7 Maxima [F]

$$\int \frac{d+ex^4}{a-cx^8}\,dx = \int -\frac{ex^4+d}{cx^8-a}\,dx$$

input
```
integrate((e*x^4+d)/(-c*x^8+a),x, algorithm="maxima")
```

output
```
-integrate((e*x^4 + d)/(c*x^8 - a), x)
```

3.4.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 625 vs. $2(220) = 440$.

Time = 0.44 (sec) , antiderivative size = 625, normalized size of antiderivative = 1.90

$$\begin{aligned}
&\int \frac{d+ex^4}{a-cx^8}\,dx \\
&= -\frac{\left(e\sqrt{-\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{5}{8}} - d\sqrt{\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}}\right)\arctan\left(\frac{2x+\sqrt{-\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}}}{\sqrt{\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{8a} \\
&\quad - \frac{\left(e\sqrt{-\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{5}{8}} - d\sqrt{\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}}\right)\arctan\left(\frac{2x-\sqrt{-\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}}}{\sqrt{\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{8a} \\
&\quad + \frac{\left(e\sqrt{\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{5}{8}} + d\sqrt{-\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}}\right)\arctan\left(\frac{2x+\sqrt{\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}}}{\sqrt{-\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{8a} \\
&\quad + \frac{\left(e\sqrt{\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{5}{8}} + d\sqrt{-\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}}\right)\arctan\left(\frac{2x-\sqrt{\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}}}{\sqrt{-\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{8a} \\
&\quad - \frac{\left(e\sqrt{-\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{5}{8}} - d\sqrt{\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}}\right)\log\left(x^2 + x\sqrt{\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}} + \left(-\frac{a}{c}\right)^{\frac{1}{4}}\right)}{16a} \\
&\quad + \frac{\left(e\sqrt{-\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{5}{8}} - d\sqrt{\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}}\right)\log\left(x^2 - x\sqrt{\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}} + \left(-\frac{a}{c}\right)^{\frac{1}{4}}\right)}{16a} \\
&\quad + \frac{\left(e\sqrt{\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{5}{8}} + d\sqrt{-\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}}\right)\log\left(x^2 + x\sqrt{-\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}} + \left(-\frac{a}{c}\right)^{\frac{1}{4}}\right)}{16a} \\
&\quad - \frac{\left(e\sqrt{\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{5}{8}} + d\sqrt{-\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}}\right)\log\left(x^2 - x\sqrt{-\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}} + \left(-\frac{a}{c}\right)^{\frac{1}{4}}\right)}{16a}
\end{aligned}$$

input
```
integrate((e*x^4+d)/(-c*x^8+a),x, algorithm="giac")
```

output
```
-1/8*(e*sqrt(-sqrt(2) + 2)*(-a/c)^(5/8) - d*sqrt(sqrt(2) + 2)*(-a/c)^(1/8)
)*arctan((2*x + sqrt(-sqrt(2) + 2)*(-a/c)^(1/8))/(sqrt(sqrt(2) + 2)*(-a/c)
^(1/8)))/a - 1/8*(e*sqrt(-sqrt(2) + 2)*(-a/c)^(5/8) - d*sqrt(sqrt(2) + 2)*
(-a/c)^(1/8))*arctan((2*x - sqrt(-sqrt(2) + 2)*(-a/c)^(1/8))/(sqrt(sqrt(2)
 + 2)*(-a/c)^(1/8)))/a + 1/8*(e*sqrt(sqrt(2) + 2)*(-a/c)^(5/8) + d*sqrt(-s
qrt(2) + 2)*(-a/c)^(1/8))*arctan((2*x + sqrt(sqrt(2) + 2)*(-a/c)^(1/8))/(s
qrt(-sqrt(2) + 2)*(-a/c)^(1/8)))/a + 1/8*(e*sqrt(sqrt(2) + 2)*(-a/c)^(5/8)
 + d*sqrt(-sqrt(2) + 2)*(-a/c)^(1/8))*arctan((2*x - sqrt(sqrt(2) + 2)*(-a/
c)^(1/8))/(sqrt(-sqrt(2) + 2)*(-a/c)^(1/8)))/a - 1/16*(e*sqrt(-sqrt(2) + 2
)*(-a/c)^(5/8) - d*sqrt(sqrt(2) + 2)*(-a/c)^(1/8))*log(x^2 + x*sqrt(sqrt(2
) + 2)*(-a/c)^(1/8) + (-a/c)^(1/4))/a + 1/16*(e*sqrt(-sqrt(2) + 2)*(-a/c)^
(5/8) - d*sqrt(sqrt(2) + 2)*(-a/c)^(1/8))*log(x^2 - x*sqrt(sqrt(2) + 2)*(-
a/c)^(1/8) + (-a/c)^(1/4))/a + 1/16*(e*sqrt(sqrt(2) + 2)*(-a/c)^(5/8) + d*
sqrt(-sqrt(2) + 2)*(-a/c)^(1/8))*log(x^2 + x*sqrt(-sqrt(2) + 2)*(-a/c)^(1/
8) + (-a/c)^(1/4))/a - 1/16*(e*sqrt(sqrt(2) + 2)*(-a/c)^(5/8) + d*sqrt(-sq
rt(2) + 2)*(-a/c)^(1/8))*log(x^2 - x*sqrt(-sqrt(2) + 2)*(-a/c)^(1/8) + (-a
/c)^(1/4))/a
```

3.4.9 Mupad [B] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 2438, normalized size of antiderivative = 7.41

$$\int \frac{d+ex^4}{a-cx^8}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^4)/(a - c*x^8),x)
```

output
```
(atan((a^3*e^6*x + c^3*d^6*x - a*c^2*d^4*e^2*x - a^2*c*d^2*e^4*x + (2*d*e*
x*(a^2*e^4*(a^7*c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) + 4*a^4*c^4*d^3*e + 4
*a^5*c^3*d*e^3 + 6*a*c*d^2*e^2*(a^7*c^5)^(1/2)))/(a^3*c^2))/(a*c^3*d^5*((a
^2*e^4*(a^7*c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) + 4*a^4*c^4*d^3*e + 4*a^5
*c^3*d*e^3 + 6*a*c*d^2*e^2*(a^7*c^5)^(1/2))/(a^7*c^5))^(1/4) + a^5*c^3*e*(
(a^2*e^4*(a^7*c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) + 4*a^4*c^4*d^3*e + 4*a
^5*c^3*d*e^3 + 6*a*c*d^2*e^2*(a^7*c^5)^(1/2))/(a^7*c^5))^(5/4) + 2*a^2*c^2
*d^3*e^2*((a^2*e^4*(a^7*c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) + 4*a^4*c^4*d
^3*e + 4*a^5*c^3*d*e^3 + 6*a*c*d^2*e^2*(a^7*c^5)^(1/2))/(a^7*c^5))^(1/4) -
 3*a^3*c*d*e^4*((a^2*e^4*(a^7*c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) + 4*a^4
*c^4*d^3*e + 4*a^5*c^3*d*e^3 + 6*a*c*d^2*e^2*(a^7*c^5)^(1/2))/(a^7*c^5))^(
1/4)))*((a^2*e^4*(a^7*c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) + 4*a^4*c^4*d^3
*e + 4*a^5*c^3*d*e^3 + 6*a*c*d^2*e^2*(a^7*c^5)^(1/2))/(a^7*c^5))^(1/4))/4
- (atan((a*c^2*d^4*e^2*x - c^3*d^6*x - a^3*e^6*x + a^2*c*d^2*e^4*x + (2*d*
e*x*(a^2*e^4*(a^7*c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) - 4*a^4*c^4*d^3*e -
 4*a^5*c^3*d*e^3 + 6*a*c*d^2*e^2*(a^7*c^5)^(1/2)))/(a^3*c^2))/(a*c^3*d^5*(
-(a^2*e^4*(a^7*c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) - 4*a^4*c^4*d^3*e - 4*
a^5*c^3*d*e^3 + 6*a*c*d^2*e^2*(a^7*c^5)^(1/2))/(a^7*c^5))^(1/4) + a^5*c^3*
e*(-(a^2*e^4*(a^7*c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) - 4*a^4*c^4*d^3*e -
 4*a^5*c^3*d*e^3 + 6*a*c*d^2*e^2*(a^7*c^5)^(1/2))/(a^7*c^5))^(5/4) + 2*...
```

3.4. $\int \frac{d+ex^4}{a-cx^8}\,dx$

3.5 $\int \frac{d+ex^4}{d^2+bx^4+e^2x^8} \, dx$

3.5.1 Optimal result

Integrand size = 26, antiderivative size = 791

$$\int \frac{d+ex^4}{d^2+bx^4+e^2x^8}\,dx = -\frac{\arctan\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}-2\sqrt{e}x}{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{-b+2de}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{-b+2de}}} - \frac{\arctan\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{-b+2de}}-2\sqrt{e}x}{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}} + \frac{\arctan\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}+2\sqrt{e}x}{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{-b+2de}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{-b+2de}}} + \frac{\arctan\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{-b+2de}}+2\sqrt{e}x}{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}} - \frac{\log\left(\sqrt{d}-\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}x+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}} + \frac{\log\left(\sqrt{d}+\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}x+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}} - \frac{\log\left(\sqrt{d}-\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{-b+2de}}x+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{-b+2de}}} + \frac{\log\left(\sqrt{d}+\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{-b+2de}}x+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{-b+2de}}}$$

output
```
-1/4*arctan((-2*x*e^(1/2)+(2*d^(1/2)*e^(1/2)+(2*d*e-b)^(1/2))^(1/2))/(2*d^
(1/2)*e^(1/2)-(2*d*e-b)^(1/2))^(1/2))/d^(1/2)/(2*d^(1/2)*e^(1/2)-(2*d*e-b)
^(1/2))^(1/2)+1/4*arctan((2*x*e^(1/2)+(2*d^(1/2)*e^(1/2)+(2*d*e-b)^(1/2))^
(1/2))/(2*d^(1/2)*e^(1/2)-(2*d*e-b)^(1/2))^(1/2))/d^(1/2)/(2*d^(1/2)*e^(1/
2)-(2*d*e-b)^(1/2))^(1/2)-1/8*ln(d^(1/2)+x^2*e^(1/2)-x*(2*d^(1/2)*e^(1/2)-
(2*d*e-b)^(1/2))^(1/2))/d^(1/2)/(2*d^(1/2)*e^(1/2)-(2*d*e-b)^(1/2))^(1/2)+
1/8*ln(d^(1/2)+x^2*e^(1/2)+x*(2*d^(1/2)*e^(1/2)-(2*d*e-b)^(1/2))^(1/2))/d^
(1/2)/(2*d^(1/2)*e^(1/2)-(2*d*e-b)^(1/2))^(1/2)-1/4*arctan((-2*x*e^(1/2)+(
2*d^(1/2)*e^(1/2)-(2*d*e-b)^(1/2))^(1/2))/(2*d^(1/2)*e^(1/2)+(2*d*e-b)^(1/
2))^(1/2))/d^(1/2)/(2*d^(1/2)*e^(1/2)+(2*d*e-b)^(1/2))^(1/2)+1/4*arctan((2
*x*e^(1/2)+(2*d^(1/2)*e^(1/2)-(2*d*e-b)^(1/2))^(1/2))/(2*d^(1/2)*e^(1/2)+(
2*d*e-b)^(1/2))^(1/2))/d^(1/2)/(2*d^(1/2)*e^(1/2)+(2*d*e-b)^(1/2))^(1/2)-1
/8*ln(d^(1/2)+x^2*e^(1/2)-x*(2*d^(1/2)*e^(1/2)+(2*d*e-b)^(1/2))^(1/2))/d^(
1/2)/(2*d^(1/2)*e^(1/2)+(2*d*e-b)^(1/2))^(1/2)+1/8*ln(d^(1/2)+x^2*e^(1/2)+
x*(2*d^(1/2)*e^(1/2)+(2*d*e-b)^(1/2))^(1/2))/d^(1/2)/(2*d^(1/2)*e^(1/2)+(2
*d*e-b)^(1/2))^(1/2)
```

3.5.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.08

$$\int \frac{d+ex^4}{d^2+bx^4+e^2x^8}\,dx = \frac{1}{4}\text{RootSum}\left[d^2+b\#1^4 + e^2\#1^8\&, \frac{d\log(x-\#1)+e\log(x-\#1)\#1^4}{b\#1^3+2e^2\#1^7}\&\right]$$

input
```
Integrate[(d + e*x^4)/(d^2 + b*x^4 + e^2*x^8),x]
```

output
```
RootSum[d^2 + b*#1^4 + e^2*#1^8 & , (d*Log[x - #1] + e*Log[x - #1]*#1^4)/(
b*#1^3 + 2*e^2*#1^7) & ]/4
```

3.5.3 Rubi [A] (verified)

Time = 1.30 (sec) , antiderivative size = 1001, normalized size of antiderivative = 1.27, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {1749, 1407, 27, 1142, 25, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^4}{bx^4+d^2+e^2x^8}\,dx$$

$$\Big\downarrow \text{1749}$$

$$\frac{\int \frac{1}{x^4-\frac{\sqrt{2de-b}x^2}{e}+\frac{d}{e}}dx}{2e}+\frac{\int \frac{1}{x^4+\frac{\sqrt{2de-b}x^2}{e}+\frac{d}{e}}dx}{2e}$$

$$\Big\downarrow \text{1407}$$

$$\frac{\frac{e\int \frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}-\sqrt{e}x}{\sqrt{e}\left(x^2-\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}x}{\sqrt{e}}+\frac{\sqrt{d}}{\sqrt{e}}\right)}dx}{2\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}}+\frac{e\int \frac{\sqrt{e}x+\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}}{\sqrt{e}\left(x^2+\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}x}{\sqrt{e}}+\frac{\sqrt{d}}{\sqrt{e}}\right)}dx}{2\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}}}{2e}+\frac{\frac{e\int \frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-b}}-\sqrt{e}x}{\sqrt{e}\left(x^2-\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-b}}x}{\sqrt{e}}+\frac{\sqrt{d}}{\sqrt{e}}\right)}dx}{2\sqrt{d}\sqrt{\sqrt{2de-b}+2\sqrt{d}\sqrt{e}}}+\frac{e\int \frac{\sqrt{e}x+\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-b}}}{\sqrt{e}\left(x^2+\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-b}}x}{\sqrt{e}}+\frac{\sqrt{d}}{\sqrt{e}}\right)}dx}{2\sqrt{d}\sqrt{\sqrt{2de-b}+2\sqrt{d}\sqrt{e}}}}{2e}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\sqrt{e}\int \frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}-\sqrt{e}x}{x^2-\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}x}{\sqrt{e}}+\frac{\sqrt{d}}{\sqrt{e}}}dx}{2\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}}+\frac{\sqrt{e}\int \frac{\sqrt{e}x+\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}}{x^2+\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}x}{\sqrt{e}}+\frac{\sqrt{d}}{\sqrt{e}}}dx}{2\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}}}{2e}+\frac{\frac{\sqrt{e}\int \frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-b}}-\sqrt{e}x}{x^2-\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-b}}x}{\sqrt{e}}+\frac{\sqrt{d}}{\sqrt{e}}}dx}{2\sqrt{d}\sqrt{\sqrt{2de-b}+2\sqrt{d}\sqrt{e}}}+\frac{\sqrt{e}\int \frac{\sqrt{e}x+\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-b}}}{x^2+\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-b}}x}{\sqrt{e}}+\frac{\sqrt{d}}{\sqrt{e}}}dx}{2\sqrt{d}\sqrt{\sqrt{2de-b}+2\sqrt{d}\sqrt{e}}}}{2e}$$

$$\Big\downarrow \text{1142}$$

$$\frac{\frac{\sqrt{e}\left(\frac{1}{2}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}\int \frac{1}{x^2-\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}x}{\sqrt{e}}+\frac{\sqrt{d}}{\sqrt{e}}}dx-\frac{1}{2}\sqrt{e}\int -\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}-2\sqrt{e}x}{\sqrt{e}\left(x^2-\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}x}{\sqrt{e}}+\frac{\sqrt{d}}{\sqrt{e}}\right)}dx\right)}{2\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}}+\frac{\sqrt{e}\left(\frac{1}{2}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}\int \frac{\cdots}{x^2\cdots}\cdots\right.}{\cdots}}{2e}$$

$$\frac{\frac{\sqrt{e}\left(\frac{1}{2}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-b}}\int \frac{1}{x^2-\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-b}}x}{\sqrt{e}}+\frac{\sqrt{d}}{\sqrt{e}}}dx-\frac{1}{2}\sqrt{e}\int -\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-b}}-2\sqrt{e}x}{\sqrt{e}\left(x^2-\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-b}}x}{\sqrt{e}}+\frac{\sqrt{d}}{\sqrt{e}}\right)}dx\right)}{2\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-b}}}+\frac{\sqrt{e}\left(\frac{1}{2}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-b}}\int \frac{\cdots}{x^2\cdots}\cdots\right.}{\cdots}}{2e}$$

$\downarrow$ 25

$$\frac{\frac{\sqrt{e}\left(\frac{1}{2}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}\int \frac{1}{x^2-\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}x}{\sqrt{e}}+\frac{\sqrt{d}}{\sqrt{e}}}dx+\frac{1}{2}\sqrt{e}\int \frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}-2\sqrt{e}x}{\sqrt{e}\left(x^2-\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}x}{\sqrt{e}}+\frac{\sqrt{d}}{\sqrt{e}}\right)}dx\right)}{2\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}}+\frac{\sqrt{e}\left(\frac{1}{2}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}\int \frac{\cdots}{x^2+\cdots}\cdots\right.}{\cdots}}{2e}$$

$$\frac{\frac{\sqrt{e}\left(\frac{1}{2}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-b}}\int \frac{1}{x^2-\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-b}}x}{\sqrt{e}}+\frac{\sqrt{d}}{\sqrt{e}}}dx+\frac{1}{2}\sqrt{e}\int \frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-b}}-2\sqrt{e}x}{\sqrt{e}\left(x^2-\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-b}}x}{\sqrt{e}}+\frac{\sqrt{d}}{\sqrt{e}}\right)}dx\right)}{2\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-b}}}+\frac{\sqrt{e}\left(\frac{1}{2}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-b}}\int \frac{\cdots}{x^2+\cdots}\cdots\right.}{\cdots}}{2e}$$

$\downarrow$ 27

$$\frac{\frac{\sqrt{e}\left(\frac{1}{2}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}\int \frac{1}{x^2-\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}x}{\sqrt{e}}+\frac{\sqrt{d}}{\sqrt{e}}}dx+\frac{1}{2}\int \frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}-2\sqrt{e}x}{x^2-\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}x}{\sqrt{e}}+\frac{\sqrt{d}}{\sqrt{e}}}dx\right)}{2\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}}+\frac{\sqrt{e}\left(\frac{1}{2}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}\int \frac{\cdots}{x^2+\frac{\sqrt{2\sqrt{d}\sqrt{e}-\cdots}}{\cdots}}\cdots\right.}{2\sqrt{d}\cdots}}{2e}$$

$$\frac{\frac{\sqrt{e}\left(\frac{1}{2}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-b}}\int \frac{1}{x^2-\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-b}}x}{\sqrt{e}}+\frac{\sqrt{d}}{\sqrt{e}}}dx+\frac{1}{2}\int \frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-b}}-2\sqrt{e}x}{x^2-\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-b}}x}{\sqrt{e}}+\frac{\sqrt{d}}{\sqrt{e}}}dx\right)}{2\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-b}}}+\frac{\sqrt{e}\left(\frac{1}{2}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-b}}\int \frac{\cdots}{x^2+\frac{\sqrt{2\sqrt{d}\sqrt{e}+\cdots}}{\cdots}}\cdots\right.}{2\sqrt{d}\cdots}}{2e}$$

$\downarrow$ 1083

3.5. $\int \frac{d+ex^4}{d^2+bx^4+e^2x^8}\,dx$

$$\frac{\frac{\sqrt{e}\left(\frac{1}{2}\int \frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}-2\sqrt{e}x}{x^2-\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}x}{\sqrt{e}}+\frac{\sqrt{d}}{\sqrt{e}}}dx-\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}\int \frac{1}{-\left(2x-\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}}{\sqrt{e}}\right)^2-\frac{2\sqrt{d}\sqrt{e}+\sqrt{2de-b}}{e}}d\left(2x-\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}}{\sqrt{e}}\right)\right)}{2\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}}+\frac{\sqrt{e}\cdots}{\cdots}}{2e}$$

$$\frac{\frac{\sqrt{e}\left(\frac{1}{2}\int \frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-b}}-2\sqrt{e}x}{x^2-\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-b}}x}{\sqrt{e}}+\frac{\sqrt{d}}{\sqrt{e}}}dx-\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-b}}\int \frac{1}{-\left(2x-\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-b}}}{\sqrt{e}}\right)^2-\frac{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}{e}}d\left(2x-\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-b}}}{\sqrt{e}}\right)\right)}{2\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-b}}}+\frac{\sqrt{e}\cdots}{\cdots}}{2e}$$

$\downarrow$ 217

$$\frac{\frac{\sqrt{e}\left(\frac{\sqrt{e}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}\arctan\left(\frac{\sqrt{e}\left(2x-\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}}{\sqrt{e}}\right)}{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-b}}}\right)}{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-b}}}+\frac{1}{2}\int \frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}-2\sqrt{e}x}{x^2-\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}x}{\sqrt{e}}+\frac{\sqrt{d}}{\sqrt{e}}}dx\right)}{2\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}}+\frac{\sqrt{e}\left(\frac{\sqrt{e}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}\arctan\left(\frac{\sqrt{e}\left(2x\cdots\right.}{\cdots}\right.}{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de}\cdots}}\right.}{2\cdots}}{2e}$$

$$\frac{\frac{\sqrt{e}\left(\frac{\sqrt{e}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-b}}\arctan\left(\frac{\sqrt{e}\left(2x-\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-b}}}{\sqrt{e}}\right)}{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}}\right)}{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}}+\frac{1}{2}\int \frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-b}}-2\sqrt{e}x}{x^2-\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-b}}x}{\sqrt{e}}+\frac{\sqrt{d}}{\sqrt{e}}}dx\right)}{2\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-b}}}+\frac{\sqrt{e}\left(\frac{\sqrt{e}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-b}}\arctan\left(\frac{\sqrt{e}\left(2x\cdots\right.}{\cdots}\right.}{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de}\cdots}}\right.}{2\cdots}}{2e}$$

$\downarrow$ 1103

3.5. $\int \frac{d+ex^4}{d^2+bx^4+e^2x^8}\,dx$

$$\frac{\frac{\sqrt{e}\left(\frac{\sqrt{e}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}\arctan\left(\frac{\sqrt{e}\left(2x-\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}}{\sqrt{e}}\right)}{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-b}}}\right)}{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-b}}}-\frac{1}{2}\sqrt{e}\log\left(\sqrt{e}x^2-\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}x+\sqrt{d}\right)\right)}{2\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}}+\frac{\sqrt{e}\left(\frac{\sqrt{e}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}\,\mathrm{arcta}\ldots}{\sqrt{2\ldots}}\ldots}{\ldots}}{2e}$$

$$+\frac{\frac{\sqrt{e}\left(\frac{\sqrt{e}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-b}}\arctan\left(\frac{\sqrt{e}\left(2x-\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-b}}}{\sqrt{e}}\right)}{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}}\right)}{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}}-\frac{1}{2}\sqrt{e}\log\left(\sqrt{e}x^2-\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-b}}x+\sqrt{d}\right)\right)}{2\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-b}}}+\frac{\sqrt{e}\left(\frac{\sqrt{e}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-b}}\,\mathrm{arcta}\ldots}{\sqrt{2\ldots}}\ldots}{\ldots}}{2e}$$

input
```
Int[(d + e*x^4)/(d^2 + b*x^4 + e^2*x^8),x]
```

output
```
((Sqrt[e]*((Sqrt[e]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[-b + 2*d*e]]*ArcTan[(Sqr
t[e]*(-(Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[-b + 2*d*e]]/Sqrt[e]) + 2*x))/Sqrt[2
*Sqrt[d]*Sqrt[e] + Sqrt[-b + 2*d*e]]])/Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[-b +
2*d*e]] - (Sqrt[e]*Log[Sqrt[d] - Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[-b + 2*d*e]
]*x + Sqrt[e]*x^2])/2))/(2*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[-b + 2*d*
e]]) + (Sqrt[e]*((Sqrt[e]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[-b + 2*d*e]]*ArcTa
n[(Sqrt[e]*(Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[-b + 2*d*e]]/Sqrt[e] + 2*x))/Sqr
t[2*Sqrt[d]*Sqrt[e] + Sqrt[-b + 2*d*e]]])/Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[-b
 + 2*d*e]] + (Sqrt[e]*Log[Sqrt[d] + Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[-b + 2*d
*e]]*x + Sqrt[e]*x^2])/2))/(2*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[-b + 2
*d*e]]))/(2*e) + ((Sqrt[e]*((Sqrt[e]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[-b + 2*
d*e]]*ArcTan[(Sqrt[e]*(-(Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[-b + 2*d*e]]/Sqrt[e
]) + 2*x))/Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[-b + 2*d*e]]])/Sqrt[2*Sqrt[d]*Sqr
t[e] - Sqrt[-b + 2*d*e]] - (Sqrt[e]*Log[Sqrt[d] - Sqrt[2*Sqrt[d]*Sqrt[e] +
 Sqrt[-b + 2*d*e]]*x + Sqrt[e]*x^2])/2))/(2*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e]
 + Sqrt[-b + 2*d*e]]) + (Sqrt[e]*((Sqrt[e]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[-
b + 2*d*e]]*ArcTan[(Sqrt[e]*(Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[-b + 2*d*e]]/Sq
rt[e] + 2*x))/Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[-b + 2*d*e]]])/Sqrt[2*Sqrt[d]*
Sqrt[e] - Sqrt[-b + 2*d*e]] + (Sqrt[e]*Log[Sqrt[d] + Sqrt[2*Sqrt[d]*Sqrt[e
] + Sqrt[-b + 2*d*e]]*x + Sqrt[e]*x^2])/2))/(2*Sqrt[d]*Sqrt[2*Sqrt[d]*S...
```

3.5. $\int \frac{d+ex^4}{d^2+bx^4+e^2x^8}\,dx$

3.5.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1407
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   Int[(r - x)/(q - r*
x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(r + x)/(q + r*x + x^2), x], x]]]
 /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

rule 1749
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e
+ q*x^(n/2) + x^n, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x^(n/2) +
 x^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[2*(d/e) - b/c,
0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

3.5.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.17 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.07

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(e^{2}\_Z^{8}+\_Z^{4}b+d^{2}\right)}\frac{\left(\_R^{4}e+d\right)\ln\left(x-\_R\right)}{2\_R^{7}e^{2}+\_R^{3}b}\right)}{4}$	53
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(e^{2}\_Z^{8}+\_Z^{4}b+d^{2}\right)}\frac{\left(\_R^{4}e+d\right)\ln\left(x-\_R\right)}{2\_R^{7}e^{2}+\_R^{3}b}\right)}{4}$	53

input
```
int((e*x^4+d)/(e^2*x^8+b*x^4+d^2),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum((_R^4*e+d)/(2*_R^7*e^2+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*e^2+_Z^4*b+
d^2))
```

3.5.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2461 vs. 2(581) = 1162.

Time = 0.32 (sec) , antiderivative size = 2461, normalized size of antiderivative = 3.11

$$\int \frac{d+ex^4}{d^2+bx^4+e^2x^8}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^4+d)/(e^2*x^8+b*x^4+d^2),x, algorithm="fricas")
```

output
```
1/4*sqrt(sqrt(1/2)*sqrt(-((4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e -
 b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4)) + b)/(4*d^4*e^2 +
4*b*d^3*e + b^2*d^2)))*log(e*x + 1/2*(2*d*e - (4*d^4*e^2 + 4*b*d^3*e + b^2
*d^2)*sqrt(-(2*d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4)
) + b)*sqrt(sqrt(1/2)*sqrt(-((4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*
e - b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4)) + b)/(4*d^4*e^2
 + 4*b*d^3*e + b^2*d^2)))) - 1/4*sqrt(sqrt(1/2)*sqrt(-((4*d^4*e^2 + 4*b*d^
3*e + b^2*d^2)*sqrt(-(2*d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e +
 b^3*d^4)) + b)/(4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)))*log(e*x - 1/2*(2*d*e -
 (4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e - b)/(8*d^7*e^3 + 12*b*d^6
*e^2 + 6*b^2*d^5*e + b^3*d^4)) + b)*sqrt(sqrt(1/2)*sqrt(-((4*d^4*e^2 + 4*b
*d^3*e + b^2*d^2)*sqrt(-(2*d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*
e + b^3*d^4)) + b)/(4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)))) + 1/4*sqrt(-sqrt(1
/2)*sqrt(-((4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e - b)/(8*d^7*e^3
+ 12*b*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4)) + b)/(4*d^4*e^2 + 4*b*d^3*e + b^2
*d^2)))*log(e*x + 1/2*(2*d*e - (4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)*sqrt(-(2*
d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4)) + b)*sqrt(-sq
rt(1/2)*sqrt(-((4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e - b)/(8*d^7*
e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4)) + b)/(4*d^4*e^2 + 4*b*d^3*e +
 b^2*d^2)))) - 1/4*sqrt(-sqrt(1/2)*sqrt(-((4*d^4*e^2 + 4*b*d^3*e + b^2*...
```

3.5.6 Sympy [A] (verification not implemented)

Time = 19.01 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.17

$$\int \frac{d + ex^4}{d^2 + bx^4 + e^2x^8}\,dx$$
$$= \text{RootSum}\left(t^8 \cdot \left(65536b^4d^2 + 524288b^3d^3e + 1572864b^2d^4e^2 + 2097152bd^5e^3 + 1048576d^6e^4\right) + t^4 \cdot \left(256b^3\right.\right.$$

input
```
integrate((e*x**4+d)/(e**2*x**8+b*x**4+d**2),x)
```

output
```
RootSum(_t**8*(65536*b**4*d**2 + 524288*b**3*d**3*e + 1572864*b**2*d**4*e*
*2 + 2097152*b*d**5*e**3 + 1048576*d**6*e**4) + _t**4*(256*b**3 + 1024*b**
2*d*e + 1024*b*d**2*e**2) + e**2, Lambda(_t, _t*log(x + (1024*_t**5*b**2*d
**2 + 4096*_t**5*b*d**3*e + 4096*_t**5*d**4*e**2 + 4*_t*b + 4*_t*d*e)/e)))
```

3.5.7 Maxima [F]

$$\int \frac{d+ex^4}{d^2+bx^4+e^2x^8}\,dx = \int \frac{ex^4+d}{e^2x^8+bx^4+d^2}\,dx$$

input
```
integrate((e*x^4+d)/(e^2*x^8+b*x^4+d^2),x, algorithm="maxima")
```

output
```
integrate((e*x^4 + d)/(e^2*x^8 + b*x^4 + d^2), x)
```

3.5.8 Giac [F]

$$\int \frac{d+ex^4}{d^2+bx^4+e^2x^8}\,dx = \int \frac{ex^4+d}{e^2x^8+bx^4+d^2}\,dx$$

input
```
integrate((e*x^4+d)/(e^2*x^8+b*x^4+d^2),x, algorithm="giac")
```

output
```
integrate((e*x^4 + d)/(e^2*x^8 + b*x^4 + d^2), x)
```

3.5.9 Mupad [B] (verification not implemented)

Time = 9.72 (sec) , antiderivative size = 10409, normalized size of antiderivative = 13.16

$$\int \frac{d+ex^4}{d^2+bx^4+e^2x^8}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^4)/(b*x^4 + d^2 + e^2*x^8),x)
```

output

```
2*atan(((x*(32*b*d^5*e^13 - 4*b^4*d^2*e^10 + 24*b^3*d^3*e^11 - 48*b^2*d^4*
e^12) + (-(b^3 + ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d
*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e
^2)))^(1/4)*((x*(65536*d^9*e^15 - 32768*b*d^8*e^14 + 1024*b^7*d^2*e^8 - 20
48*b^6*d^3*e^9 - 10240*b^5*d^4*e^10 + 20480*b^4*d^5*e^11 + 32768*b^3*d^6*e
^12 - 65536*b^2*d^7*e^13) - (-(b^3 + ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4
*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^
5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*(262144*d^10*e^15 - 262144*b*d^9*e^14 + 40
96*b^7*d^3*e^8 - 4096*b^6*d^4*e^9 - 49152*b^5*d^5*e^10 + 49152*b^4*d^6*e^1
1 + 196608*b^3*d^7*e^12 - 196608*b^2*d^8*e^13)*1i)*(-(b^3 + ((b - 2*d*e)*(
b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4
+ 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(3/4)*1i - 256*d^7*e^14 +
 256*b*d^6*e^13 + 16*b^4*d^3*e^10 - 64*b^3*d^4*e^11)*1i)*(-(b^3 + ((b - 2*
d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^
6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4) + (x*(32*b*d^
5*e^13 - 4*b^4*d^2*e^10 + 24*b^3*d^3*e^11 - 48*b^2*d^4*e^12) + (-(b^3 + ((
b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 +
 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*((x*(65
536*d^9*e^15 - 32768*b*d^8*e^14 + 1024*b^7*d^2*e^8 - 2048*b^6*d^3*e^9 - 10
240*b^5*d^4*e^10 + 20480*b^4*d^5*e^11 + 32768*b^3*d^6*e^12 - 65536*b^2*...
```

3.5. $\int \frac{d+ex^4}{d^2+bx^4+e^2x^8}\,dx$

3.6 $\int \frac{d+ex^4}{d^2+fx^4+e^2x^8}\,dx$

3.6.1 Optimal result

Integrand size = 26, antiderivative size = 791

$$\begin{aligned}\int \frac{d+ex^4}{d^2+fx^4+e^2x^8}\,dx = &-\frac{\arctan\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}-2\sqrt{e}x}{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}}-\frac{\arctan\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}-2\sqrt{e}x}{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}\\
&+\frac{\arctan\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}+2\sqrt{e}x}{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}}+\frac{\arctan\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}+2\sqrt{e}x}{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}\\
&-\frac{\log\left(\sqrt{d}-\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}\,x+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}\\
&+\frac{\log\left(\sqrt{d}+\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}\,x+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}\\
&-\frac{\log\left(\sqrt{d}-\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}\,x+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}}\\
&+\frac{\log\left(\sqrt{d}+\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}\,x+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}}\end{aligned}$$

output
```
-1/4*arctan((-2*x*e^(1/2)+(2*d^(1/2)*e^(1/2)+(2*d*e-f)^(1/2))^(1/2))/(2*d^
(1/2)*e^(1/2)-(2*d*e-f)^(1/2))^(1/2))/d^(1/2)/(2*d^(1/2)*e^(1/2)-(2*d*e-f)
^(1/2))^(1/2)+1/4*arctan((2*x*e^(1/2)+(2*d^(1/2)*e^(1/2)+(2*d*e-f)^(1/2))^
(1/2))/(2*d^(1/2)*e^(1/2)-(2*d*e-f)^(1/2))^(1/2))/d^(1/2)/(2*d^(1/2)*e^(1/
2)-(2*d*e-f)^(1/2))^(1/2)-1/8*ln(d^(1/2)+x^2*e^(1/2)-x*(2*d^(1/2)*e^(1/2)-
(2*d*e-f)^(1/2))^(1/2))/d^(1/2)/(2*d^(1/2)*e^(1/2)-(2*d*e-f)^(1/2))^(1/2)+
1/8*ln(d^(1/2)+x^2*e^(1/2)+x*(2*d^(1/2)*e^(1/2)-(2*d*e-f)^(1/2))^(1/2))/d^
(1/2)/(2*d^(1/2)*e^(1/2)-(2*d*e-f)^(1/2))^(1/2)-1/4*arctan((-2*x*e^(1/2)+(
2*d^(1/2)*e^(1/2)-(2*d*e-f)^(1/2))^(1/2))/(2*d^(1/2)*e^(1/2)+(2*d*e-f)^(1/
2))^(1/2))/d^(1/2)/(2*d^(1/2)*e^(1/2)+(2*d*e-f)^(1/2))^(1/2)+1/4*arctan((2
*x*e^(1/2)+(2*d^(1/2)*e^(1/2)-(2*d*e-f)^(1/2))^(1/2))/(2*d^(1/2)*e^(1/2)+(
2*d*e-f)^(1/2))^(1/2))/d^(1/2)/(2*d^(1/2)*e^(1/2)+(2*d*e-f)^(1/2))^(1/2)-1
/8*ln(d^(1/2)+x^2*e^(1/2)-x*(2*d^(1/2)*e^(1/2)+(2*d*e-f)^(1/2))^(1/2))/d^(
1/2)/(2*d^(1/2)*e^(1/2)+(2*d*e-f)^(1/2))^(1/2)+1/8*ln(d^(1/2)+x^2*e^(1/2)+
x*(2*d^(1/2)*e^(1/2)+(2*d*e-f)^(1/2))^(1/2))/d^(1/2)/(2*d^(1/2)*e^(1/2)+(2
*d*e-f)^(1/2))^(1/2)
```

3.6.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.08

$$\int \frac{d+ex^4}{d^2+fx^4+e^2x^8}\,dx = \frac{1}{4}\text{RootSum}\left[d^2+f\#1^4+e^2\#1^8\&,\frac{d\log(x-\#1)+e\log(x-\#1)\#1^4}{f\#1^3+2e^2\#1^7}\&\right]$$

input
```
Integrate[(d + e*x^4)/(d^2 + f*x^4 + e^2*x^8),x]
```

output
```
RootSum[d^2 + f*#1^4 + e^2*#1^8 & , (d*Log[x - #1] + e*Log[x - #1]*#1^4)/(
f*#1^3 + 2*e^2*#1^7) & ]/4
```

3.6.3 Rubi [A] (verified)

Time = 1.31 (sec) , antiderivative size = 1001, normalized size of antiderivative = 1.27, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {1749, 1407, 27, 1142, 25, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^4}{d^2+e^2x^8+fx^4}\,dx$$

$$\Big\downarrow \text{1749}$$

$$\frac{\int \frac{1}{x^4-\frac{\sqrt{2de-f}x^2}{e}+\frac{d}{e}}dx}{2e}+\frac{\int \frac{1}{x^4+\frac{\sqrt{2de-f}x^2}{e}+\frac{d}{e}}dx}{2e}$$

$$\Big\downarrow \text{1407}$$

$$\frac{\frac{e\int \frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}-\sqrt{e}x}{\sqrt{e}\left(x^2-\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}x}{\sqrt{e}}+\frac{\sqrt{d}}{\sqrt{e}}\right)}dx}{2\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}+\frac{e\int \frac{\sqrt{e}x+\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}{\sqrt{e}\left(x^2+\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}x}{\sqrt{e}}+\frac{\sqrt{d}}{\sqrt{e}}\right)}dx}{2\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}}{2e}+\frac{\frac{e\int \frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}-\sqrt{e}x}{\sqrt{e}\left(x^2-\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}x}{\sqrt{e}}+\frac{\sqrt{d}}{\sqrt{e}}\right)}dx}{2\sqrt{d}\sqrt{\sqrt{2de-f}+2\sqrt{d}\sqrt{e}}}+\frac{e\int \frac{\sqrt{e}x+\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}}{\sqrt{e}\left(x^2+\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}x}{\sqrt{e}}+\frac{\sqrt{d}}{\sqrt{e}}\right)}dx}{2\sqrt{d}\sqrt{\sqrt{2de-f}+2\sqrt{d}\sqrt{e}}}}{2e}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\sqrt{e}\int \frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}-\sqrt{e}x}{x^2-\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}x}{\sqrt{e}}+\frac{\sqrt{d}}{\sqrt{e}}}dx}{2\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}+\frac{\sqrt{e}\int \frac{\sqrt{e}x+\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}{x^2+\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}x}{\sqrt{e}}+\frac{\sqrt{d}}{\sqrt{e}}}dx}{2\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}}{2e}+\frac{\frac{\sqrt{e}\int \frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}-\sqrt{e}x}{x^2-\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}x}{\sqrt{e}}+\frac{\sqrt{d}}{\sqrt{e}}}dx}{2\sqrt{d}\sqrt{\sqrt{2de-f}+2\sqrt{d}\sqrt{e}}}+\frac{\sqrt{e}\int \frac{\sqrt{e}x+\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}}{x^2+\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}x}{\sqrt{e}}+\frac{\sqrt{d}}{\sqrt{e}}}dx}{2\sqrt{d}\sqrt{\sqrt{2de-f}+2\sqrt{d}\sqrt{e}}}}{2e}$$

$$\Big\downarrow \text{1142}$$

$$\frac{\frac{\sqrt{e}\left(\frac{1}{2}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}\int \frac{1}{x^2-\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}\,x}{\sqrt{e}}+\frac{\sqrt{d}}{\sqrt{e}}}dx-\frac{1}{2}\sqrt{e}\int -\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}-2\sqrt{e}x}{\sqrt{e}\left(x^2-\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}\,x}{\sqrt{e}}+\frac{\sqrt{d}}{\sqrt{e}}\right)}dx\right)}{2\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}+\frac{\sqrt{e}\left(\frac{1}{2}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}\int \cdots\right)}{\cdots}}{2e}$$

$$\frac{\frac{\sqrt{e}\left(\frac{1}{2}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}\int \frac{1}{x^2-\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}\,x}{\sqrt{e}}+\frac{\sqrt{d}}{\sqrt{e}}}dx-\frac{1}{2}\sqrt{e}\int -\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}-2\sqrt{e}x}{\sqrt{e}\left(x^2-\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}\,x}{\sqrt{e}}+\frac{\sqrt{d}}{\sqrt{e}}\right)}dx\right)}{2\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}}+\frac{\sqrt{e}\left(\frac{1}{2}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}\int \cdots\right)}{\cdots}}{2e}$$

$\big\downarrow$ 25

$$\frac{\frac{\sqrt{e}\left(\frac{1}{2}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}\int \frac{1}{x^2-\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}\,x}{\sqrt{e}}+\frac{\sqrt{d}}{\sqrt{e}}}dx+\frac{1}{2}\sqrt{e}\int \frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}-2\sqrt{e}x}{\sqrt{e}\left(x^2-\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}\,x}{\sqrt{e}}+\frac{\sqrt{d}}{\sqrt{e}}\right)}dx\right)}{2\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}+\frac{\sqrt{e}\left(\frac{1}{2}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}\int \frac{\cdots}{x^2 \cdots}\cdots\right)}{\cdots}}{2e}$$

$$\frac{\frac{\sqrt{e}\left(\frac{1}{2}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}\int \frac{1}{x^2-\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}\,x}{\sqrt{e}}+\frac{\sqrt{d}}{\sqrt{e}}}dx+\frac{1}{2}\sqrt{e}\int \frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}-2\sqrt{e}x}{\sqrt{e}\left(x^2-\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}\,x}{\sqrt{e}}+\frac{\sqrt{d}}{\sqrt{e}}\right)}dx\right)}{2\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}}+\frac{\sqrt{e}\left(\frac{1}{2}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}\int \frac{\cdots}{x^2 \cdots}\cdots\right)}{\cdots}}{2e}$$

$\big\downarrow$ 27

$$\frac{\frac{\sqrt{e}\left(\frac{1}{2}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}\int \frac{1}{x^2-\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}\,x}{\sqrt{e}}+\frac{\sqrt{d}}{\sqrt{e}}}dx+\frac{1}{2}\int \frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}-2\sqrt{e}x}{x^2-\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}\,x}{\sqrt{e}}+\frac{\sqrt{d}}{\sqrt{e}}}dx\right)}{2\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}+\frac{\sqrt{e}\left(\frac{1}{2}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}\int \frac{\cdots}{x^2+\frac{\sqrt{2\sqrt{d}\sqrt{e}\cdots}}{\cdots}}\cdots\right)}{2\sqrt{d}\cdots}}{2e}$$

$$\frac{\frac{\sqrt{e}\left(\frac{1}{2}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}\int \frac{1}{x^2-\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}\,x}{\sqrt{e}}+\frac{\sqrt{d}}{\sqrt{e}}}dx+\frac{1}{2}\int \frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}-2\sqrt{e}x}{x^2-\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}\,x}{\sqrt{e}}+\frac{\sqrt{d}}{\sqrt{e}}}dx\right)}{2\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}}+\frac{\sqrt{e}\left(\frac{1}{2}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}\int \frac{\cdots}{x^2+\frac{\sqrt{2\sqrt{d}\sqrt{e}\cdots}}{\cdots}}\cdots\right)}{2\sqrt{d}\cdots}}{2e}$$

$\big\downarrow$ 1083

3.6. $\int \frac{d+ex^4}{d^2+fx^4+e^2x^8}\,dx$

$$\frac{\frac{\sqrt{e}\left(\frac{1}{2}\int \frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}-2\sqrt{e}x}{x^2-\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}x}{\sqrt{e}}+\frac{\sqrt{d}}{\sqrt{e}}}dx-\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}\int \frac{1}{-\left(2x-\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}{\sqrt{e}}\right)^2-\frac{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}{e}}d\left(2x-\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}{\sqrt{e}}\right)\right)}{2\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}+}{2e}$$

$$\frac{\frac{\sqrt{e}\left(\frac{1}{2}\int \frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}-2\sqrt{e}x}{x^2-\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}x}{\sqrt{e}}+\frac{\sqrt{d}}{\sqrt{e}}}dx-\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}\int \frac{1}{-\left(2x-\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}}{\sqrt{e}}\right)^2-\frac{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}{e}}d\left(2x-\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}}{\sqrt{e}}\right)\right)}{2\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}}+}{2e}$$

$$\Big\downarrow \; 217$$

$$\frac{\frac{\sqrt{e}\left(\frac{\sqrt{e}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}\arctan\left(\frac{\sqrt{e}\left(2x-\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}{\sqrt{e}}\right)}{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}}\right)}{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}}+\frac{1}{2}\int \frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}-2\sqrt{e}x}{x^2-\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}x}{\sqrt{e}}+\frac{\sqrt{d}}{\sqrt{e}}}dx\right)}{2\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}+}{2e}$$

$$\frac{\frac{\sqrt{e}\left(\frac{\sqrt{e}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}\arctan\left(\frac{\sqrt{e}\left(2x-\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}}{\sqrt{e}}\right)}{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}\right)}{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}+\frac{1}{2}\int \frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}-2\sqrt{e}x}{x^2-\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}x}{\sqrt{e}}+\frac{\sqrt{d}}{\sqrt{e}}}dx\right)}{2\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}}+}{2e}$$

$$\Big\downarrow \; 1103$$

3.6. $\int \frac{d+ex^4}{d^2+fx^4+e^2x^8}\,dx$

$$\frac{\frac{\sqrt{e}\left(\frac{\sqrt{e}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}\arctan\left(\frac{\sqrt{e}\left(2x-\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}{\sqrt{e}}\right)}{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}}\right)}{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}}-\frac{1}{2}\sqrt{e}\log\left(\sqrt{e}x^2-\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}x+\sqrt{d}\right)\right)}{2\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}+\frac{\sqrt{e}\left(\frac{\sqrt{e}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}\,\mathrm{arc}\cdots}{\sqrt{2\cdots}}\cdots\right.}{\cdots}}{2e}$$

$$\frac{\frac{\sqrt{e}\left(\frac{\sqrt{e}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}\arctan\left(\frac{\sqrt{e}\left(2x-\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}}{\sqrt{e}}\right)}{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}\right)}{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}-\frac{1}{2}\sqrt{e}\log\left(\sqrt{e}x^2-\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}x+\sqrt{d}\right)\right)}{2\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}}+\frac{\sqrt{e}\left(\frac{\sqrt{e}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}\,\mathrm{arc}\cdots}{\sqrt{2\cdots}}\cdots\right.}{\cdots}}{2e}$$

input

```
Int[(d + e*x^4)/(d^2 + f*x^4 + e^2*x^8),x]
```

output

```
((Sqrt[e]*((Sqrt[e]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e - f]]*ArcTan[(Sqrt
[e]*(-(Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e - f]]/Sqrt[e]) + 2*x))/Sqrt[2*S
qrt[d]*Sqrt[e] + Sqrt[2*d*e - f]]])/Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e -
f]] - (Sqrt[e]*Log[Sqrt[d] - Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e - f]]*x +
 Sqrt[e]*x^2])/2))/(2*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e - f]]) +
 (Sqrt[e]*((Sqrt[e]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e - f]]*ArcTan[(Sqrt
[e]*(Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e - f]]/Sqrt[e] + 2*x))/Sqrt[2*Sqrt
[d]*Sqrt[e] + Sqrt[2*d*e - f]]])/Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e - f]]
 + (Sqrt[e]*Log[Sqrt[d] + Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e - f]]*x + Sq
rt[e]*x^2])/2))/(2*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e - f]]))/(2*
e) + ((Sqrt[e]*((Sqrt[e]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e - f]]*ArcTan[
(Sqrt[e]*(-(Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e - f]]/Sqrt[e]) + 2*x))/Sqr
t[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e - f]]])/Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d
*e - f]] - (Sqrt[e]*Log[Sqrt[d] - Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e - f]
]*x + Sqrt[e]*x^2])/2))/(2*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e - f
]]) + (Sqrt[e]*((Sqrt[e]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e - f]]*ArcTan[
(Sqrt[e]*(Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e - f]]/Sqrt[e] + 2*x))/Sqrt[2
*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e - f]]])/Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e
- f]] + (Sqrt[e]*Log[Sqrt[d] + Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e - f]]*x
 + Sqrt[e]*x^2])/2))/(2*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e - f...
```

3.6. $\int \frac{d+ex^4}{d^2+fx^4+e^2x^8}\,dx$

3.6.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1407
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   Int[(r - x)/(q - r*
x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(r + x)/(q + r*x + x^2), x], x]]]
 /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

rule 1749
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e
+ q*x^(n/2) + x^n, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x^(n/2) +
 x^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[2*(d/e) - b/c,
0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

3.6.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.07

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(e^{2}\_Z^{8}+f\_Z^{4}+d^{2}\right)}\frac{\left(\_R^{4}e+d\right)\ln\left(x-\_R\right)}{2\_R^{7}e^{2}+\_R^{3}f}\right)}{4}$	53
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(e^{2}\_Z^{8}+f\_Z^{4}+d^{2}\right)}\frac{\left(\_R^{4}e+d\right)\ln\left(x-\_R\right)}{2\_R^{7}e^{2}+\_R^{3}f}\right)}{4}$	53

input
```
int((e*x^4+d)/(e^2*x^8+f*x^4+d^2),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum((_R^4*e+d)/(2*_R^7*e^2+_R^3*f)*ln(x-_R),_R=RootOf(_Z^8*e^2+_Z^4*f+
d^2))
```

3.6.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2461 vs. 2(581) = 1162.

Time = 0.31 (sec) , antiderivative size = 2461, normalized size of antiderivative = 3.11

$$\int \frac{d+ex^4}{d^2+fx^4+e^2x^8}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^4+d)/(e^2*x^8+f*x^4+d^2),x, algorithm="fricas")
```

output
```
1/4*sqrt(sqrt(1/2)*sqrt(-((4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e -
 f)/(8*d^7*e^3 + 12*d^6*e^2*f + 6*d^5*e*f^2 + d^4*f^3)) + f)/(4*d^4*e^2 +
4*d^3*e*f + d^2*f^2)))*log(e*x + 1/2*(2*d*e - (4*d^4*e^2 + 4*d^3*e*f + d^2
*f^2)*sqrt(-(2*d*e - f)/(8*d^7*e^3 + 12*d^6*e^2*f + 6*d^5*e*f^2 + d^4*f^3)
) + f)*sqrt(sqrt(1/2)*sqrt(-((4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*
e - f)/(8*d^7*e^3 + 12*d^6*e^2*f + 6*d^5*e*f^2 + d^4*f^3)) + f)/(4*d^4*e^2
 + 4*d^3*e*f + d^2*f^2)))) - 1/4*sqrt(sqrt(1/2)*sqrt(-((4*d^4*e^2 + 4*d^3*
e*f + d^2*f^2)*sqrt(-(2*d*e - f)/(8*d^7*e^3 + 12*d^6*e^2*f + 6*d^5*e*f^2 +
 d^4*f^3)) + f)/(4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)))*log(e*x - 1/2*(2*d*e -
 (4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e - f)/(8*d^7*e^3 + 12*d^6*e
^2*f + 6*d^5*e*f^2 + d^4*f^3)) + f)*sqrt(sqrt(1/2)*sqrt(-((4*d^4*e^2 + 4*d
^3*e*f + d^2*f^2)*sqrt(-(2*d*e - f)/(8*d^7*e^3 + 12*d^6*e^2*f + 6*d^5*e*f^
2 + d^4*f^3)) + f)/(4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)))) + 1/4*sqrt(-sqrt(1
/2)*sqrt(-((4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e - f)/(8*d^7*e^3
+ 12*d^6*e^2*f + 6*d^5*e*f^2 + d^4*f^3)) + f)/(4*d^4*e^2 + 4*d^3*e*f + d^2
*f^2)))*log(e*x + 1/2*(2*d*e - (4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)*sqrt(-(2*
d*e - f)/(8*d^7*e^3 + 12*d^6*e^2*f + 6*d^5*e*f^2 + d^4*f^3)) + f)*sqrt(-sq
rt(1/2)*sqrt(-((4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e - f)/(8*d^7*
e^3 + 12*d^6*e^2*f + 6*d^5*e*f^2 + d^4*f^3)) + f)/(4*d^4*e^2 + 4*d^3*e*f +
 d^2*f^2)))) - 1/4*sqrt(-sqrt(1/2)*sqrt(-((4*d^4*e^2 + 4*d^3*e*f + d^2*...
```

3.6.6 Sympy [A] (verification not implemented)

Time = 6.28 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.17

$$\int \frac{d+ex^4}{d^2+fx^4+e^2x^8}\,dx$$
$$= \text{RootSum}\left(t^8\cdot\left(1048576d^6e^4+2097152d^5e^3f+1572864d^4e^2f^2+524288d^3ef^3+65536d^2f^4\right)+t^4\cdot\left(102\right.\right.$$

input
```
integrate((e*x**4+d)/(e**2*x**8+f*x**4+d**2),x)
```

output
```
RootSum(_t**8*(1048576*d**6*e**4 + 2097152*d**5*e**3*f + 1572864*d**4*e**2
*f**2 + 524288*d**3*e*f**3 + 65536*d**2*f**4) + _t**4*(1024*d**2*e**2*f +
1024*d*e*f**2 + 256*f**3) + e**2, Lambda(_t, _t*log(x + (4096*_t**5*d**4*e
**2 + 4096*_t**5*d**3*e*f + 1024*_t**5*d**2*f**2 + 4*_t*d*e + 4*_t*f)/e)))
```

3.6.7 Maxima [F]

$$\int \frac{d+ex^4}{d^2+fx^4+e^2x^8}\,dx = \int \frac{ex^4+d}{e^2x^8+fx^4+d^2}\,dx$$

input
```
integrate((e*x^4+d)/(e^2*x^8+f*x^4+d^2),x, algorithm="maxima")
```

output
```
integrate((e*x^4 + d)/(e^2*x^8 + f*x^4 + d^2), x)
```

3.6.8 Giac [F]

$$\int \frac{d+ex^4}{d^2+fx^4+e^2x^8}\,dx = \int \frac{ex^4+d}{e^2x^8+fx^4+d^2}\,dx$$

input
```
integrate((e*x^4+d)/(e^2*x^8+f*x^4+d^2),x, algorithm="giac")
```

output
```
integrate((e*x^4 + d)/(e^2*x^8 + f*x^4 + d^2), x)
```

3.6.9 Mupad [B] (verification not implemented)

Time = 9.78 (sec) , antiderivative size = 10411, normalized size of antiderivative = 13.16

$$\int \frac{d+ex^4}{d^2+fx^4+e^2x^8}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^4)/(f*x^4 + d^2 + e^2*x^8),x)
```

output
```
2*atan((((-(f^3 + ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*
f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*
f^2)))^(1/4)*((x*(65536*d^9*e^15 - 32768*d^8*e^14*f + 1024*d^2*e^8*f^7 - 2
048*d^3*e^9*f^6 - 10240*d^4*e^10*f^5 + 20480*d^5*e^11*f^4 + 32768*d^6*e^12
*f^3 - 65536*d^7*e^13*f^2) - (-(f^3 + ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) +
4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5
*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*(262144*d^10*e^15 - 262144*d^9*e^14*f + 4
096*d^3*e^8*f^7 - 4096*d^4*e^9*f^6 - 49152*d^5*e^10*f^5 + 49152*d^6*e^11*f
^4 + 196608*d^7*e^12*f^3 - 196608*d^8*e^13*f^2)*1i)*(-(f^3 + ((f - 2*d*e)*
(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4
 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(3/4)*1i - 256*d^7*e^14
+ 256*d^6*e^13*f + 16*d^3*e^10*f^4 - 64*d^4*e^11*f^3)*1i + x*(32*d^5*e^13*
f - 4*d^2*e^10*f^4 + 24*d^3*e^11*f^3 - 48*d^4*e^12*f^2))*(-(f^3 + ((f - 2*
d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^
2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4) + ((-(f^3 + (
(f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e
^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*((x*(6
5536*d^9*e^15 - 32768*d^8*e^14*f + 1024*d^2*e^8*f^7 - 2048*d^3*e^9*f^6 - 1
0240*d^4*e^10*f^5 + 20480*d^5*e^11*f^4 + 32768*d^6*e^12*f^3 - 65536*d^7*e^
13*f^2) + (-(f^3 + ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*...
```

3.6. $\int \frac{d+ex^4}{d^2+fx^4+e^2x^8}\,dx$

3.7 $\int \frac{d+ex^4}{d^2-bx^4+e^2x^8}\,dx$

3.7.1 Optimal result

Integrand size = 27, antiderivative size = 349

$$\int \frac{d+ex^4}{d^2-bx^4+e^2x^8}\,dx = -\frac{\sqrt{e}\arctan\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{\sqrt{b-2de}-\sqrt{b+2de}}}\right)}{\sqrt{2}\sqrt{b-2de}\sqrt{\sqrt{b-2de}-\sqrt{b+2de}}} - \frac{\sqrt{e}\arctan\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{\sqrt{b-2de}+\sqrt{b+2de}}}\right)}{\sqrt{2}\sqrt{b-2de}\sqrt{\sqrt{b-2de}+\sqrt{b+2de}}} - \frac{\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{\sqrt{b-2de}-\sqrt{b+2de}}}\right)}{\sqrt{2}\sqrt{b-2de}\sqrt{\sqrt{b-2de}-\sqrt{b+2de}}} - \frac{\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{\sqrt{b-2de}+\sqrt{b+2de}}}\right)}{\sqrt{2}\sqrt{b-2de}\sqrt{\sqrt{b-2de}+\sqrt{b+2de}}}$$

output
```
-1/2*arctan(x*2^(1/2)*e^(1/2)/((-2*d*e+b)^(1/2)-(2*d*e+b)^(1/2))^(1/2))*e^
(1/2)*2^(1/2)/(-2*d*e+b)^(1/2)/((-2*d*e+b)^(1/2)-(2*d*e+b)^(1/2))^(1/2)-1/
2*arctanh(x*2^(1/2)*e^(1/2)/((-2*d*e+b)^(1/2)-(2*d*e+b)^(1/2))^(1/2))*e^(1
/2)*2^(1/2)/(-2*d*e+b)^(1/2)/((-2*d*e+b)^(1/2)-(2*d*e+b)^(1/2))^(1/2)-1/2*
arctan(x*2^(1/2)*e^(1/2)/((-2*d*e+b)^(1/2)+(2*d*e+b)^(1/2))^(1/2))*e^(1/2)
*2^(1/2)/(-2*d*e+b)^(1/2)/((-2*d*e+b)^(1/2)+(2*d*e+b)^(1/2))^(1/2)-1/2*arc
tanh(x*2^(1/2)*e^(1/2)/((-2*d*e+b)^(1/2)+(2*d*e+b)^(1/2))^(1/2))*e^(1/2)*2
^(1/2)/(-2*d*e+b)^(1/2)/((-2*d*e+b)^(1/2)+(2*d*e+b)^(1/2))^(1/2)
```

3.7.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.20

$$\int \frac{d+ex^4}{d^2-bx^4+e^2x^8}\,dx = \frac{1}{4}\text{RootSum}\left[d^2-b\#1^4+e^2\#1^8\&,\frac{d\log(x-\#1)+e\log(x-\#1)\#1^4}{-b\#1^3+2e^2\#1^7}\&\right]$$

input
```
Integrate[(d + e*x^4)/(d^2 - b*x^4 + e^2*x^8),x]
```

output
```
RootSum[d^2 - b*#1^4 + e^2*#1^8 & , (d*Log[x - #1] + e*Log[x - #1]*#1^4)/(
-(b*#1^3) + 2*e^2*#1^7) & ]/4
```

3.7.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1749, 1406, 216, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^4}{-bx^4+d^2+e^2x^8}\,dx$$

$$\Big\downarrow \; 1749$$

$$\frac{\int \frac{1}{x^4-\frac{\sqrt{b+2de}x^2}{e}+\frac{d}{e}}dx}{2e}+\frac{\int \frac{1}{x^4+\frac{\sqrt{b+2de}x^2}{e}+\frac{d}{e}}dx}{2e}$$

$$\Big\downarrow \; 1406$$

$$\frac{\frac{e\int \frac{1}{x^2-\frac{\sqrt{b-2de}+\sqrt{b+2de}}{2e}}dx}{\sqrt{b-2de}}-\frac{e\int \frac{1}{x^2+\frac{\sqrt{b-2de}-\sqrt{b+2de}}{2e}}dx}{\sqrt{b-2de}}}{2e}+\frac{\frac{e\int \frac{1}{x^2-\frac{\sqrt{b-2de}-\sqrt{b+2de}}{2e}}dx}{\sqrt{b-2de}}-\frac{e\int \frac{1}{x^2+\frac{\sqrt{b-2de}+\sqrt{b+2de}}{2e}}dx}{\sqrt{b-2de}}}{2e}$$

$$\Big\downarrow \; 216$$

$$\frac{\frac{e\int \frac{1}{x^2-\frac{\sqrt{b-2de}-\sqrt{b+2de}}{2e}}dx}{\sqrt{b-2de}}-\frac{\sqrt{2}e^{3/2}\arctan\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{\sqrt{b-2de}+\sqrt{b+2de}}}\right)}{\sqrt{b-2de}\sqrt{\sqrt{b-2de}+\sqrt{b+2de}}}}{2e}+\frac{\frac{e\int \frac{1}{x^2-\frac{\sqrt{b-2de}+\sqrt{b+2de}}{2e}}dx}{\sqrt{b-2de}}-\frac{\sqrt{2}e^{3/2}\arctan\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{\sqrt{b-2de}-\sqrt{b+2de}}}\right)}{\sqrt{b-2de}\sqrt{\sqrt{b-2de}-\sqrt{b+2de}}}}{2e}$$

$$\Big\downarrow \text{ 220}$$

$$\frac{-\frac{\sqrt{2}e^{3/2}\arctan\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{\sqrt{b-2de}+\sqrt{b+2de}}}\right)}{\sqrt{b-2de}\sqrt{\sqrt{b-2de}+\sqrt{b+2de}}}-\frac{\sqrt{2}e^{3/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{\sqrt{b-2de}-\sqrt{b+2de}}}\right)}{\sqrt{b-2de}\sqrt{\sqrt{b-2de}-\sqrt{b+2de}}}}{2e}+\frac{-\frac{\sqrt{2}e^{3/2}\arctan\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{\sqrt{b-2de}-\sqrt{b+2de}}}\right)}{\sqrt{b-2de}\sqrt{\sqrt{b-2de}-\sqrt{b+2de}}}-\frac{\sqrt{2}e^{3/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{\sqrt{b-2de}+\sqrt{b+2de}}}\right)}{\sqrt{b-2de}\sqrt{\sqrt{b-2de}+\sqrt{b+2de}}}}{2e}$$

input
```
Int[(d + e*x^4)/(d^2 - b*x^4 + e^2*x^8),x]
```

output
```
(-((Sqrt[2]*e^(3/2)*ArcTan[(Sqrt[2]*Sqrt[e]*x)/Sqrt[Sqrt[b - 2*d*e] + Sqrt
[b + 2*d*e]]])/(Sqrt[b - 2*d*e]*Sqrt[Sqrt[b - 2*d*e] + Sqrt[b + 2*d*e]]))
- (Sqrt[2]*e^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[e]*x)/Sqrt[Sqrt[b - 2*d*e] - Sqrt
[b + 2*d*e]]])/(Sqrt[b - 2*d*e]*Sqrt[Sqrt[b - 2*d*e] - Sqrt[b + 2*d*e]]))/
(2*e) + (-((Sqrt[2]*e^(3/2)*ArcTan[(Sqrt[2]*Sqrt[e]*x)/Sqrt[Sqrt[b - 2*d*e
] - Sqrt[b + 2*d*e]]])/(Sqrt[b - 2*d*e]*Sqrt[Sqrt[b - 2*d*e] - Sqrt[b + 2*
d*e]])) - (Sqrt[2]*e^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[e]*x)/Sqrt[Sqrt[b - 2*d*e
] + Sqrt[b + 2*d*e]]])/(Sqrt[b - 2*d*e]*Sqrt[Sqrt[b - 2*d*e] + Sqrt[b + 2*
d*e]]))/(2*e)
```

3.7.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

rule 1749
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e
+ q*x^(n/2) + x^n, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x^(n/2) +
 x^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[2*(d/e) - b/c,
0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

3.7.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.16

method	result	size
default	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(e^{2}\_Z^{8}-\_Z^{4}b+d^{2}\right)}\frac{\left(-\_R^{4}e-d\right)\ln\left(x-\_R\right)}{-2\_R^{7}e^{2}+\_R^{3}b}\right)}{4}$	57
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(e^{2}\_Z^{8}-\_Z^{4}b+d^{2}\right)}\frac{\left(-\_R^{4}e-d\right)\ln\left(x-\_R\right)}{-2\_R^{7}e^{2}+\_R^{3}b}\right)}{4}$	57

input
```
int((e*x^4+d)/(e^2*x^8-b*x^4+d^2),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum((-_R^4*e-d)/(-2*_R^7*e^2+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*e^2-_Z^4*
b+d^2))
```

3.7.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2453 vs. 2(261) = 522.

Time = 0.33 (sec) , antiderivative size = 2453, normalized size of antiderivative = 7.03

$$\int \frac{d+ex^4}{d^2-bx^4+e^2x^8}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^4+d)/(e^2*x^8-b*x^4+d^2),x, algorithm="fricas")
```

output
```
1/4*sqrt(sqrt(1/2)*sqrt(((4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e +
b)/(8*d^7*e^3 - 12*b*d^6*e^2 + 6*b^2*d^5*e - b^3*d^4)) + b)/(4*d^4*e^2 - 4
*b*d^3*e + b^2*d^2)))*log(e*x + 1/2*(2*d*e + (4*d^4*e^2 - 4*b*d^3*e + b^2*
d^2)*sqrt(-(2*d*e + b)/(8*d^7*e^3 - 12*b*d^6*e^2 + 6*b^2*d^5*e - b^3*d^4))
 - b)*sqrt(sqrt(1/2)*sqrt(((4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e
+ b)/(8*d^7*e^3 - 12*b*d^6*e^2 + 6*b^2*d^5*e - b^3*d^4)) + b)/(4*d^4*e^2 -
 4*b*d^3*e + b^2*d^2)))) - 1/4*sqrt(sqrt(1/2)*sqrt(((4*d^4*e^2 - 4*b*d^3*e
 + b^2*d^2)*sqrt(-(2*d*e + b)/(8*d^7*e^3 - 12*b*d^6*e^2 + 6*b^2*d^5*e - b^
3*d^4)) + b)/(4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)))*log(e*x - 1/2*(2*d*e + (4
*d^4*e^2 - 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e + b)/(8*d^7*e^3 - 12*b*d^6*e^
2 + 6*b^2*d^5*e - b^3*d^4)) - b)*sqrt(sqrt(1/2)*sqrt(((4*d^4*e^2 - 4*b*d^3
*e + b^2*d^2)*sqrt(-(2*d*e + b)/(8*d^7*e^3 - 12*b*d^6*e^2 + 6*b^2*d^5*e -
b^3*d^4)) + b)/(4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)))) + 1/4*sqrt(-sqrt(1/2)*
sqrt(((4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e + b)/(8*d^7*e^3 - 12*
b*d^6*e^2 + 6*b^2*d^5*e - b^3*d^4)) + b)/(4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)
))*log(e*x + 1/2*(2*d*e + (4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e +
 b)/(8*d^7*e^3 - 12*b*d^6*e^2 + 6*b^2*d^5*e - b^3*d^4)) - b)*sqrt(-sqrt(1/
2)*sqrt(((4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e + b)/(8*d^7*e^3 -
12*b*d^6*e^2 + 6*b^2*d^5*e - b^3*d^4)) + b)/(4*d^4*e^2 - 4*b*d^3*e + b^2*d
^2)))) - 1/4*sqrt(-sqrt(1/2)*sqrt(((4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)*sq...
```

3.7.6 Sympy [A] (verification not implemented)

Time = 19.11 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.39

$$\int \frac{d+ex^4}{d^2-bx^4+e^2x^8}\,dx$$
$$= \operatorname{RootSum}\left(t^8 \cdot \left(65536b^4d^2 - 524288b^3d^3e + 1572864b^2d^4e^2 - 2097152bd^5e^3 + 1048576d^6e^4\right) + t^4\left(-256b^3\right.\right.$$

input
```
integrate((e*x**4+d)/(e**2*x**8-b*x**4+d**2),x)
```

output
```
RootSum(_t**8*(65536*b**4*d**2 - 524288*b**3*d**3*e + 1572864*b**2*d**4*e*
*2 - 2097152*b*d**5*e**3 + 1048576*d**6*e**4) + _t**4*(-256*b**3 + 1024*b*
*2*d*e - 1024*b*d**2*e**2) + e**2, Lambda(_t, _t*log(x + (1024*_t**5*b**2*
d**2 - 4096*_t**5*b*d**3*e + 4096*_t**5*d**4*e**2 - 4*_t*b + 4*_t*d*e)/e))
)
```

3.7.7 Maxima [F]

$$\int \frac{d+ex^4}{d^2-bx^4+e^2x^8}\,dx=\int \frac{ex^4+d}{e^2x^8-bx^4+d^2}\,dx$$

input
```
integrate((e*x^4+d)/(e^2*x^8-b*x^4+d^2),x, algorithm="maxima")
```

output
```
integrate((e*x^4 + d)/(e^2*x^8 - b*x^4 + d^2), x)
```

3.7.8 Giac [F]

$$\int \frac{d+ex^4}{d^2-bx^4+e^2x^8}\,dx=\int \frac{ex^4+d}{e^2x^8-bx^4+d^2}\,dx$$

input
```
integrate((e*x^4+d)/(e^2*x^8-b*x^4+d^2),x, algorithm="giac")
```

output
```
integrate((e*x^4 + d)/(e^2*x^8 - b*x^4 + d^2), x)
```

3.7.9 Mupad [B] (verification not implemented)

Time = 9.78 (sec) , antiderivative size = 10337, normalized size of antiderivative = 29.62

$$\int \frac{d+ex^4}{d^2-bx^4+e^2x^8}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^4)/(d^2 - b*x^4 + e^2*x^8),x)
```

output
```
2*atan(((x*(32*b*d^5*e^13 + 4*b^4*d^2*e^10 + 24*b^3*d^3*e^11 + 48*b^2*d^4*
e^12) - ((b^3 + ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*
e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^
2)))^(1/4)*((x*(65536*d^9*e^15 + 32768*b*d^8*e^14 - 1024*b^7*d^2*e^8 - 204
8*b^6*d^3*e^9 + 10240*b^5*d^4*e^10 + 20480*b^4*d^5*e^11 - 32768*b^3*d^6*e^
12 - 65536*b^2*d^7*e^13) - ((b^3 + ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b
*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*
e^3 + 24*b^2*d^4*e^2)))^(1/4)*(262144*d^10*e^15 + 262144*b*d^9*e^14 - 4096
*b^7*d^3*e^8 - 4096*b^6*d^4*e^9 + 49152*b^5*d^5*e^10 + 49152*b^4*d^6*e^11
- 196608*b^3*d^7*e^12 - 196608*b^2*d^8*e^13)*1i)*((b^3 + ((b - 2*d*e)^5*(b
 + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8
*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(3/4)*1i - 256*d^7*e^14 - 25
6*b*d^6*e^13 + 16*b^4*d^3*e^10 + 64*b^3*d^4*e^11)*1i)*((b^3 + ((b - 2*d*e)
^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^
4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4) + (x*(32*b*d^5*e^
13 + 4*b^4*d^2*e^10 + 24*b^3*d^3*e^11 + 48*b^2*d^4*e^12) - ((b^3 + ((b - 2
*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d
^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*((x*(65536*d
^9*e^15 + 32768*b*d^8*e^14 - 1024*b^7*d^2*e^8 - 2048*b^6*d^3*e^9 + 10240*b
^5*d^4*e^10 + 20480*b^4*d^5*e^11 - 32768*b^3*d^6*e^12 - 65536*b^2*d^7*e...
```

3.8 $\int \frac{d+ex^4}{d^2-fx^4+e^2x^8}\,dx$

3.8.1 Optimal result

Integrand size = 27, antiderivative size = 751

$$\begin{aligned}\int \frac{d+ex^4}{d^2-fx^4+e^2x^8}\,dx = &-\frac{\arctan\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}-2\sqrt{e}x}{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}}-\frac{\arctan\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}-2\sqrt{e}x}{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}\\
&+\frac{\arctan\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}+2\sqrt{e}x}{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}}+\frac{\arctan\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}+2\sqrt{e}x}{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}\\
&-\frac{\log\left(\sqrt{d}-\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}x+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}\\
&+\frac{\log\left(\sqrt{d}+\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}x+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}\\
&-\frac{\log\left(\sqrt{d}-\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}x+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}}\\
&+\frac{\log\left(\sqrt{d}+\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}x+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}}\end{aligned}$$

output
```
-1/4*arctan((-2*x*e^(1/2)+(2*d^(1/2)*e^(1/2)+(2*d*e+f)^(1/2))^(1/2))/(2*d^
(1/2)*e^(1/2)-(2*d*e+f)^(1/2))^(1/2))/d^(1/2)/(2*d^(1/2)*e^(1/2)-(2*d*e+f)
^(1/2))^(1/2)+1/4*arctan((2*x*e^(1/2)+(2*d^(1/2)*e^(1/2)+(2*d*e+f)^(1/2))^
(1/2))/(2*d^(1/2)*e^(1/2)-(2*d*e+f)^(1/2))^(1/2))/d^(1/2)/(2*d^(1/2)*e^(1/
2)-(2*d*e+f)^(1/2))^(1/2)-1/8*ln(d^(1/2)+x^2*e^(1/2)-x*(2*d^(1/2)*e^(1/2)-
(2*d*e+f)^(1/2))^(1/2))/d^(1/2)/(2*d^(1/2)*e^(1/2)-(2*d*e+f)^(1/2))^(1/2)+
1/8*ln(d^(1/2)+x^2*e^(1/2)+x*(2*d^(1/2)*e^(1/2)-(2*d*e+f)^(1/2))^(1/2))/d^
(1/2)/(2*d^(1/2)*e^(1/2)-(2*d*e+f)^(1/2))^(1/2)-1/4*arctan((-2*x*e^(1/2)+(
2*d^(1/2)*e^(1/2)-(2*d*e+f)^(1/2))^(1/2))/(2*d^(1/2)*e^(1/2)+(2*d*e+f)^(1/
2))^(1/2))/d^(1/2)/(2*d^(1/2)*e^(1/2)+(2*d*e+f)^(1/2))^(1/2)+1/4*arctan((2
*x*e^(1/2)+(2*d^(1/2)*e^(1/2)-(2*d*e+f)^(1/2))^(1/2))/(2*d^(1/2)*e^(1/2)+(
2*d*e+f)^(1/2))^(1/2))/d^(1/2)/(2*d^(1/2)*e^(1/2)+(2*d*e+f)^(1/2))^(1/2)-1
/8*ln(d^(1/2)+x^2*e^(1/2)-x*(2*d^(1/2)*e^(1/2)+(2*d*e+f)^(1/2))^(1/2))/d^(
1/2)/(2*d^(1/2)*e^(1/2)+(2*d*e+f)^(1/2))^(1/2)+1/8*ln(d^(1/2)+x^2*e^(1/2)+
x*(2*d^(1/2)*e^(1/2)+(2*d*e+f)^(1/2))^(1/2))/d^(1/2)/(2*d^(1/2)*e^(1/2)+(2
*d*e+f)^(1/2))^(1/2)
```

3.8.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.09

$$\int \frac{d+ex^4}{d^2-fx^4+e^2x^8}\,dx = \frac{1}{4}\text{RootSum}\left[d^2-f\#1^4 + e^2\#1^8\&,\frac{d\log(x-\#1)+e\log(x-\#1)\#1^4}{-f\#1^3+2e^2\#1^7}\&\right]$$

input
```
Integrate[(d + e*x^4)/(d^2 - f*x^4 + e^2*x^8),x]
```

output
```
RootSum[d^2 - f*#1^4 + e^2*#1^8 & , (d*Log[x - #1] + e*Log[x - #1]*#1^4)/(
-(f*#1^3) + 2*e^2*#1^7) & ]/4
```

3.8.3 Rubi [A] (verified)

Time = 1.35 (sec) , antiderivative size = 953, normalized size of antiderivative = 1.27, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1749, 1407, 27, 1142, 25, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^4}{d^2+e^2x^8-fx^4}dx$$

$$\Big\downarrow \text{ 1749}$$

$$\frac{\int \frac{1}{x^4-\frac{\sqrt{2de+f}x^2}{e}+\frac{d}{e}}dx}{2e}+\frac{\int \frac{1}{x^4+\frac{\sqrt{2de+f}x^2}{e}+\frac{d}{e}}dx}{2e}$$

$$\Big\downarrow \text{ 1407}$$

$$\frac{\frac{e\int \frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}-\sqrt{e}x}{\sqrt{e}\left(x^2-\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}x}{\sqrt{e}}+\frac{\sqrt{d}}{\sqrt{e}}\right)}dx}{2\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}+\frac{e\int \frac{\sqrt{e}x+\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}{\sqrt{e}\left(x^2+\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}x}{\sqrt{e}}+\frac{\sqrt{d}}{\sqrt{e}}\right)}dx}{2\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}}{2e}+\frac{\frac{e\int \frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}-\sqrt{e}x}{\sqrt{e}\left(x^2-\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}x}{\sqrt{e}}+\frac{\sqrt{d}}{\sqrt{e}}\right)}dx}{2\sqrt{d}\sqrt{\sqrt{2de+f}+2\sqrt{d}\sqrt{e}}}+\frac{e\int \frac{\sqrt{e}x+\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}}{\sqrt{e}\left(x^2+\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}x}{\sqrt{e}}+\frac{\sqrt{d}}{\sqrt{e}}\right)}dx}{2\sqrt{d}\sqrt{\sqrt{2de+f}+2\sqrt{d}\sqrt{e}}}}{2e}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\sqrt{e}\int \frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}-\sqrt{e}x}{x^2-\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}x}{\sqrt{e}}+\frac{\sqrt{d}}{\sqrt{e}}}dx}{2\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}+\frac{\sqrt{e}\int \frac{\sqrt{e}x+\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}{x^2+\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}x}{\sqrt{e}}+\frac{\sqrt{d}}{\sqrt{e}}}dx}{2\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}}{2e}+\frac{\frac{\sqrt{e}\int \frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}-\sqrt{e}x}{x^2-\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}x}{\sqrt{e}}+\frac{\sqrt{d}}{\sqrt{e}}}dx}{2\sqrt{d}\sqrt{\sqrt{2de+f}+2\sqrt{d}\sqrt{e}}}+\frac{\sqrt{e}\int \frac{\sqrt{e}x+\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}}{x^2+\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}x}{\sqrt{e}}+\frac{\sqrt{d}}{\sqrt{e}}}dx}{2\sqrt{d}\sqrt{\sqrt{2de+f}+2\sqrt{d}\sqrt{e}}}}{2e}$$

$$\Big\downarrow \text{ 1142}$$

$$\frac{\frac{\sqrt{e}\left(\frac{1}{2}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}\int\frac{1}{x^2-\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}x}{\sqrt{e}}+\frac{\sqrt{d}}{\sqrt{e}}}dx-\frac{1}{2}\sqrt{e}\int-\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}-2\sqrt{e}x}{\sqrt{e}\left(x^2-\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}x}{\sqrt{e}}+\frac{\sqrt{d}}{\sqrt{e}}\right)}dx\right)}{2\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}+\frac{\sqrt{e}\left(\frac{1}{2}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}\int\cdots\right.}{\cdots}}{2e}$$

$$\frac{\frac{\sqrt{e}\left(\frac{1}{2}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}\int\frac{1}{x^2-\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}x}{\sqrt{e}}+\frac{\sqrt{d}}{\sqrt{e}}}dx-\frac{1}{2}\sqrt{e}\int-\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}-2\sqrt{e}x}{\sqrt{e}\left(x^2-\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}x}{\sqrt{e}}+\frac{\sqrt{d}}{\sqrt{e}}\right)}dx\right)}{2\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}}+\frac{\sqrt{e}\left(\frac{1}{2}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}\int\cdots\right.}{\cdots}}{2e}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\sqrt{e}\left(\frac{1}{2}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}\int\frac{1}{x^2-\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}x}{\sqrt{e}}+\frac{\sqrt{d}}{\sqrt{e}}}dx+\frac{1}{2}\sqrt{e}\int\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}-2\sqrt{e}x}{\sqrt{e}\left(x^2-\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}x}{\sqrt{e}}+\frac{\sqrt{d}}{\sqrt{e}}\right)}dx\right)}{2\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}+\frac{\sqrt{e}\left(\frac{1}{2}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}\int\frac{\cdots}{x^2-\cdots}\right.}{\cdots}}{2e}$$

$$\frac{\frac{\sqrt{e}\left(\frac{1}{2}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}\int\frac{1}{x^2-\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}x}{\sqrt{e}}+\frac{\sqrt{d}}{\sqrt{e}}}dx+\frac{1}{2}\sqrt{e}\int\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}-2\sqrt{e}x}{\sqrt{e}\left(x^2-\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}x}{\sqrt{e}}+\frac{\sqrt{d}}{\sqrt{e}}\right)}dx\right)}{2\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}}+\frac{\sqrt{e}\left(\frac{1}{2}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}\int\frac{\cdots}{x^2+\cdots}\right.}{\cdots}}{2e}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\sqrt{e}\left(\frac{1}{2}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}\int\frac{1}{x^2-\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}x}{\sqrt{e}}+\frac{\sqrt{d}}{\sqrt{e}}}dx+\frac{1}{2}\int\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}-2\sqrt{e}x}{x^2-\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}x}{\sqrt{e}}+\frac{\sqrt{d}}{\sqrt{e}}}dx\right)}{2\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}+\frac{\sqrt{e}\left(\frac{1}{2}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}\int\frac{\cdots}{x^2+\frac{\sqrt{2\sqrt{d}\sqrt{e}\cdots}}{\cdots}}\right.}{2\sqrt{d}\cdots}}{2e}$$

$$\frac{\frac{\sqrt{e}\left(\frac{1}{2}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}\int\frac{1}{x^2-\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}x}{\sqrt{e}}+\frac{\sqrt{d}}{\sqrt{e}}}dx+\frac{1}{2}\int\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}-2\sqrt{e}x}{x^2-\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}x}{\sqrt{e}}+\frac{\sqrt{d}}{\sqrt{e}}}dx\right)}{2\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}}+\frac{\sqrt{e}\left(\frac{1}{2}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}\int\frac{\cdots}{x^2+\frac{\sqrt{2\sqrt{d}\sqrt{e}\cdots}}{\cdots}}\right.}{2\sqrt{d}\cdots}}{2e}$$

$$\Big\downarrow \; 1083$$

3.8. $\int \frac{d+ex^4}{d^2-fx^4+e^2x^8}\,dx$

$$\frac{\frac{\sqrt{e}\left(\frac{1}{2}\int \frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}-2\sqrt{e}x}{x^2-\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}x}{\sqrt{e}}+\frac{\sqrt{d}}{\sqrt{e}}}dx-\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}\int \frac{1}{-\left(2x-\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}{\sqrt{e}}\right)^2-\frac{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}{e}}d\left(2x-\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}{\sqrt{e}}\right)\right)}{2\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}+\cdots}{2e}$$

$$\frac{\frac{\sqrt{e}\left(\frac{1}{2}\int \frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}-2\sqrt{e}x}{x^2-\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}x}{\sqrt{e}}+\frac{\sqrt{d}}{\sqrt{e}}}dx-\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}\int \frac{1}{-\left(2x-\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}}{\sqrt{e}}\right)^2-\frac{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}{e}}d\left(2x-\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}}{\sqrt{e}}\right)\right)}{2\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}}+\cdots}{2e}$$

$$\Big\downarrow \; 217$$

$$\frac{\frac{\sqrt{e}\left(\frac{\sqrt{e}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}\arctan\left(\frac{\sqrt{e}\left(2x-\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}{\sqrt{e}}\right)}{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}}\right)}{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}}+\frac{1}{2}\int \frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}-2\sqrt{e}x}{x^2-\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}x}{\sqrt{e}}+\frac{\sqrt{d}}{\sqrt{e}}}dx\right)}{2\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}+\cdots}{2e}$$

$$\frac{\frac{\sqrt{e}\left(\frac{\sqrt{e}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}\arctan\left(\frac{\sqrt{e}\left(2x-\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}}{\sqrt{e}}\right)}{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}\right)}{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}+\frac{1}{2}\int \frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}-2\sqrt{e}x}{x^2-\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}x}{\sqrt{e}}+\frac{\sqrt{d}}{\sqrt{e}}}dx\right)}{2\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}}+\cdots}{2e}$$

$$\Big\downarrow \; 1103$$

3.8. $\int \frac{d+ex^4}{d^2-fx^4+e^2x^8}\,dx$

$$\frac{\frac{\sqrt{e}\left(\frac{\sqrt{e}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}\arctan\left(\frac{\sqrt{e}\left(2x-\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}{\sqrt{e}}\right)}{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}}\right)}{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}}-\frac{1}{2}\sqrt{e}\log\left(\sqrt{e}x^2-\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}\,x+\sqrt{d}\right)\right)}{2\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}+\frac{\sqrt{e}\left(\frac{\sqrt{e}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}\,\mathrm{arc}\ldots}{\sqrt{2\ldots}}\right.}{\ldots}}{2e}$$

$$\frac{\frac{\sqrt{e}\left(\frac{\sqrt{e}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}\arctan\left(\frac{\sqrt{e}\left(2x-\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}}{\sqrt{e}}\right)}{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}\right)}{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}-\frac{1}{2}\sqrt{e}\log\left(\sqrt{e}x^2-\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}\,x+\sqrt{d}\right)\right)}{2\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}}+\frac{\sqrt{e}\left(\frac{\sqrt{e}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}\,\mathrm{arc}\ldots}{\sqrt{2\ldots}}\right.}{\ldots}}{2e}$$

input
```
Int[(d + e*x^4)/(d^2 - f*x^4 + e^2*x^8),x]
```

output
```
((Sqrt[e]*((Sqrt[e]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e + f]]*ArcTan[(Sqrt
[e]*(-(Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e + f]]/Sqrt[e]) + 2*x))/Sqrt[2*S
qrt[d]*Sqrt[e] + Sqrt[2*d*e + f]]])/Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e +
f]] - (Sqrt[e]*Log[Sqrt[d] - Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e + f]]*x +
 Sqrt[e]*x^2])/2))/(2*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e + f]]) +
 (Sqrt[e]*((Sqrt[e]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e + f]]*ArcTan[(Sqrt
[e]*(Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e + f]]/Sqrt[e] + 2*x))/Sqrt[2*Sqrt
[d]*Sqrt[e] + Sqrt[2*d*e + f]]])/Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e + f]]
 + (Sqrt[e]*Log[Sqrt[d] + Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e + f]]*x + Sq
rt[e]*x^2])/2))/(2*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e + f]]))/(2*
e) + ((Sqrt[e]*((Sqrt[e]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e + f]]*ArcTan[
(Sqrt[e]*(-(Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e + f]]/Sqrt[e]) + 2*x))/Sqr
t[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e + f]]])/Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d
*e + f]] - (Sqrt[e]*Log[Sqrt[d] - Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e + f]
]*x + Sqrt[e]*x^2])/2))/(2*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e + f
]]) + (Sqrt[e]*((Sqrt[e]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e + f]]*ArcTan[
(Sqrt[e]*(Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e + f]]/Sqrt[e] + 2*x))/Sqrt[2
*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e + f]]])/Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e
+ f]] + (Sqrt[e]*Log[Sqrt[d] + Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e + f]]*x
 + Sqrt[e]*x^2])/2))/(2*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e + f...
```

3.8. $\int \frac{d+ex^4}{d^2-fx^4+e^2x^8}\,dx$

3.8.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1407
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   Int[(r - x)/(q - r*
x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(r + x)/(q + r*x + x^2), x], x]]]
 /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

rule 1749
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e
+ q*x^(n/2) + x^n, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x^(n/2) +
 x^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[2*(d/e) - b/c,
0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

3.8.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.13 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.07

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(e^{2}\_Z^{8}-f\_Z^{4}+d^{2}\right)}\frac{\left(\_R^{4}e+d\right)\ln\left(x-\_R\right)}{2\_R^{7}e^{2}-\_R^{3}f}\right)}{4}$	55
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(e^{2}\_Z^{8}-f\_Z^{4}+d^{2}\right)}\frac{\left(\_R^{4}e+d\right)\ln\left(x-\_R\right)}{2\_R^{7}e^{2}-\_R^{3}f}\right)}{4}$	55

input
```
int((e*x^4+d)/(e^2*x^8-f*x^4+d^2),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum((_R^4*e+d)/(2*_R^7*e^2-_R^3*f)*ln(x-_R),_R=RootOf(_Z^8*e^2-_Z^4*f+
d^2))
```

3.8.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2453 vs. 2(541) = 1082.

Time = 0.33 (sec) , antiderivative size = 2453, normalized size of antiderivative = 3.27

$$\int \frac{d+ex^4}{d^2-fx^4+e^2x^8}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^4+d)/(e^2*x^8-f*x^4+d^2),x, algorithm="fricas")
```

output
```
1/4*sqrt(sqrt(1/2)*sqrt(((4*d^4*e^2 - 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e +
f)/(8*d^7*e^3 - 12*d^6*e^2*f + 6*d^5*e*f^2 - d^4*f^3)) + f)/(4*d^4*e^2 - 4
*d^3*e*f + d^2*f^2)))*log(e*x + 1/2*(2*d*e + (4*d^4*e^2 - 4*d^3*e*f + d^2*
f^2)*sqrt(-(2*d*e + f)/(8*d^7*e^3 - 12*d^6*e^2*f + 6*d^5*e*f^2 - d^4*f^3))
 - f)*sqrt(sqrt(1/2)*sqrt(((4*d^4*e^2 - 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e
+ f)/(8*d^7*e^3 - 12*d^6*e^2*f + 6*d^5*e*f^2 - d^4*f^3)) + f)/(4*d^4*e^2 -
 4*d^3*e*f + d^2*f^2)))) - 1/4*sqrt(sqrt(1/2)*sqrt(((4*d^4*e^2 - 4*d^3*e*f
 + d^2*f^2)*sqrt(-(2*d*e + f)/(8*d^7*e^3 - 12*d^6*e^2*f + 6*d^5*e*f^2 - d^
4*f^3)) + f)/(4*d^4*e^2 - 4*d^3*e*f + d^2*f^2)))*log(e*x - 1/2*(2*d*e + (4
*d^4*e^2 - 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e + f)/(8*d^7*e^3 - 12*d^6*e^2*
f + 6*d^5*e*f^2 - d^4*f^3)) - f)*sqrt(sqrt(1/2)*sqrt(((4*d^4*e^2 - 4*d^3*e
*f + d^2*f^2)*sqrt(-(2*d*e + f)/(8*d^7*e^3 - 12*d^6*e^2*f + 6*d^5*e*f^2 -
d^4*f^3)) + f)/(4*d^4*e^2 - 4*d^3*e*f + d^2*f^2)))) + 1/4*sqrt(-sqrt(1/2)*
sqrt(((4*d^4*e^2 - 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e + f)/(8*d^7*e^3 - 12*
d^6*e^2*f + 6*d^5*e*f^2 - d^4*f^3)) + f)/(4*d^4*e^2 - 4*d^3*e*f + d^2*f^2)
))*log(e*x + 1/2*(2*d*e + (4*d^4*e^2 - 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e +
 f)/(8*d^7*e^3 - 12*d^6*e^2*f + 6*d^5*e*f^2 - d^4*f^3)) - f)*sqrt(-sqrt(1/
2)*sqrt(((4*d^4*e^2 - 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e + f)/(8*d^7*e^3 -
12*d^6*e^2*f + 6*d^5*e*f^2 - d^4*f^3)) + f)/(4*d^4*e^2 - 4*d^3*e*f + d^2*f
^2)))) - 1/4*sqrt(-sqrt(1/2)*sqrt(((4*d^4*e^2 - 4*d^3*e*f + d^2*f^2)*sq...
```

3.8.6 Sympy [A] (verification not implemented)

Time = 6.38 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.18

$$\int \frac{d+ex^4}{d^2-fx^4+e^2x^8}\,dx$$
$$= \text{RootSum}\left(t^8\cdot\left(1048576d^6e^4-2097152d^5e^3f+1572864d^4e^2f^2-524288d^3ef^3+65536d^2f^4\right)+t^4\left(-102\right.\right.$$

input
```
integrate((e*x**4+d)/(e**2*x**8-f*x**4+d**2),x)
```

output
```
RootSum(_t**8*(1048576*d**6*e**4 - 2097152*d**5*e**3*f + 1572864*d**4*e**2
*f**2 - 524288*d**3*e*f**3 + 65536*d**2*f**4) + _t**4*(-1024*d**2*e**2*f +
 1024*d*e*f**2 - 256*f**3) + e**2, Lambda(_t, _t*log(x + (4096*_t**5*d**4*
e**2 - 4096*_t**5*d**3*e*f + 1024*_t**5*d**2*f**2 + 4*_t*d*e - 4*_t*f)/e))
)
```

3.8.7 Maxima [F]

$$\int \frac{d+ex^4}{d^2-fx^4+e^2x^8}\,dx=\int \frac{ex^4+d}{e^2x^8-fx^4+d^2}\,dx$$

input
```
integrate((e*x^4+d)/(e^2*x^8-f*x^4+d^2),x, algorithm="maxima")
```

output
```
integrate((e*x^4 + d)/(e^2*x^8 - f*x^4 + d^2), x)
```

3.8.8 Giac [F]

$$\int \frac{d+ex^4}{d^2-fx^4+e^2x^8}\,dx=\int \frac{ex^4+d}{e^2x^8-fx^4+d^2}\,dx$$

input
```
integrate((e*x^4+d)/(e^2*x^8-f*x^4+d^2),x, algorithm="giac")
```

output
```
integrate((e*x^4 + d)/(e^2*x^8 - f*x^4 + d^2), x)
```

3.8.9 Mupad [B] (verification not implemented)

Time = 9.85 (sec) , antiderivative size = 10343, normalized size of antiderivative = 13.77

$$\int \frac{d+ex^4}{d^2-fx^4+e^2x^8}\,dx=\text{Too large to display}$$

input
```
int((d + e*x^4)/(d^2 - f*x^4 + e^2*x^8),x)
```

output
```
2*atan(((((f^3 + ((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f
^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f
^2)))^(1/4)*((x*(65536*d^9*e^15 + 32768*d^8*e^14*f - 1024*d^2*e^8*f^7 - 20
48*d^3*e^9*f^6 + 10240*d^4*e^10*f^5 + 20480*d^5*e^11*f^4 - 32768*d^6*e^12*
f^3 - 65536*d^7*e^13*f^2) - ((f^3 + ((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*
d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e
^3*f + 24*d^4*e^2*f^2)))^(1/4)*(262144*d^10*e^15 + 262144*d^9*e^14*f - 409
6*d^3*e^8*f^7 - 4096*d^4*e^9*f^6 + 49152*d^5*e^10*f^5 + 49152*d^6*e^11*f^4
 - 196608*d^7*e^12*f^3 - 196608*d^8*e^13*f^2)*1i)*((f^3 + ((f - 2*d*e)^5*(
f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 -
8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(3/4)*1i - 256*d^7*e^14 - 2
56*d^6*e^13*f + 16*d^3*e^10*f^4 + 64*d^4*e^11*f^3)*1i - x*(32*d^5*e^13*f +
 4*d^2*e^10*f^4 + 24*d^3*e^11*f^3 + 48*d^4*e^12*f^2))*((f^3 + ((f - 2*d*e)
^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^
4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4) + (((f^3 + ((f -
2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 +
d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*((x*(65536*
d^9*e^15 + 32768*d^8*e^14*f - 1024*d^2*e^8*f^7 - 2048*d^3*e^9*f^6 + 10240*
d^4*e^10*f^5 + 20480*d^5*e^11*f^4 - 32768*d^6*e^12*f^3 - 65536*d^7*e^13*f^
2) + ((f^3 + ((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^...
```

3.8. $\int \frac{d+ex^4}{d^2-fx^4+e^2x^8}\,dx$

3.9 $\int \frac{1+x^4}{1+bx^4+x^8}\,dx$

3.9.1 Optimal result

Integrand size = 18, antiderivative size = 411

$$\int \frac{1+x^4}{1+bx^4+x^8}\,dx = -\frac{\arctan\left(\frac{\sqrt{2-\sqrt{2-b}}-2x}{\sqrt{2+\sqrt{2-b}}}\right)}{4\sqrt{2+\sqrt{2-b}}}-\frac{\arctan\left(\frac{\sqrt{2+\sqrt{2-b}}-2x}{\sqrt{2-\sqrt{2-b}}}\right)}{4\sqrt{2-\sqrt{2-b}}}+\frac{\arctan\left(\frac{\sqrt{2-\sqrt{2-b}}+2x}{\sqrt{2+\sqrt{2-b}}}\right)}{4\sqrt{2+\sqrt{2-b}}}+\frac{\arctan\left(\frac{\sqrt{2+\sqrt{2-b}}+2x}{\sqrt{2-\sqrt{2-b}}}\right)}{4\sqrt{2-\sqrt{2-b}}}-\frac{\log\left(1-\sqrt{2-\sqrt{2-b}}x+x^2\right)}{8\sqrt{2-\sqrt{2-b}}}+\frac{\log\left(1+\sqrt{2-\sqrt{2-b}}x+x^2\right)}{8\sqrt{2-\sqrt{2-b}}}-\frac{\log\left(1-\sqrt{2+\sqrt{2-b}}x+x^2\right)}{8\sqrt{2+\sqrt{2-b}}}+\frac{\log\left(1+\sqrt{2+\sqrt{2-b}}x+x^2\right)}{8\sqrt{2+\sqrt{2-b}}}$$

output
```
-1/4*arctan((-2*x+(2+(2-b)^(1/2))^(1/2))/(2-(2-b)^(1/2))^(1/2))/(2-(2-b)^(
1/2))^(1/2)+1/4*arctan((2*x+(2+(2-b)^(1/2))^(1/2))/(2-(2-b)^(1/2))^(1/2))/
(2-(2-b)^(1/2))^(1/2)-1/8*ln(1+x^2-x*(2-(2-b)^(1/2))^(1/2))/(2-(2-b)^(1/2)
)^(1/2)+1/8*ln(1+x^2+x*(2-(2-b)^(1/2))^(1/2))/(2-(2-b)^(1/2))^(1/2)-1/4*ar
ctan((-2*x+(2-(2-b)^(1/2))^(1/2))/(2+(2-b)^(1/2))^(1/2))/(2+(2-b)^(1/2))^(
1/2)+1/4*arctan((2*x+(2-(2-b)^(1/2))^(1/2))/(2+(2-b)^(1/2))^(1/2))/(2+(2-b
)^(1/2))^(1/2)-1/8*ln(1+x^2-x*(2+(2-b)^(1/2))^(1/2))/(2+(2-b)^(1/2))^(1/2)
+1/8*ln(1+x^2+x*(2+(2-b)^(1/2))^(1/2))/(2+(2-b)^(1/2))^(1/2)
```

3.9.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.13

$$\int \frac{1+x^4}{1+bx^4+x^8}\,dx = \frac{1}{4}\text{RootSum}\left[1+b\#1^4+\#1^8\&, \frac{\log(x-\#1)+\log(x-\#1)\#1^4}{b\#1^3+2\#1^7}\&\right]$$

input
```
Integrate[(1 + x^4)/(1 + b*x^4 + x^8),x]
```

output
```
RootSum[1 + b*#1^4 + #1^8 & , (Log[x - #1] + Log[x - #1]*#1^4)/(b*#1^3 + 2
*#1^7) & ]/4
```

3.9.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 497, normalized size of antiderivative = 1.21, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {1749, 1407, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4+1}{bx^4+x^8+1}\,dx$$

$$\Big\downarrow \ 1749$$

$$\frac{1}{2}\int \frac{1}{x^4-\sqrt{2-b}x^2+1}dx+\frac{1}{2}\int \frac{1}{x^4+\sqrt{2-b}x^2+1}dx$$

$$\Big\downarrow \ 1407$$

$$\frac{1}{2}\left(\frac{\int \frac{\sqrt{2-\sqrt{2-b}}-x}{x^2-\sqrt{2-\sqrt{2-b}}x+1}dx}{2\sqrt{2-\sqrt{2-b}}}+\frac{\int \frac{x+\sqrt{2-\sqrt{2-b}}}{x^2+\sqrt{2-\sqrt{2-b}}x+1}dx}{2\sqrt{2-\sqrt{2-b}}}\right)+\frac{1}{2}\left(\frac{\int \frac{\sqrt{\sqrt{2-b}+2}-x}{x^2-\sqrt{\sqrt{2-b}+2}x+1}dx}{2\sqrt{\sqrt{2-b}+2}}+\frac{\int \frac{x+\sqrt{\sqrt{2-b}+2}}{x^2+\sqrt{\sqrt{2-b}+2}x+1}dx}{2\sqrt{\sqrt{2-b}+2}}\right)$$

$$\Big\downarrow \ 1142$$

$$\frac{1}{2}\left(\frac{\frac{1}{2}\sqrt{2-\sqrt{2-b}}\int \frac{1}{x^2-\sqrt{2-\sqrt{2-b}}x+1}dx-\frac{1}{2}\int -\frac{\sqrt{2-\sqrt{2-b}}-2x}{x^2-\sqrt{2-\sqrt{2-b}}x+1}dx}{2\sqrt{2-\sqrt{2-b}}}+\frac{\frac{1}{2}\sqrt{2-\sqrt{2-b}}\int \frac{1}{x^2+\sqrt{2-\sqrt{2-b}}x+1}dx+\frac{1}{2}\int \cdots}{2\sqrt{2-\sqrt{2-b}}}\cdots\right.$$
$$\frac{1}{2}\left(\frac{\frac{1}{2}\sqrt{\sqrt{2-b}+2}\int \frac{1}{x^2-\sqrt{\sqrt{2-b}+2}x+1}dx-\frac{1}{2}\int -\frac{\sqrt{\sqrt{2-b}+2}-2x}{x^2-\sqrt{\sqrt{2-b}+2}x+1}dx}{2\sqrt{\sqrt{2-b}+2}}+\frac{\frac{1}{2}\sqrt{\sqrt{2-b}+2}\int \frac{1}{x^2+\sqrt{\sqrt{2-b}+2}x+1}dx+\frac{1}{2}\int \cdots}{2\sqrt{\sqrt{2-b}+2}}\cdots\right.$$

$\downarrow$ 25

$$\frac{1}{2}\left(\frac{\frac{1}{2}\sqrt{2-\sqrt{2-b}}\int \frac{1}{x^2-\sqrt{2-\sqrt{2-b}}x+1}dx+\frac{1}{2}\int \frac{\sqrt{2-\sqrt{2-b}}-2x}{x^2-\sqrt{2-\sqrt{2-b}}x+1}dx}{2\sqrt{2-\sqrt{2-b}}}+\frac{\frac{1}{2}\sqrt{2-\sqrt{2-b}}\int \frac{1}{x^2+\sqrt{2-\sqrt{2-b}}x+1}dx+\frac{1}{2}\int \frac{2\cdots}{x^2\cdots}}{2\sqrt{2-\sqrt{2-b}}}\cdots\right.$$
$$\frac{1}{2}\left(\frac{\frac{1}{2}\sqrt{\sqrt{2-b}+2}\int \frac{1}{x^2-\sqrt{\sqrt{2-b}+2}x+1}dx+\frac{1}{2}\int \frac{\sqrt{\sqrt{2-b}+2}-2x}{x^2-\sqrt{\sqrt{2-b}+2}x+1}dx}{2\sqrt{\sqrt{2-b}+2}}+\frac{\frac{1}{2}\sqrt{\sqrt{2-b}+2}\int \frac{1}{x^2+\sqrt{\sqrt{2-b}+2}x+1}dx+\frac{1}{2}\int \frac{2\cdots}{x^2\cdots}}{2\sqrt{\sqrt{2-b}+2}}\cdots\right.$$

$\downarrow$ 1083

$$\frac{1}{2}\left(\frac{\frac{1}{2}\int \frac{\sqrt{2-\sqrt{2-b}}-2x}{x^2-\sqrt{2-\sqrt{2-b}}x+1}dx-\sqrt{2-\sqrt{2-b}}\int \frac{1}{-\left(2x-\sqrt{2-\sqrt{2-b}}\right)^2-\sqrt{2-b}-2}d\left(2x-\sqrt{2-\sqrt{2-b}}\right)}{2\sqrt{2-\sqrt{2-b}}}+\frac{\frac{1}{2}\int \frac{2x+\sqrt{2-\sqrt{2-}\cdots}}{x^2+\sqrt{2-\sqrt{2-b}}x\cdots}}{\cdots}\right.$$
$$\frac{1}{2}\left(\frac{\frac{1}{2}\int \frac{\sqrt{\sqrt{2-b}+2}-2x}{x^2-\sqrt{\sqrt{2-b}+2}x+1}dx-\sqrt{\sqrt{2-b}+2}\int \frac{1}{-\left(2x-\sqrt{\sqrt{2-b}+2}\right)^2+\sqrt{2-b}-2}d\left(2x-\sqrt{\sqrt{2-b}+2}\right)}{2\sqrt{\sqrt{2-b}+2}}+\frac{\frac{1}{2}\int \frac{2x+\sqrt{\sqrt{2-b}+\cdots}}{x^2+\sqrt{\sqrt{2-b}+2}x\cdots}}{\cdots}\right.$$

$\downarrow$ 217

$$\frac{1}{2}\left(\frac{\frac{1}{2}\int \frac{\sqrt{2-\sqrt{2-b}}-2x}{x^2-\sqrt{2-\sqrt{2-b}}x+1}dx+\frac{\sqrt{2-\sqrt{2-b}}\arctan\left(\frac{2x-\sqrt{2-\sqrt{2-b}}}{\sqrt{\sqrt{2-b}+2}}\right)}{\sqrt{\sqrt{2-b}+2}}}{2\sqrt{2-\sqrt{2-b}}}+\frac{\frac{1}{2}\int \frac{2x+\sqrt{2-\sqrt{2-b}}}{x^2+\sqrt{2-\sqrt{2-b}}x+1}dx+\frac{\sqrt{2-\sqrt{2-b}}\arctan\left(\frac{\sqrt{2-\sqrt{2-b}}+2x}{\sqrt{\sqrt{2-b}+2}}\right)}{\sqrt{\sqrt{2-b}+2}}}{2\sqrt{2-\sqrt{2-b}}}\cdots\right.$$
$$\frac{1}{2}\left(\frac{\frac{1}{2}\int \frac{\sqrt{\sqrt{2-b}+2}-2x}{x^2-\sqrt{\sqrt{2-b}+2}x+1}dx+\frac{\sqrt{\sqrt{2-b}+2}\arctan\left(\frac{2x-\sqrt{\sqrt{2-b}+2}}{\sqrt{2-\sqrt{2-b}}}\right)}{\sqrt{2-\sqrt{2-b}}}}{2\sqrt{\sqrt{2-b}+2}}+\frac{\frac{1}{2}\int \frac{2x+\sqrt{\sqrt{2-b}+2}}{x^2+\sqrt{\sqrt{2-b}+2}x+1}dx+\frac{\sqrt{\sqrt{2-b}+2}\arctan\left(\frac{\sqrt{\sqrt{2-b}+2}+2x}{\sqrt{2-\sqrt{2-b}}}\right)}{\sqrt{2-\sqrt{2-b}}}}{2\sqrt{\sqrt{2-b}+2}}\cdots\right.$$

$\downarrow$ 1103

3.9. $\int \frac{1+x^4}{1+bx^4+x^8}\,dx$

$$\frac{1}{2}\left(\frac{\frac{\sqrt{2-\sqrt{2-b}}\arctan\left(\frac{2x-\sqrt{2-\sqrt{2-b}}}{\sqrt{\sqrt{2-b}+2}}\right)}{\sqrt{\sqrt{2-b}+2}}-\frac{1}{2}\log\left(-\sqrt{2-\sqrt{2-b}}x+x^2+1\right)}{2\sqrt{2-\sqrt{2-b}}}+\frac{\frac{\sqrt{2-\sqrt{2-b}}\arctan\left(\frac{\sqrt{2-\sqrt{2-b}}+2x}{\sqrt{\sqrt{2-b}+2}}\right)}{\sqrt{\sqrt{2-b}+2}}+\frac{1}{2}\log\left(\sqrt{2\cdots\right.}{2\sqrt{2-\sqrt{2-b}}}\right.$$

$$\frac{1}{2}\left(\frac{\frac{\sqrt{\sqrt{2-b}+2}\arctan\left(\frac{2x-\sqrt{\sqrt{2-b}+2}}{\sqrt{2-\sqrt{2-b}}}\right)}{\sqrt{2-\sqrt{2-b}}}-\frac{1}{2}\log\left(-\sqrt{\sqrt{2-b}+2}x+x^2+1\right)}{2\sqrt{\sqrt{2-b}+2}}+\frac{\frac{\sqrt{\sqrt{2-b}+2}\arctan\left(\frac{\sqrt{\sqrt{2-b}+2}+2x}{\sqrt{2-\sqrt{2-b}}}\right)}{\sqrt{2-\sqrt{2-b}}}+\frac{1}{2}\log\left(\sqrt{\cdots\right.}{2\sqrt{\sqrt{2-b}+2}}\right.$$

input
```
Int[(1 + x^4)/(1 + b*x^4 + x^8),x]
```

output
```
(((Sqrt[2 - Sqrt[2 - b]]*ArcTan[(-Sqrt[2 - Sqrt[2 - b]] + 2*x)/Sqrt[2 + Sq
rt[2 - b]]])/Sqrt[2 + Sqrt[2 - b]] - Log[1 - Sqrt[2 - Sqrt[2 - b]]*x + x^2
]/2)/(2*Sqrt[2 - Sqrt[2 - b]]) + ((Sqrt[2 - Sqrt[2 - b]]*ArcTan[(Sqrt[2 -
Sqrt[2 - b]] + 2*x)/Sqrt[2 + Sqrt[2 - b]]])/Sqrt[2 + Sqrt[2 - b]] + Log[1
+ Sqrt[2 - Sqrt[2 - b]]*x + x^2]/2)/(2*Sqrt[2 - Sqrt[2 - b]]))/2 + (((Sqrt
[2 + Sqrt[2 - b]]*ArcTan[(-Sqrt[2 + Sqrt[2 - b]] + 2*x)/Sqrt[2 - Sqrt[2 -
b]]])/Sqrt[2 - Sqrt[2 - b]] - Log[1 - Sqrt[2 + Sqrt[2 - b]]*x + x^2]/2)/(2
*Sqrt[2 + Sqrt[2 - b]]) + ((Sqrt[2 + Sqrt[2 - b]]*ArcTan[(Sqrt[2 + Sqrt[2
- b]] + 2*x)/Sqrt[2 - Sqrt[2 - b]]])/Sqrt[2 - Sqrt[2 - b]] + Log[1 + Sqrt[
2 + Sqrt[2 - b]]*x + x^2]/2)/(2*Sqrt[2 + Sqrt[2 - b]]))/2
```

3.9.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1407
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   Int[(r - x)/(q - r*
x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(r + x)/(q + r*x + x^2), x], x]]]
 /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

rule 1749
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e
+ q*x^(n/2) + x^n, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x^(n/2) +
 x^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[2*(d/e) - b/c,
0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

3.9.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.10

method	result	size
default	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{8}+\_Z^{4}b+1\right)}\frac{\left(\_R^{4}+1\right)\ln\left(x-\_R\right)}{2\_R^{7}+\_R^{3}b}\right)}{4}$	42
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{8}+\_Z^{4}b+1\right)}\frac{\left(\_R^{4}+1\right)\ln\left(x-\_R\right)}{2\_R^{7}+\_R^{3}b}\right)}{4}$	42

input
```
int((x^4+1)/(x^8+b*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum((_R^4+1)/(2*_R^7+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8+_Z^4*b+1))
```

3.9.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1177 vs. $2(321) = 642$.

Time = 0.28 (sec) , antiderivative size = 1177, normalized size of antiderivative = 2.86

$$\int \frac{1+x^4}{1+bx^4+x^8}\,dx = \text{Too large to display}$$

input
```
integrate((x^4+1)/(x^8+b*x^4+1),x, algorithm="fricas")
```

output
```
-1/4*sqrt(sqrt(1/2)*sqrt(-((b^2 + 4*b + 4)*sqrt((b - 2)/(b^3 + 6*b^2 + 12*
b + 8)) + b)/(b^2 + 4*b + 4)))*log(1/2*((b^2 + 4*b + 4)*sqrt((b - 2)/(b^3
+ 6*b^2 + 12*b + 8)) - b - 2)*sqrt(sqrt(1/2)*sqrt(-((b^2 + 4*b + 4)*sqrt((
b - 2)/(b^3 + 6*b^2 + 12*b + 8)) + b)/(b^2 + 4*b + 4))) + x) + 1/4*sqrt(sq
rt(1/2)*sqrt(-((b^2 + 4*b + 4)*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b + 8)) + b)
/(b^2 + 4*b + 4)))*log(-1/2*((b^2 + 4*b + 4)*sqrt((b - 2)/(b^3 + 6*b^2 + 1
2*b + 8)) - b - 2)*sqrt(sqrt(1/2)*sqrt(-((b^2 + 4*b + 4)*sqrt((b - 2)/(b^3
 + 6*b^2 + 12*b + 8)) + b)/(b^2 + 4*b + 4))) + x) - 1/4*sqrt(-sqrt(1/2)*sq
rt(-((b^2 + 4*b + 4)*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b + 8)) + b)/(b^2 + 4*
b + 4)))*log(1/2*((b^2 + 4*b + 4)*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b + 8)) -
 b - 2)*sqrt(-sqrt(1/2)*sqrt(-((b^2 + 4*b + 4)*sqrt((b - 2)/(b^3 + 6*b^2 +
 12*b + 8)) + b)/(b^2 + 4*b + 4))) + x) + 1/4*sqrt(-sqrt(1/2)*sqrt(-((b^2
+ 4*b + 4)*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b + 8)) + b)/(b^2 + 4*b + 4)))*l
og(-1/2*((b^2 + 4*b + 4)*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b + 8)) - b - 2)*s
qrt(-sqrt(1/2)*sqrt(-((b^2 + 4*b + 4)*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b + 8
)) + b)/(b^2 + 4*b + 4))) + x) + 1/4*sqrt(sqrt(1/2)*sqrt(((b^2 + 4*b + 4)*
sqrt((b - 2)/(b^3 + 6*b^2 + 12*b + 8)) - b)/(b^2 + 4*b + 4)))*log(1/2*((b^
2 + 4*b + 4)*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b + 8)) + b + 2)*sqrt(sqrt(1/2
)*sqrt(((b^2 + 4*b + 4)*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b + 8)) - b)/(b^2 +
 4*b + 4))) + x) - 1/4*sqrt(sqrt(1/2)*sqrt(((b^2 + 4*b + 4)*sqrt((b - 2...
```

3.9.6 Sympy [A] (verification not implemented)

Time = 1.81 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.18

$$\int \frac{1+x^4}{1+bx^4+x^8}\,dx$$
$$= \operatorname{RootSum}\left(t^8\cdot\left(65536b^4+524288b^3+1572864b^2+2097152b+1048576\right)+t^4\cdot\left(256b^3+1024b^2+1024b\right)\right.$$

input
```
integrate((x**4+1)/(x**8+b*x**4+1),x)
```

output
```
RootSum(_t**8*(65536*b**4 + 524288*b**3 + 1572864*b**2 + 2097152*b + 10485
76) + _t**4*(256*b**3 + 1024*b**2 + 1024*b) + 1, Lambda(_t, _t*log(1024*_t
**5*b**2 + 4096*_t**5*b + 4096*_t**5 + 4*_t*b + 4*_t + x)))
```

3.9.7 Maxima [F]

$$\int \frac{1+x^4}{1+bx^4+x^8}\,dx = \int \frac{x^4+1}{x^8+bx^4+1}\,dx$$

input
```
integrate((x^4+1)/(x^8+b*x^4+1),x, algorithm="maxima")
```

output
```
integrate((x^4 + 1)/(x^8 + b*x^4 + 1), x)
```

3.9.8 Giac [F]

$$\int \frac{1+x^4}{1+bx^4+x^8}\,dx = \int \frac{x^4+1}{x^8+bx^4+1}\,dx$$

input
```
integrate((x^4+1)/(x^8+b*x^4+1),x, algorithm="giac")
```

output
```
integrate((x^4 + 1)/(x^8 + b*x^4 + 1), x)
```

3.9.9 Mupad [B] (verification not implemented)

Time = 9.56 (sec) , antiderivative size = 5341, normalized size of antiderivative = 13.00

$$\int \frac{1+x^4}{1+bx^4+x^8}\,dx = \text{Too large to display}$$

input
```
int((x^4 + 1)/(b*x^4 + x^8 + 1),x)
```

output
```
- atan((((-(4*b + ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24
*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*(((-(4*b + ((b - 2)*(b + 2)^5)^(1/2) + 4*
b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*(262144*b + 196
608*b^2 - 196608*b^3 - 49152*b^4 + 49152*b^5 + 4096*b^6 - 4096*b^7 - 26214
4) + x*(32768*b + 65536*b^2 - 32768*b^3 - 20480*b^4 + 10240*b^5 + 2048*b^6
 - 1024*b^7 - 65536))*(-(4*b + ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(5
12*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(3/4) - 256*b + 64*b^3 - 16*b^4 +
256) + x*(32*b - 48*b^2 + 24*b^3 - 4*b^4))*(-(4*b + ((b - 2)*(b + 2)^5)^(1
/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*1i - ((
-(4*b + ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b
^3 + b^4 + 16)))^(1/4)*(((-(4*b + ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)
/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*(262144*b + 196608*b^2 -
196608*b^3 - 49152*b^4 + 49152*b^5 + 4096*b^6 - 4096*b^7 - 262144) - x*(32
768*b + 65536*b^2 - 32768*b^3 - 20480*b^4 + 10240*b^5 + 2048*b^6 - 1024*b^
7 - 65536))*(-(4*b + ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b +
 24*b^2 + 8*b^3 + b^4 + 16)))^(3/4) - 256*b + 64*b^3 - 16*b^4 + 256) - x*(
32*b - 48*b^2 + 24*b^3 - 4*b^4))*(-(4*b + ((b - 2)*(b + 2)^5)^(1/2) + 4*b^
2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*1i)/(((-(4*b + ((
b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 +
 16)))^(1/4)*(((-(4*b + ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(...
```

3.9. $\int \frac{1+x^4}{1+bx^4+x^8}\,dx$

3.10 $\int \frac{1+x^4}{1+3x^4+x^8} \, dx$

3.10.1 Optimal result

Integrand size = 18, antiderivative size = 451

$$\int \frac{1+x^4}{1+3x^4+x^8}\,dx = -\frac{\sqrt[4]{3+\sqrt{5}}\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}} + \frac{\sqrt[4]{3+\sqrt{5}}\arctan\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}} - \frac{\sqrt[4]{3-\sqrt{5}}\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}} + \frac{\sqrt[4]{3-\sqrt{5}}\arctan\left(1+\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}} - \frac{\sqrt[4]{3+\sqrt{5}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{4\,2^{3/4}\sqrt{5}} + \frac{\sqrt[4]{3+\sqrt{5}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{4\,2^{3/4}\sqrt{5}} - \frac{\sqrt[4]{3-\sqrt{5}}\log\left(\sqrt{2\left(3+\sqrt{5}\right)}-2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+2x^2\right)}{4\,2^{3/4}\sqrt{5}} + \frac{\sqrt[4]{3-\sqrt{5}}\log\left(\sqrt{2\left(3+\sqrt{5}\right)}+2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+2x^2\right)}{4\,2^{3/4}\sqrt{5}}$$

output
```
1/20*arctan(-1+2^(3/4)*x/(3+5^(1/2))^(1/4))*(3-5^(1/2))^(1/4)*2^(1/4)*5^(1
/2)+1/20*arctan(1+2^(3/4)*x/(3+5^(1/2))^(1/4))*(3-5^(1/2))^(1/4)*2^(1/4)*5
^(1/2)-1/40*ln(2*x^2-2*2^(1/4)*x*(3+5^(1/2))^(1/4)+5^(1/2)+1)*(3-5^(1/2))^
(1/4)*2^(1/4)*5^(1/2)+1/40*ln(2*x^2+2*2^(1/4)*x*(3+5^(1/2))^(1/4)+5^(1/2)+
1)*(3-5^(1/2))^(1/4)*2^(1/4)*5^(1/2)+1/20*arctan(-1+2^(3/4)*x/(3-5^(1/2))^
(1/4))*(3+5^(1/2))^(1/4)*2^(1/4)*5^(1/2)+1/20*arctan(1+2^(3/4)*x/(3-5^(1/2
))^(1/4))*(3+5^(1/2))^(1/4)*2^(1/4)*5^(1/2)-1/40*ln(2*x^2-2*2^(1/4)*x*(3-5
^(1/2))^(1/4)+5^(1/2)-1)*(3+5^(1/2))^(1/4)*2^(1/4)*5^(1/2)+1/40*ln(2*x^2+2
*2^(1/4)*x*(3-5^(1/2))^(1/4)+5^(1/2)-1)*(3+5^(1/2))^(1/4)*2^(1/4)*5^(1/2)
```

3.10.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.12

$$\int \frac{1+x^4}{1+3x^4+x^8}\,dx = \frac{1}{4}\text{RootSum}\left[1+3\#1^4+\#1^8\&,\frac{\log(x-\#1)+\log(x-\#1)\#1^4}{3\#1^3+2\#1^7}\&\right]$$

input
```
Integrate[(1 + x^4)/(1 + 3*x^4 + x^8),x]
```

output
```
RootSum[1 + 3*#1^4 + #1^8 & , (Log[x - #1] + Log[x - #1]*#1^4)/(3*#1^3 + 2
*#1^7) & ]/4
```

3.10.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 477, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1750, 755, 27, 1476, 1082, 217, 1479, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4+1}{x^8+3x^4+1}\,dx$$

$$\Big\downarrow \text{1750}$$

$$\frac{1}{10}\left(5-\sqrt{5}\right)\int \frac{1}{x^4+\frac{1}{2}\left(3-\sqrt{5}\right)}dx+\frac{1}{10}\left(5+\sqrt{5}\right)\int \frac{1}{x^4+\frac{1}{2}\left(3+\sqrt{5}\right)}dx$$

$$\Big\downarrow \; 755$$

$$\frac{1}{10}\left(5-\sqrt{5}\right)\left(\frac{\int \frac{2\left(\sqrt{3-\sqrt{5}}-\sqrt{2}x^2\right)}{2x^4-\sqrt{5}+3}dx}{2\sqrt{3-\sqrt{5}}}+\frac{\int \frac{2\left(\sqrt{2}x^2+\sqrt{3-\sqrt{5}}\right)}{2x^4-\sqrt{5}+3}dx}{2\sqrt{3-\sqrt{5}}}\right)+\frac{1}{10}\left(5+\sqrt{5}\right)\left(\frac{\int \frac{2\left(\sqrt{3+\sqrt{5}}-\sqrt{2}x^2\right)}{2x^4+\sqrt{5}+3}dx}{2\sqrt{3+\sqrt{5}}}+\frac{\int \frac{2\left(\sqrt{2}x^2+\sqrt{3+\sqrt{5}}\right)}{2x^4+\sqrt{5}+3}dx}{2\sqrt{3+\sqrt{5}}}\right)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{10}\left(5-\sqrt{5}\right)\left(\frac{\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{2x^4-\sqrt{5}+3}dx}{\sqrt{3-\sqrt{5}}}+\frac{\int \frac{\sqrt{2}x^2+\sqrt{3-\sqrt{5}}}{2x^4-\sqrt{5}+3}dx}{\sqrt{3-\sqrt{5}}}\right)+\frac{1}{10}\left(5+\sqrt{5}\right)\left(\frac{\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{2x^4+\sqrt{5}+3}dx}{\sqrt{3+\sqrt{5}}}+\frac{\int \frac{\sqrt{2}x^2+\sqrt{3+\sqrt{5}}}{2x^4+\sqrt{5}+3}dx}{\sqrt{3+\sqrt{5}}}\right)$$

$$\Big\downarrow \; 1476$$

$$\frac{1}{10}\left(5-\sqrt{5}\right)\left(\frac{\frac{\int \frac{1}{x^2-\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}}dx}{2\sqrt{2}}+\frac{\int \frac{1}{x^2+\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}}dx}{2\sqrt{2}}}{\sqrt{3-\sqrt{5}}}+\frac{\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{2x^4-\sqrt{5}+3}dx}{\sqrt{3-\sqrt{5}}}\right)+\frac{1}{10}\left(5+\sqrt{5}\right)\left(\frac{\frac{\int \frac{1}{x^2-\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\sqrt{2}}+\frac{\int \frac{1}{x^2+\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\sqrt{2}}}{\sqrt{3+\sqrt{5}}}+\frac{\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{2x^4+\sqrt{5}+3}dx}{\sqrt{3+\sqrt{5}}}\right)$$

$$\Big\downarrow \; 1082$$

3.10. $\int \frac{1+x^4}{1+3x^4+x^8}\,dx$

$$\frac{1}{10}\left(5-\sqrt{5}\right)\left(\frac{\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{2x^4-\sqrt{5}+3}dx}{\sqrt{3-\sqrt{5}}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)^2-1}d\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}-\frac{\int \frac{1}{-\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)^2-1}d\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+\right.}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}}{\sqrt{3-\sqrt{5}}}\right.$$

$$\frac{1}{10}\left(5+\sqrt{5}\right)\left(\frac{\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{2x^4+\sqrt{5}+3}dx}{\sqrt{3+\sqrt{5}}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)^2-1}d\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\int \frac{1}{-\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)^2-1}d\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+\right.}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{\sqrt{3+\sqrt{5}}}\right.$$

$$\downarrow 217$$

$$\frac{1}{10}\left(5-\sqrt{5}\right)\left(\frac{\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{2x^4-\sqrt{5}+3}dx}{\sqrt{3-\sqrt{5}}}+\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}}{\sqrt{3-\sqrt{5}}}\right)+$$
$$\frac{1}{10}\left(5+\sqrt{5}\right)\left(\frac{\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{2x^4+\sqrt{5}+3}dx}{\sqrt{3+\sqrt{5}}}+\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{\sqrt{3+\sqrt{5}}}\right)$$

3.10. $\int \frac{1+x^4}{1+3x^4+x^8}\,dx$

$$\Big\downarrow \; 1479$$

$$\frac{1}{10}\left(5-\sqrt{5}\right)\left(\frac{-\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\int -\frac{\sqrt[4]{2\left(3-\sqrt{5}\right)}-2x}{x^2-\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}}dx-\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\int -\frac{2x+\sqrt[4]{2\left(3-\cdots\right.}}{x^2+\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\cdots}\cdots}{\sqrt{3-\sqrt{5}}}\cdots\right.$$

$$\frac{1}{10}\left(5+\sqrt{5}\right)\left(\frac{-\frac{\int -\frac{\sqrt[4]{2\left(3+\sqrt{5}\right)}-2x}{x^2-\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\;2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\int -\frac{2x+\sqrt[4]{2\left(3+\sqrt{5}\right)}}{x^2+\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\;2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{\sqrt{3+\sqrt{5}}}+\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}\cdots}{\sqrt{3\cdots}}\cdots\right.$$

$$\Big\downarrow \; 25$$

$$\frac{1}{10}\left(5-\sqrt{5}\right)\left(\frac{\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\int \frac{\sqrt[4]{2\left(3-\sqrt{5}\right)}-2x}{x^2-\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}}dx+\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\int \frac{2x+\sqrt[4]{2\left(3-\sqrt{5}\right)}}{x^2+\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3\cdots\right.}}\cdots}{\sqrt{3-\sqrt{5}}}\cdots\right.$$

$$\frac{1}{10}\left(5+\sqrt{5}\right)\left(\frac{\frac{\int \frac{\sqrt[4]{2\left(3+\sqrt{5}\right)}-2x}{x^2-\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\;2^{3/4}\sqrt[4]{3+\sqrt{5}}}+\frac{\int \frac{2x+\sqrt[4]{2\left(3+\sqrt{5}\right)}}{x^2+\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\;2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{\sqrt{3+\sqrt{5}}}+\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\text{arc}\cdots}{\cdots}}{\sqrt{3+\sqrt{\cdots}}}\cdots\right.$$

3.10. $\int \frac{1+x^4}{1+3x^4+x^8}\,dx$

$$\downarrow 1103$$

$$\frac{1}{10}\left(5-\sqrt{5}\right)\left(\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}}{\sqrt{3-\sqrt{5}}}+\frac{\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{\ }\right.}{}\right.$$

$$\frac{1}{10}\left(5+\sqrt{5}\right)\left(\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{\sqrt{3+\sqrt{5}}}+\frac{\frac{\log\left(2x^2+2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\log\left(2x^2-\right.}{}}{\sqrt{3+\sqrt{5}}}\right.$$

input
```
Int[(1 + x^4)/(1 + 3*x^4 + x^8),x]
```

output
```
((5 - Sqrt[5])*((-(ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)]/(2^(3/4)*(3
 - Sqrt[5])^(1/4))) + ArcTan[1 + (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)]/(2^(3/4)
*(3 - Sqrt[5])^(1/4)))/Sqrt[3 - Sqrt[5]] + (-1/4*(((3 + Sqrt[5])/2)^(1/4)*
Log[Sqrt[2*(3 - Sqrt[5])] - 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2]) + (((3 +
 Sqrt[5])/2)^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])] + 2*(2*(3 - Sqrt[5]))^(1/4)*x
 + 2*x^2])/4)/Sqrt[3 - Sqrt[5]]))/10 + ((5 + Sqrt[5])*((-(ArcTan[1 - (2^(3
/4)*x)/(3 + Sqrt[5])^(1/4)]/(2^(3/4)*(3 + Sqrt[5])^(1/4))) + ArcTan[1 + (2
^(3/4)*x)/(3 + Sqrt[5])^(1/4)]/(2^(3/4)*(3 + Sqrt[5])^(1/4)))/Sqrt[3 + Sqr
t[5]] + (-1/2*Log[Sqrt[2*(3 + Sqrt[5])] - 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*
x^2]/(2^(3/4)*(3 + Sqrt[5])^(1/4)) + Log[Sqrt[2*(3 + Sqrt[5])] + 2*(2*(3 +
 Sqrt[5]))^(1/4)*x + 2*x^2]/(2*2^(3/4)*(3 + Sqrt[5])^(1/4)))/Sqrt[3 + Sqrt
[5]]))/10
```

3.10. $\int \frac{1+x^4}{1+3x^4+x^8}\,dx$

3.10.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1750
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && G
tQ[b^2 - 4*a*c, 0]
```

3.10.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.09

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}+3\_Z^{4}+1\right)}\frac{\left(\_R^{4}+1\right)\ln\left(x-\_R\right)}{2\_R^{7}+3\_R^{3}}\right)}{4}$	42
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}+3\_Z^{4}+1\right)}\frac{\left(\_R^{4}+1\right)\ln\left(x-\_R\right)}{2\_R^{7}+3\_R^{3}}\right)}{4}$	42

input
```
int((x^4+1)/(x^8+3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum((_R^4+1)/(2*_R^7+3*_R^3)*ln(x-_R),_R=RootOf(_Z^8+3*_Z^4+1))
```

3.10.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 389, normalized size of antiderivative = 0.86

$$\int \frac{1+x^4}{1+3x^4+x^8}\,dx = \frac{1}{40}\sqrt{10}\sqrt{\sqrt{2}\sqrt{\sqrt{5}-3}}\log\left(\sqrt{10}\sqrt{\sqrt{2}\sqrt{\sqrt{5}-3}}\left(\sqrt{5}+5\right)+20\,x\right)$$
$$-\frac{1}{40}\sqrt{10}\sqrt{\sqrt{2}\sqrt{\sqrt{5}-3}}\log\left(-\sqrt{10}\sqrt{\sqrt{2}\sqrt{\sqrt{5}-3}}\left(\sqrt{5}+5\right)+20\,x\right)$$
$$+\frac{1}{40}\sqrt{10}\sqrt{-\sqrt{2}\sqrt{\sqrt{5}-3}}\log\left(\sqrt{10}\sqrt{-\sqrt{2}\sqrt{\sqrt{5}-3}}\left(\sqrt{5}+5\right)+20\,x\right)$$
$$-\frac{1}{40}\sqrt{10}\sqrt{-\sqrt{2}\sqrt{\sqrt{5}-3}}\log\left(-\sqrt{10}\sqrt{-\sqrt{2}\sqrt{\sqrt{5}-3}}\left(\sqrt{5}+5\right)+20\,x\right)$$
$$-\frac{1}{40}\sqrt{10}\sqrt{\sqrt{2}\sqrt{-\sqrt{5}-3}}\log\left(\sqrt{10}\sqrt{\sqrt{2}\sqrt{-\sqrt{5}-3}}\left(\sqrt{5}-5\right)+20\,x\right)$$
$$+\frac{1}{40}\sqrt{10}\sqrt{\sqrt{2}\sqrt{-\sqrt{5}-3}}\log\left(-\sqrt{10}\sqrt{\sqrt{2}\sqrt{-\sqrt{5}-3}}\left(\sqrt{5}-5\right)+20\,x\right)$$
$$-\frac{1}{40}\sqrt{10}\sqrt{-\sqrt{2}\sqrt{-\sqrt{5}-3}}\log\left(\sqrt{10}\sqrt{-\sqrt{2}\sqrt{-\sqrt{5}-3}}\left(\sqrt{5}-5\right)+20\,x\right)$$
$$+\frac{1}{40}\sqrt{10}\sqrt{-\sqrt{2}\sqrt{-\sqrt{5}-3}}\log\left(-\sqrt{10}\sqrt{-\sqrt{2}\sqrt{-\sqrt{5}-3}}\left(\sqrt{5}-5\right)+20\,x\right)$$

input
```
integrate((x^4+1)/(x^8+3*x^4+1),x, algorithm="fricas")
```

output
```
1/40*sqrt(10)*sqrt(sqrt(2)*sqrt(sqrt(5) - 3))*log(sqrt(10)*sqrt(sqrt(2)*sq
rt(sqrt(5) - 3))*(sqrt(5) + 5) + 20*x) - 1/40*sqrt(10)*sqrt(sqrt(2)*sqrt(s
qrt(5) - 3))*log(-sqrt(10)*sqrt(sqrt(2)*sqrt(sqrt(5) - 3))*(sqrt(5) + 5) +
 20*x) + 1/40*sqrt(10)*sqrt(-sqrt(2)*sqrt(sqrt(5) - 3))*log(sqrt(10)*sqrt(
-sqrt(2)*sqrt(sqrt(5) - 3))*(sqrt(5) + 5) + 20*x) - 1/40*sqrt(10)*sqrt(-sq
rt(2)*sqrt(sqrt(5) - 3))*log(-sqrt(10)*sqrt(-sqrt(2)*sqrt(sqrt(5) - 3))*(s
qrt(5) + 5) + 20*x) - 1/40*sqrt(10)*sqrt(sqrt(2)*sqrt(-sqrt(5) - 3))*log(s
qrt(10)*sqrt(sqrt(2)*sqrt(-sqrt(5) - 3))*(sqrt(5) - 5) + 20*x) + 1/40*sqrt
(10)*sqrt(sqrt(2)*sqrt(-sqrt(5) - 3))*log(-sqrt(10)*sqrt(sqrt(2)*sqrt(-sqr
t(5) - 3))*(sqrt(5) - 5) + 20*x) - 1/40*sqrt(10)*sqrt(-sqrt(2)*sqrt(-sqrt(
5) - 3))*log(sqrt(10)*sqrt(-sqrt(2)*sqrt(-sqrt(5) - 3))*(sqrt(5) - 5) + 20
*x) + 1/40*sqrt(10)*sqrt(-sqrt(2)*sqrt(-sqrt(5) - 3))*log(-sqrt(10)*sqrt(-
sqrt(2)*sqrt(-sqrt(5) - 3))*(sqrt(5) - 5) + 20*x)
```

3.10.6 Sympy [A] (verification not implemented)

Time = 0.86 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.05

$$\int \frac{1+x^4}{1+3x^4+x^8}\,dx = \operatorname{RootSum}\left(40960000t^8+19200t^4+1,\left(t\mapsto t\log\left(25600t^5+16t+x\right)\right)\right)$$

input
```
integrate((x**4+1)/(x**8+3*x**4+1),x)
```

output
```
RootSum(40960000*_t**8 + 19200*_t**4 + 1, Lambda(_t, _t*log(25600*_t**5 +
16*_t + x)))
```

3.10.7 Maxima [F]

$$\int \frac{1+x^4}{1+3x^4+x^8}\,dx = \int \frac{x^4+1}{x^8+3\,x^4+1}\,dx$$

input
```
integrate((x^4+1)/(x^8+3*x^4+1),x, algorithm="maxima")
```

output
```
integrate((x^4 + 1)/(x^8 + 3*x^4 + 1), x)
```

3.10.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.53

$$\int \frac{1+x^4}{1+3x^4+x^8}\,dx = \frac{1}{80}\left(\pi+4\arctan\left(x\sqrt{\sqrt{5}+1}+1\right)\right)\sqrt{5\sqrt{5}+5} - \frac{1}{80}\left(\pi+4\arctan\left(-x\sqrt{\sqrt{5}+1}+1\right)\right)\sqrt{5\sqrt{5}+5} + \frac{1}{80}\left(\pi+4\arctan\left(x\sqrt{\sqrt{5}-1}-1\right)\right)\sqrt{5\sqrt{5}-5} - \frac{1}{80}\left(\pi+4\arctan\left(-x\sqrt{\sqrt{5}-1}-1\right)\right)\sqrt{5\sqrt{5}-5} + \frac{1}{40}\sqrt{5\sqrt{5}-5}\log\left(16900\left(x+\sqrt{\sqrt{5}+1}\right)^2+16900x^2\right) - \frac{1}{40}\sqrt{5\sqrt{5}-5}\log\left(16900\left(x-\sqrt{\sqrt{5}+1}\right)^2+16900x^2\right) + \frac{1}{40}\sqrt{5\sqrt{5}+5}\log\left(2500\left(x+\sqrt{\sqrt{5}-1}\right)^2+2500x^2\right) - \frac{1}{40}\sqrt{5\sqrt{5}+5}\log\left(2500\left(x-\sqrt{\sqrt{5}-1}\right)^2+2500x^2\right)$$

input
```
integrate((x^4+1)/(x^8+3*x^4+1),x, algorithm="giac")
```

output
```
1/80*(pi + 4*arctan(x*sqrt(sqrt(5) + 1) + 1))*sqrt(5*sqrt(5) + 5) - 1/80*(
pi + 4*arctan(-x*sqrt(sqrt(5) + 1) + 1))*sqrt(5*sqrt(5) + 5) + 1/80*(pi +
4*arctan(x*sqrt(sqrt(5) - 1) - 1))*sqrt(5*sqrt(5) - 5) - 1/80*(pi + 4*arct
an(-x*sqrt(sqrt(5) - 1) - 1))*sqrt(5*sqrt(5) - 5) + 1/40*sqrt(5*sqrt(5) -
5)*log(16900*(x + sqrt(sqrt(5) + 1))^2 + 16900*x^2) - 1/40*sqrt(5*sqrt(5)
- 5)*log(16900*(x - sqrt(sqrt(5) + 1))^2 + 16900*x^2) + 1/40*sqrt(5*sqrt(5
) + 5)*log(2500*(x + sqrt(sqrt(5) - 1))^2 + 2500*x^2) - 1/40*sqrt(5*sqrt(5
) + 5)*log(2500*(x - sqrt(sqrt(5) - 1))^2 + 2500*x^2)
```

3.10.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.02

$$\int \frac{1+x^4}{1+3x^4+x^8}\,dx$$
$$= \frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{7\,2^{3/4}\,x\,\left(-\sqrt{5}-3\right)^{1/4}}{2\left(2\sqrt{2}\sqrt{-\sqrt{5}-3}+\sqrt{2}\sqrt{5}\sqrt{-\sqrt{5}-3}\right)}+\frac{3\,2^{3/4}\sqrt{5}\,x\left(-\sqrt{5}-3\right)^{1/4}}{2\left(2\sqrt{2}\sqrt{-\sqrt{5}-3}+\sqrt{2}\sqrt{5}\sqrt{-\sqrt{5}-3}\right)}\right)\left(-\sqrt{5}-3\right)^{1/4}}{20}$$
$$-\frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\left(-\sqrt{5}-3\right)^{1/4}7\mathrm{i}}{2\left(2\sqrt{2}\sqrt{-\sqrt{5}-3}+\sqrt{2}\sqrt{5}\sqrt{-\sqrt{5}-3}\right)}+\frac{2^{3/4}\sqrt{5}\,x\left(-\sqrt{5}-3\right)^{1/4}3\mathrm{i}}{2\left(2\sqrt{2}\sqrt{-\sqrt{5}-3}+\sqrt{2}\sqrt{5}\sqrt{-\sqrt{5}-3}\right)}\right)\left(-\sqrt{5}-3\right)^{1/4}1\mathrm{i}}{20}$$
$$-\frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{7\,2^{3/4}\,x\left(\sqrt{5}-3\right)^{1/4}}{2\left(2\sqrt{2}\sqrt{\sqrt{5}-3}-\sqrt{2}\sqrt{5}\sqrt{\sqrt{5}-3}\right)}-\frac{3\,2^{3/4}\sqrt{5}\,x\left(\sqrt{5}-3\right)^{1/4}}{2\left(2\sqrt{2}\sqrt{\sqrt{5}-3}-\sqrt{2}\sqrt{5}\sqrt{\sqrt{5}-3}\right)}\right)\left(\sqrt{5}-3\right)^{1/4}}{20}$$
$$+\frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\left(\sqrt{5}-3\right)^{1/4}7\mathrm{i}}{2\left(2\sqrt{2}\sqrt{\sqrt{5}-3}-\sqrt{2}\sqrt{5}\sqrt{\sqrt{5}-3}\right)}-\frac{2^{3/4}\sqrt{5}\,x\left(\sqrt{5}-3\right)^{1/4}3\mathrm{i}}{2\left(2\sqrt{2}\sqrt{\sqrt{5}-3}-\sqrt{2}\sqrt{5}\sqrt{\sqrt{5}-3}\right)}\right)\left(\sqrt{5}-3\right)^{1/4}1\mathrm{i}}{20}$$

input
```
int((x^4 + 1)/(3*x^4 + x^8 + 1),x)
```

output
```
(2^(3/4)*5^(1/2)*atan((7*2^(3/4)*x*(- 5^(1/2) - 3)^(1/4))/(2*(2*2^(1/2)*(-
 5^(1/2) - 3)^(1/2) + 2^(1/2)*5^(1/2)*(- 5^(1/2) - 3)^(1/2))) + (3*2^(3/4)
*5^(1/2)*x*(- 5^(1/2) - 3)^(1/4))/(2*(2*2^(1/2)*(- 5^(1/2) - 3)^(1/2) + 2^
(1/2)*5^(1/2)*(- 5^(1/2) - 3)^(1/2))))*(- 5^(1/2) - 3)^(1/4))/20 - (2^(3/4
)*5^(1/2)*atan((2^(3/4)*x*(- 5^(1/2) - 3)^(1/4)*7i)/(2*(2*2^(1/2)*(- 5^(1/
2) - 3)^(1/2) + 2^(1/2)*5^(1/2)*(- 5^(1/2) - 3)^(1/2))) + (2^(3/4)*5^(1/2)
*x*(- 5^(1/2) - 3)^(1/4)*3i)/(2*(2*2^(1/2)*(- 5^(1/2) - 3)^(1/2) + 2^(1/2)
*5^(1/2)*(- 5^(1/2) - 3)^(1/2))))*(- 5^(1/2) - 3)^(1/4)*1i)/20 - (2^(3/4)*
5^(1/2)*atan((7*2^(3/4)*x*(5^(1/2) - 3)^(1/4))/(2*(2*2^(1/2)*(5^(1/2) - 3)
^(1/2) - 2^(1/2)*5^(1/2)*(5^(1/2) - 3)^(1/2))) - (3*2^(3/4)*5^(1/2)*x*(5^(
1/2) - 3)^(1/4))/(2*(2*2^(1/2)*(5^(1/2) - 3)^(1/2) - 2^(1/2)*5^(1/2)*(5^(1
/2) - 3)^(1/2))))*(5^(1/2) - 3)^(1/4))/20 + (2^(3/4)*5^(1/2)*atan((2^(3/4)
*x*(5^(1/2) - 3)^(1/4)*7i)/(2*(2*2^(1/2)*(5^(1/2) - 3)^(1/2) - 2^(1/2)*5^(
1/2)*(5^(1/2) - 3)^(1/2))) - (2^(3/4)*5^(1/2)*x*(5^(1/2) - 3)^(1/4)*3i)/(2
*(2*2^(1/2)*(5^(1/2) - 3)^(1/2) - 2^(1/2)*5^(1/2)*(5^(1/2) - 3)^(1/2))))*(
5^(1/2) - 3)^(1/4)*1i)/20
```

3.11 $\int \frac{1+x^4}{1+2x^4+x^8} \, dx$

3.11.1 Optimal result

Integrand size = 18, antiderivative size = 85

$$\int \frac{1+x^4}{1+2x^4+x^8} \, dx = -\frac{\arctan\left(1-\sqrt{2}x\right)}{2\sqrt{2}} + \frac{\arctan\left(1+\sqrt{2}x\right)}{2\sqrt{2}} - \frac{\log\left(1-\sqrt{2}x+x^2\right)}{4\sqrt{2}} + \frac{\log\left(1+\sqrt{2}x+x^2\right)}{4\sqrt{2}}$$

output
```
1/4*arctan(-1+x*2^(1/2))*2^(1/2)+1/4*arctan(1+x*2^(1/2))*2^(1/2)-1/8*ln(1+
x^2-x*2^(1/2))*2^(1/2)+1/8*ln(1+x^2+x*2^(1/2))*2^(1/2)
```

3.11.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.75

$$\int \frac{1+x^4}{1+2x^4+x^8} \, dx = \frac{-2\arctan\left(1-\sqrt{2}x\right) + 2\arctan\left(1+\sqrt{2}x\right) - \log\left(1-\sqrt{2}x+x^2\right) + \log\left(1+\sqrt{2}x+x^2\right)}{4\sqrt{2}}$$

input
```
Integrate[(1 + x^4)/(1 + 2*x^4 + x^8),x]
```

output
```
(-2*ArcTan[1 - Sqrt[2]*x] + 2*ArcTan[1 + Sqrt[2]*x] - Log[1 - Sqrt[2]*x +
x^2] + Log[1 + Sqrt[2]*x + x^2])/(4*Sqrt[2])
```

3.11.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1380, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4+1}{x^8+2x^4+1}\,dx$$

$$\Big\downarrow \text{ 1380}$$

$$\int \frac{1}{x^4+1}dx$$

$$\Big\downarrow \text{ 755}$$

$$\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx+\frac{1}{2}\int \frac{x^2+1}{x^4+1}dx$$

$$\Big\downarrow \text{ 1476}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2-\sqrt{2}x+1}dx+\frac{1}{2}\int \frac{1}{x^2+\sqrt{2}x+1}dx\right)+\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx+\frac{1}{2}\left(\frac{\int \frac{1}{-\left(1-\sqrt{2}x\right)^2-1}d\left(1-\sqrt{2}x\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\left(\sqrt{2}x+1\right)^2-1}d\left(\sqrt{2}x+1\right)}{\sqrt{2}}\right)$$

$$\Big\downarrow \text{ 217}$$

$$\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)$$

$$\Big\downarrow \text{ 1479}$$

$$\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}x+1\right)}{x^2+\sqrt{2}x+1}dx}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}x+1\right)}{x^2+\sqrt{2}x+1}dx}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)$$

$$\downarrow 27$$

$$\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}x+1}{x^2+\sqrt{2}x+1}dx\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)$$

$$\downarrow 1103$$

$$\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(x^2+\sqrt{2}x+1\right)}{2\sqrt{2}}-\frac{\log\left(x^2-\sqrt{2}x+1\right)}{2\sqrt{2}}\right)$$

input
```
Int[(1 + x^4)/(1 + 2*x^4 + x^8),x]
```

output
```
(-(ArcTan[1 - Sqrt[2]*x]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*x]/Sqrt[2])/2 + (-1
/2*Log[1 - Sqrt[2]*x + x^2]/Sqrt[2] + Log[1 + Sqrt[2]*x + x^2]/(2*Sqrt[2])
)/2
```

3.11.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.11.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.26

method	result	size
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{4}+1\right)}\frac{\ln\left(x-\_R\right)}{\_R^{3}}\right)}{4}$	22
default	$\frac{\sqrt{2}\,\left(\ln\left(\frac{1+x^{2}+x\sqrt{2}}{1+x^{2}-x\sqrt{2}}\right)+2\arctan\left(x\sqrt{2}+1\right)+2\arctan\left(x\sqrt{2}-1\right)\right)}{8}$	52

input
```
int((x^4+1)/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(1/_R^3*ln(x-_R),_R=RootOf(_Z^4+1))
```

3.11.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.72

$$\int \frac{1+x^4}{1+2x^4+x^8}\,dx = \left(\frac{1}{8}i+\frac{1}{8}\right)\sqrt{2}\log\left(2\,x+(i+1)\,\sqrt{2}\right) - \left(\frac{1}{8}i-\frac{1}{8}\right)\sqrt{2}\log\left(2\,x-(i-1)\,\sqrt{2}\right) + \left(\frac{1}{8}i-\frac{1}{8}\right)\sqrt{2}\log\left(2\,x+(i-1)\,\sqrt{2}\right) - \left(\frac{1}{8}i+\frac{1}{8}\right)\sqrt{2}\log\left(2\,x-(i+1)\,\sqrt{2}\right)$$

input
```
integrate((x^4+1)/(x^8+2*x^4+1),x, algorithm="fricas")
```

output
```
(1/8*I + 1/8)*sqrt(2)*log(2*x + (I + 1)*sqrt(2)) - (1/8*I - 1/8)*sqrt(2)*l
og(2*x - (I - 1)*sqrt(2)) + (1/8*I - 1/8)*sqrt(2)*log(2*x + (I - 1)*sqrt(2
)) - (1/8*I + 1/8)*sqrt(2)*log(2*x - (I + 1)*sqrt(2))
```

3.11.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

$$\int \frac{1+x^4}{1+2x^4+x^8}\,dx = -\frac{\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{8} + \frac{\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{8} + \frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x-1\right)}{4} + \frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x+1\right)}{4}$$

input
```
integrate((x**4+1)/(x**8+2*x**4+1),x)
```

output
```
-sqrt(2)*log(x**2 - sqrt(2)*x + 1)/8 + sqrt(2)*log(x**2 + sqrt(2)*x + 1)/8
 + sqrt(2)*atan(sqrt(2)*x - 1)/4 + sqrt(2)*atan(sqrt(2)*x + 1)/4
```

3.11.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.85

$$\int \frac{1+x^4}{1+2x^4+x^8}\,dx = \frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right) + \frac{1}{8}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{1}{8}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)$$

input
```
integrate((x^4+1)/(x^8+2*x^4+1),x, algorithm="maxima")
```

output
```
1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*sqrt(2)*arctan(1/2*s
qrt(2)*(2*x - sqrt(2))) + 1/8*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/8*sqrt(
2)*log(x^2 - sqrt(2)*x + 1)
```

3.11.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.85

$$\int \frac{1+x^4}{1+2x^4+x^8}\,dx = \frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right) + \frac{1}{8}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{1}{8}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)$$

input
```
integrate((x^4+1)/(x^8+2*x^4+1),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*sqrt(2)*arctan(1/2*s
qrt(2)*(2*x - sqrt(2))) + 1/8*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/8*sqrt(
2)*log(x^2 - sqrt(2)*x + 1)
```

3.11.9 Mupad [B] (verification not implemented)

Time = 8.34 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.39

$$\int \frac{1+x^4}{1+2x^4+x^8}\,dx = \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{4}+\frac{1}{4}\mathrm{i}\right) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{4}-\frac{1}{4}\mathrm{i}\right)$$

input
```
int((x^4 + 1)/(2*x^4 + x^8 + 1),x)
```

output
```
2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(1/4 + 1i/4) + 2^(1/2)*atan(2^(1/2)*x
*(1/2 + 1i/2))*(1/4 - 1i/4)
```

3.11. $\int \frac{1+x^4}{1+2x^4+x^8}\,dx$

3.12 $\int \frac{1+x^4}{1+x^4+x^8} dx$

3.12.1 Optimal result

Integrand size = 16, antiderivative size = 140

$$\int \frac{1+x^4}{1+x^4+x^8} dx = -\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{4}\arctan\left(\sqrt{3}-2x\right) + \frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{1}{4}\arctan\left(\sqrt{3}+2x\right) - \frac{1}{8}\log\left(1-x+x^2\right) + \frac{1}{8}\log\left(1+x+x^2\right) - \frac{\log\left(1-\sqrt{3}x+x^2\right)}{8\sqrt{3}} + \frac{\log\left(1+\sqrt{3}x+x^2\right)}{8\sqrt{3}}$$

output
```
1/4*arctan(2*x-3^(1/2))+1/4*arctan(2*x+3^(1/2))-1/8*ln(x^2-x+1)+1/8*ln(x^2
+x+1)-1/12*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1/12*arctan(1/3*(1+2*x)*3^(
1/2))*3^(1/2)-1/24*ln(1+x^2-x*3^(1/2))*3^(1/2)+1/24*ln(1+x^2+x*3^(1/2))*3^
(1/2)
```

3.12.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.96

$$\int \frac{1+x^4}{1+x^4+x^8} dx = \frac{1}{48}\left(4i\sqrt{-6-6i\sqrt{3}}\arctan\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x\right) - 4i\sqrt{-6+6i\sqrt{3}}\arctan\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x\right) + 4\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right) + 4\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right) - 6\log\left(1-x+x^2\right) + 6\log\left(1+x+x^2\right)\right)$$

input
```
Integrate[(1 + x^4)/(1 + x^4 + x^8),x]
```

output
```
((4*I)*Sqrt[-6 - (6*I)*Sqrt[3]]*ArcTan[((1 - I*Sqrt[3])*x)/2] - (4*I)*Sqrt
[-6 + (6*I)*Sqrt[3]]*ArcTan[((1 + I*Sqrt[3])*x)/2] + 4*Sqrt[3]*ArcTan[(-1
+ 2*x)/Sqrt[3]] + 4*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 6*Log[1 - x + x^2]
 + 6*Log[1 + x + x^2])/48
```

3.12.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.21, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {1749, 1407, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4+1}{x^8+x^4+1}dx$$

$$\Big\downarrow \text{1749}$$

$$\frac{1}{2}\int \frac{1}{x^4-x^2+1}dx+\frac{1}{2}\int \frac{1}{x^4+x^2+1}dx$$

$$\Big\downarrow \text{1407}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1-x}{x^2-x+1}dx+\frac{1}{2}\int \frac{x+1}{x^2+x+1}dx\right)+\frac{1}{2}\left(\frac{\int \frac{\sqrt{3}-x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}+\frac{\int \frac{x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}\right)$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2-x+1}dx-\frac{1}{2}\int -\frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2+x+1}dx+\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx\right)\right)+\frac{1}{2}\left(\frac{\frac{1}{2}\sqrt{3}\int \frac{1}{x^2-\sqrt{3}x+1}dx-\frac{1}{2}\int -\frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}+\frac{\frac{1}{2}\sqrt{3}\int \frac{1}{x^2+\sqrt{3}x+1}dx+\frac{1}{2}\int \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2-x+1}dx+\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2+x+1}dx+\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx\right)\right)+\frac{1}{2}\left(\frac{\frac{1}{2}\sqrt{3}\int \frac{1}{x^2-\sqrt{3}x+1}dx+\frac{1}{2}\int \frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}+\frac{\frac{1}{2}\sqrt{3}\int \frac{1}{x^2+\sqrt{3}x+1}dx+\frac{1}{2}\int \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx-\int \frac{1}{-(2x-1)^2-3}d(2x-1)\right)+\frac{1}{2}\left(\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx-\int \frac{1}{-(2x+1)^2-3}d(2x+1\right.\right.$$
$$\frac{1}{2}\left(\frac{\frac{1}{2}\int \frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx-\sqrt{3}\int \frac{1}{-\left(2x-\sqrt{3}\right)^2-1}d\left(2x-\sqrt{3}\right)}{2\sqrt{3}}+\frac{\frac{1}{2}\int \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx-\sqrt{3}\int \frac{1}{-\left(2x+\sqrt{3}\right)^2-1}d\left(2x+\sqrt{3}\right)}{2\sqrt{3}}\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx+\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}\right)+\frac{1}{2}\left(\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx+\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)\right)+$$
$$\frac{1}{2}\left(\frac{\frac{1}{2}\int \frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx-\sqrt{3}\arctan\left(\sqrt{3}-2x\right)}{2\sqrt{3}}+\frac{\frac{1}{2}\int \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx+\sqrt{3}\arctan\left(2x+\sqrt{3}\right)}{2\sqrt{3}}\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2}\log\left(x^2-x+1\right)\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(x^2+x+1\right)\right)\right)+$$
$$\frac{1}{2}\left(\frac{-\sqrt{3}\arctan\left(\sqrt{3}-2x\right)-\frac{1}{2}\log\left(x^2-\sqrt{3}x+1\right)}{2\sqrt{3}}+\frac{\sqrt{3}\arctan\left(2x+\sqrt{3}\right)+\frac{1}{2}\log\left(x^2+\sqrt{3}x+1\right)}{2\sqrt{3}}\right)$$

input
```
Int[(1 + x^4)/(1 + x^4 + x^8),x]
```

output
```
((ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3] - Log[1 - x + x^2]/2)/2 + (ArcTan[(1
+ 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 + x + x^2]/2)/2)/2 + ((-(Sqrt[3]*ArcTan[Sq
rt[3] - 2*x]) - Log[1 - Sqrt[3]*x + x^2]/2)/(2*Sqrt[3]) + (Sqrt[3]*ArcTan[
Sqrt[3] + 2*x] + Log[1 + Sqrt[3]*x + x^2]/2)/(2*Sqrt[3]))/2
```

3.12.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1407
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   Int[(r - x)/(q - r*
x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(r + x)/(q + r*x + x^2), x], x]]]
 /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

rule 1749
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e
+ q*x^(n/2) + x^n, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x^(n/2) +
 x^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[2*(d/e) - b/c,
0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

3.12.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.14 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.64

method	result
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(9\_Z^4+3\_Z^2+1\right)}\_R\ln\left(-3\_R^3+\_R+x\right)\right)}{4}-\frac{\ln\left(4x^2-4x+4\right)}{8}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{12}+\frac{\ln\left(4x^2+4x+4\right)}{8}+$
default	$-\frac{\ln\left(x^2-x+1\right)}{8}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{12}+\frac{\ln\left(x^2+x+1\right)}{8}+\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{12}-\frac{\ln\left(1+x^2-x\sqrt{3}\right)\sqrt{3}}{24}+\frac{\arctan\left(2x-\sqrt{3}\right.}{4}$

input
```
int((x^4+1)/(x^8+x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(_R*ln(-3*_R^3+_R+x),_R=RootOf(9*_Z^4+3*_Z^2+1))-1/8*ln(4*x^2-4*x+4
)+1/12*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))+1/8*ln(4*x^2+4*x+4)+1/12*arctan
(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

3.12.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.49

$$\begin{aligned}\int \frac{1+x^4}{1+x^4+x^8}\,dx = &-\frac{1}{24}\sqrt{6}\sqrt{i\sqrt{3}-1}\log\left(\sqrt{6}\sqrt{i\sqrt{3}-1}\left(i\sqrt{3}-3\right)+12x\right)\\ &+\frac{1}{24}\sqrt{6}\sqrt{i\sqrt{3}-1}\log\left(\sqrt{6}\sqrt{i\sqrt{3}-1}\left(-i\sqrt{3}+3\right)+12x\right)\\ &+\frac{1}{24}\sqrt{6}\sqrt{-i\sqrt{3}-1}\log\left(\sqrt{6}\left(i\sqrt{3}+3\right)\sqrt{-i\sqrt{3}-1}+12x\right)\\ &-\frac{1}{24}\sqrt{6}\sqrt{-i\sqrt{3}-1}\log\left(\sqrt{6}\sqrt{-i\sqrt{3}-1}\left(-i\sqrt{3}-3\right)+12x\right)\\ &+\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)\\ &+\frac{1}{8}\log\left(x^2+x+1\right)-\frac{1}{8}\log\left(x^2-x+1\right)\end{aligned}$$

input
```
integrate((x^4+1)/(x^8+x^4+1),x, algorithm="fricas")
```

output
```
-1/24*sqrt(6)*sqrt(I*sqrt(3) - 1)*log(sqrt(6)*sqrt(I*sqrt(3) - 1)*(I*sqrt(
3) - 3) + 12*x) + 1/24*sqrt(6)*sqrt(I*sqrt(3) - 1)*log(sqrt(6)*sqrt(I*sqrt
(3) - 1)*(-I*sqrt(3) + 3) + 12*x) + 1/24*sqrt(6)*sqrt(-I*sqrt(3) - 1)*log(
sqrt(6)*(I*sqrt(3) + 3)*sqrt(-I*sqrt(3) - 1) + 12*x) - 1/24*sqrt(6)*sqrt(-
I*sqrt(3) - 1)*log(sqrt(6)*sqrt(-I*sqrt(3) - 1)*(-I*sqrt(3) - 3) + 12*x) +
 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/12*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x - 1)) + 1/8*log(x^2 + x + 1) - 1/8*log(x^2 - x + 1)
```

3.12.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.36

$$\int \frac{1+x^4}{1+x^4+x^8}\,dx = \left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x-1-\frac{\sqrt{3}i}{3}+9216\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^5\right) + \left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x-1+9216\left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^5+\frac{\sqrt{3}i}{3}\right) + \left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x+1-\frac{\sqrt{3}i}{3}+9216\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^5\right) + \left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x+1+9216\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^5+\frac{\sqrt{3}i}{3}\right) + \operatorname{RootSum}\left(2304t^4+48t^2+1,\left(t\mapsto t\log\left(9216t^5+8t+x\right)\right)\right)$$

input
```
integrate((x**4+1)/(x**8+x**4+1),x)
```

output
```
(-1/8 - sqrt(3)*I/24)*log(x - 1 - sqrt(3)*I/3 + 9216*(-1/8 - sqrt(3)*I/24)
**5) + (-1/8 + sqrt(3)*I/24)*log(x - 1 + 9216*(-1/8 + sqrt(3)*I/24)**5 + s
qrt(3)*I/3) + (1/8 - sqrt(3)*I/24)*log(x + 1 - sqrt(3)*I/3 + 9216*(1/8 - s
qrt(3)*I/24)**5) + (1/8 + sqrt(3)*I/24)*log(x + 1 + 9216*(1/8 + sqrt(3)*I/
24)**5 + sqrt(3)*I/3) + RootSum(2304*_t**4 + 48*_t**2 + 1, Lambda(_t, _t*l
og(9216*_t**5 + 8*_t + x)))
```

3.12.7 Maxima [F]

$$\int \frac{1+x^4}{1+x^4+x^8}\,dx = \int \frac{x^4+1}{x^8+x^4+1}\,dx$$

input
```
integrate((x^4+1)/(x^8+x^4+1),x, algorithm="maxima")
```

output
```
1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/12*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x - 1)) + 1/2*integrate(1/(x^4 - x^2 + 1), x) + 1/8*log(x^2 + x + 1)
 - 1/8*log(x^2 - x + 1)
```

3.12.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.77

$$\int \frac{1+x^4}{1+x^4+x^8}\,dx = \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{24}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right) - \frac{1}{24}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right) + \frac{1}{4}\arctan\left(2x+\sqrt{3}\right) + \frac{1}{4}\arctan\left(2x-\sqrt{3}\right) + \frac{1}{8}\log\left(x^2+x+1\right) - \frac{1}{8}\log\left(x^2-x+1\right)$$

input
```
integrate((x^4+1)/(x^8+x^4+1),x, algorithm="giac")
```

output
```
1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/12*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x - 1)) + 1/24*sqrt(3)*log(x^2 + sqrt(3)*x + 1) - 1/24*sqrt(3)*log(x
^2 - sqrt(3)*x + 1) + 1/4*arctan(2*x + sqrt(3)) + 1/4*arctan(2*x - sqrt(3)
) + 1/8*log(x^2 + x + 1) - 1/8*log(x^2 - x + 1)
```

3.12.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.68

$$\int \frac{1+x^4}{1+x^4+x^8}\,dx = \operatorname{atan}\left(\frac{2x}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) + \operatorname{atan}\left(\frac{2x}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) + \operatorname{atan}\left(\frac{x\,2\mathrm{i}}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{12}+\frac{1}{4}\mathrm{i}\right) + \operatorname{atan}\left(\frac{x\,2\mathrm{i}}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{12}-\frac{1}{4}\mathrm{i}\right)$$

input
```
int((x^4 + 1)/(x^4 + x^8 + 1),x)
```

output
```
atan((2*x)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/12 - 1/4) + atan((2*x)/(3^(1/2)
*1i + 1))*((3^(1/2)*1i)/12 + 1/4) + atan((x*2i)/(3^(1/2)*1i - 1))*(3^(1/2)
/12 + 1i/4) + atan((x*2i)/(3^(1/2)*1i + 1))*(3^(1/2)/12 - 1i/4)
```

3.13 $\int \frac{1+x^4}{1+x^8} \, dx$

3.13.1 Optimal result

Integrand size = 13, antiderivative size = 347

$$\begin{aligned}\int \frac{1+x^4}{1+x^8} \, dx = & -\frac{1}{4}\sqrt{\frac{1}{2}\left(2-\sqrt{2}\right)} \arctan\left(\frac{\sqrt{2-\sqrt{2}}-2x}{\sqrt{2+\sqrt{2}}}\right) \\ & -\frac{1}{4}\sqrt{\frac{1}{2}\left(2+\sqrt{2}\right)} \arctan\left(\frac{\sqrt{2+\sqrt{2}}-2x}{\sqrt{2-\sqrt{2}}}\right) \\ & +\frac{1}{4}\sqrt{\frac{1}{2}\left(2-\sqrt{2}\right)} \arctan\left(\frac{\sqrt{2-\sqrt{2}}+2x}{\sqrt{2+\sqrt{2}}}\right) \\ & +\frac{1}{4}\sqrt{\frac{1}{2}\left(2+\sqrt{2}\right)} \arctan\left(\frac{\sqrt{2+\sqrt{2}}+2x}{\sqrt{2-\sqrt{2}}}\right) \\ & -\frac{\log\left(1-\sqrt{2-\sqrt{2}}x+x^2\right)}{8\sqrt{2-\sqrt{2}}}+\frac{\log\left(1+\sqrt{2-\sqrt{2}}x+x^2\right)}{8\sqrt{2-\sqrt{2}}} \\ & -\frac{\log\left(1-\sqrt{2+\sqrt{2}}x+x^2\right)}{8\sqrt{2+\sqrt{2}}}+\frac{\log\left(1+\sqrt{2+\sqrt{2}}x+x^2\right)}{8\sqrt{2+\sqrt{2}}}\end{aligned}$$

output
```
-1/8*arctan((-2*x+(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2))*(4-2*2^(1/2))^(1/2
)+1/8*arctan((2*x+(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2))*(4-2*2^(1/2))^(1/2
)-1/8*ln(1+x^2-x*(2-2^(1/2))^(1/2))/(2-2^(1/2))^(1/2)+1/8*ln(1+x^2+x*(2-2^
(1/2))^(1/2))/(2-2^(1/2))^(1/2)-1/8*arctan((-2*x+(2+2^(1/2))^(1/2))/(2-2^(
1/2))^(1/2))*(4+2*2^(1/2))^(1/2)+1/8*arctan((2*x+(2+2^(1/2))^(1/2))/(2-2^(
1/2))^(1/2))*(4+2*2^(1/2))^(1/2)-1/8*ln(1+x^2-x*(2+2^(1/2))^(1/2))/(2+2^(1
/2))^(1/2)+1/8*ln(1+x^2+x*(2+2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)
```

3.13.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.74

$$\int \frac{1+x^4}{1+x^8}\,dx = \frac{1}{8}\Big(2\arctan\left(\sec\left(\frac{\pi}{8}\right)\left(x+\sin\left(\frac{\pi}{8}\right)\right)\right)\left(\cos\left(\frac{\pi}{8}\right)-\sin\left(\frac{\pi}{8}\right)\right)$$
$$+2\arctan\left(x\sec\left(\frac{\pi}{8}\right)-\tan\left(\frac{\pi}{8}\right)\right)\left(\cos\left(\frac{\pi}{8}\right)-\sin\left(\frac{\pi}{8}\right)\right)$$
$$+\log\left(1+x^2+2x\cos\left(\frac{\pi}{8}\right)\right)\left(\cos\left(\frac{\pi}{8}\right)-\sin\left(\frac{\pi}{8}\right)\right)$$
$$+\log\left(1+x^2-2x\cos\left(\frac{\pi}{8}\right)\right)\left(-\cos\left(\frac{\pi}{8}\right)+\sin\left(\frac{\pi}{8}\right)\right)$$
$$+2\arctan\left(\left(x-\cos\left(\frac{\pi}{8}\right)\right)\csc\left(\frac{\pi}{8}\right)\right)\left(\cos\left(\frac{\pi}{8}\right)+\sin\left(\frac{\pi}{8}\right)\right)$$
$$+2\arctan\left(\left(x+\cos\left(\frac{\pi}{8}\right)\right)\csc\left(\frac{\pi}{8}\right)\right)\left(\cos\left(\frac{\pi}{8}\right)+\sin\left(\frac{\pi}{8}\right)\right)$$
$$-\log\left(1+x^2-2x\sin\left(\frac{\pi}{8}\right)\right)\left(\cos\left(\frac{\pi}{8}\right)+\sin\left(\frac{\pi}{8}\right)\right)$$
$$+\log\left(1+x^2+2x\sin\left(\frac{\pi}{8}\right)\right)\left(\cos\left(\frac{\pi}{8}\right)+\sin\left(\frac{\pi}{8}\right)\right)\Big)$$

input
```
Integrate[(1 + x^4)/(1 + x^8),x]
```

output
```
(2*ArcTan[Sec[Pi/8]*(x + Sin[Pi/8])]*(Cos[Pi/8] - Sin[Pi/8]) + 2*ArcTan[x*
Sec[Pi/8] - Tan[Pi/8]]*(Cos[Pi/8] - Sin[Pi/8]) + Log[1 + x^2 + 2*x*Cos[Pi/
8]]*(Cos[Pi/8] - Sin[Pi/8]) + Log[1 + x^2 - 2*x*Cos[Pi/8]]*(-Cos[Pi/8] + S
in[Pi/8]) + 2*ArcTan[(x - Cos[Pi/8])*Csc[Pi/8]]*(Cos[Pi/8] + Sin[Pi/8]) +
2*ArcTan[(x + Cos[Pi/8])*Csc[Pi/8]]*(Cos[Pi/8] + Sin[Pi/8]) - Log[1 + x^2
- 2*x*Sin[Pi/8]]*(Cos[Pi/8] + Sin[Pi/8]) + Log[1 + x^2 + 2*x*Sin[Pi/8]]*(C
os[Pi/8] + Sin[Pi/8]))/8
```

3.13.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {1743, 1407, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4+1}{x^8+1}\,dx$$

$$\Big\downarrow \text{1743}$$

$$\frac{1}{2}\int \frac{1}{x^4-\sqrt{2}x^2+1}dx+\frac{1}{2}\int \frac{1}{x^4+\sqrt{2}x^2+1}dx$$

$$\downarrow 1407$$

$$\frac{1}{2}\left(\frac{\int \frac{\sqrt{2-\sqrt{2}}-x}{x^2-\sqrt{2-\sqrt{2}}x+1}dx}{2\sqrt{2-\sqrt{2}}}+\frac{\int \frac{x+\sqrt{2-\sqrt{2}}}{x^2+\sqrt{2-\sqrt{2}}x+1}dx}{2\sqrt{2-\sqrt{2}}}\right)+\frac{1}{2}\left(\frac{\int \frac{\sqrt{2+\sqrt{2}}-x}{x^2-\sqrt{2+\sqrt{2}}x+1}dx}{2\sqrt{2+\sqrt{2}}}+\frac{\int \frac{x+\sqrt{2+\sqrt{2}}}{x^2+\sqrt{2+\sqrt{2}}x+1}dx}{2\sqrt{2+\sqrt{2}}}\right)$$

$$\downarrow 1142$$

$$\frac{1}{2}\left(\frac{\frac{1}{2}\sqrt{2-\sqrt{2}}\int \frac{1}{x^2-\sqrt{2-\sqrt{2}}x+1}dx-\frac{1}{2}\int -\frac{\sqrt{2-\sqrt{2}}-2x}{x^2-\sqrt{2-\sqrt{2}}x+1}dx}{2\sqrt{2-\sqrt{2}}}+\frac{\frac{1}{2}\sqrt{2-\sqrt{2}}\int \frac{1}{x^2+\sqrt{2-\sqrt{2}}x+1}dx+\frac{1}{2}\int \frac{2x+\sqrt{2-\sqrt{2}}}{x^2+\sqrt{2-\sqrt{2}}x+1}dx}{2\sqrt{2-\sqrt{2}}}\right.$$

$$\frac{1}{2}\left(\frac{\frac{1}{2}\sqrt{2+\sqrt{2}}\int \frac{1}{x^2-\sqrt{2+\sqrt{2}}x+1}dx-\frac{1}{2}\int -\frac{\sqrt{2+\sqrt{2}}-2x}{x^2-\sqrt{2+\sqrt{2}}x+1}dx}{2\sqrt{2+\sqrt{2}}}+\frac{\frac{1}{2}\sqrt{2+\sqrt{2}}\int \frac{1}{x^2+\sqrt{2+\sqrt{2}}x+1}dx+\frac{1}{2}\int \frac{2x+\sqrt{2+\sqrt{2}}}{x^2+\sqrt{2+\sqrt{2}}x+1}dx}{2\sqrt{2+\sqrt{2}}}\right.$$

$$\downarrow 25$$

$$\frac{1}{2}\left(\frac{\frac{1}{2}\sqrt{2-\sqrt{2}}\int \frac{1}{x^2-\sqrt{2-\sqrt{2}}x+1}dx+\frac{1}{2}\int \frac{\sqrt{2-\sqrt{2}}-2x}{x^2-\sqrt{2-\sqrt{2}}x+1}dx}{2\sqrt{2-\sqrt{2}}}+\frac{\frac{1}{2}\sqrt{2-\sqrt{2}}\int \frac{1}{x^2+\sqrt{2-\sqrt{2}}x+1}dx+\frac{1}{2}\int \frac{2x+\sqrt{2-\sqrt{2}}}{x^2+\sqrt{2-\sqrt{2}}x+1}dx}{2\sqrt{2-\sqrt{2}}}\right)$$

$$\frac{1}{2}\left(\frac{\frac{1}{2}\sqrt{2+\sqrt{2}}\int \frac{1}{x^2-\sqrt{2+\sqrt{2}}x+1}dx+\frac{1}{2}\int \frac{\sqrt{2+\sqrt{2}}-2x}{x^2-\sqrt{2+\sqrt{2}}x+1}dx}{2\sqrt{2+\sqrt{2}}}+\frac{\frac{1}{2}\sqrt{2+\sqrt{2}}\int \frac{1}{x^2+\sqrt{2+\sqrt{2}}x+1}dx+\frac{1}{2}\int \frac{2x+\sqrt{2+\sqrt{2}}}{x^2+\sqrt{2+\sqrt{2}}x+1}dx}{2\sqrt{2+\sqrt{2}}}\right)$$

$$\downarrow 1083$$

$$\frac{1}{2}\left(\frac{\frac{1}{2}\int \frac{\sqrt{2-\sqrt{2}}-2x}{x^2-\sqrt{2-\sqrt{2}}x+1}dx-\sqrt{2-\sqrt{2}}\int \frac{1}{-\left(2x-\sqrt{2-\sqrt{2}}\right)^2-\sqrt{2}-2}d\left(2x-\sqrt{2-\sqrt{2}}\right)}{2\sqrt{2-\sqrt{2}}}+\frac{\frac{1}{2}\int \frac{2x+\sqrt{2-\sqrt{2}}}{x^2+\sqrt{2-\sqrt{2}}x+1}dx-\sqrt{2-\sqrt{\cdots}}}{\cdots}\right.$$

$$\frac{1}{2}\left(\frac{\frac{1}{2}\int \frac{\sqrt{2+\sqrt{2}}-2x}{x^2-\sqrt{2+\sqrt{2}}x+1}dx-\sqrt{2+\sqrt{2}}\int \frac{1}{-\left(2x-\sqrt{2+\sqrt{2}}\right)^2+\sqrt{2}-2}d\left(2x-\sqrt{2+\sqrt{2}}\right)}{2\sqrt{2+\sqrt{2}}}+\frac{\frac{1}{2}\int \frac{2x+\sqrt{2+\sqrt{2}}}{x^2+\sqrt{2+\sqrt{2}}x+1}dx-\sqrt{2+\sqrt{\cdots}}}{\cdots}\right.$$

$$\downarrow 217$$

3.13. $\int \frac{1+x^4}{1+x^8}\,dx$

$$\frac{1}{2}\left(\frac{\frac{1}{2}\int \frac{\sqrt{2-\sqrt{2}}-2x}{x^2-\sqrt{2-\sqrt{2}}x+1}dx+\sqrt{\frac{2-\sqrt{2}}{2+\sqrt{2}}}\arctan\left(\frac{2x-\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2-\sqrt{2}}}+\frac{\frac{1}{2}\int \frac{2x+\sqrt{2-\sqrt{2}}}{x^2+\sqrt{2-\sqrt{2}}x+1}dx+\sqrt{\frac{2-\sqrt{2}}{2+\sqrt{2}}}\arctan\left(\frac{2x+\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2-\sqrt{2}}}\right)+\frac{1}{2}\left(\frac{\frac{1}{2}\int \frac{\sqrt{2+\sqrt{2}}-2x}{x^2-\sqrt{2+\sqrt{2}}x+1}dx+\sqrt{\frac{2+\sqrt{2}}{2-\sqrt{2}}}\arctan\left(\frac{2x-\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2+\sqrt{2}}}+\frac{\frac{1}{2}\int \frac{2x+\sqrt{2+\sqrt{2}}}{x^2+\sqrt{2+\sqrt{2}}x+1}dx+\sqrt{\frac{2+\sqrt{2}}{2-\sqrt{2}}}\arctan\left(\frac{2x+\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2+\sqrt{2}}}\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\left(\frac{\sqrt{\frac{2-\sqrt{2}}{2+\sqrt{2}}}\arctan\left(\frac{2x-\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)-\frac{1}{2}\log\left(x^2-\sqrt{2-\sqrt{2}}x+1\right)}{2\sqrt{2-\sqrt{2}}}+\frac{\sqrt{\frac{2-\sqrt{2}}{2+\sqrt{2}}}\arctan\left(\frac{2x+\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)+\frac{1}{2}\log\left(x^2+\sqrt{2\right.}{2\sqrt{2-\sqrt{2}}}\right.$$

$$\frac{1}{2}\left(\frac{\sqrt{\frac{2+\sqrt{2}}{2-\sqrt{2}}}\arctan\left(\frac{2x-\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)-\frac{1}{2}\log\left(x^2-\sqrt{2+\sqrt{2}}x+1\right)}{2\sqrt{2+\sqrt{2}}}+\frac{\sqrt{\frac{2+\sqrt{2}}{2-\sqrt{2}}}\arctan\left(\frac{2x+\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)+\frac{1}{2}\log\left(x^2+\sqrt{2\right.}{2\sqrt{2+\sqrt{2}}}\right.$$

input
```
Int[(1 + x^4)/(1 + x^8),x]
```

output
```
((Sqrt[(2 - Sqrt[2])/(2 + Sqrt[2])]*ArcTan[(-Sqrt[2 - Sqrt[2]] + 2*x)/Sqrt
[2 + Sqrt[2]]] - Log[1 - Sqrt[2 - Sqrt[2]]*x + x^2]/2)/(2*Sqrt[2 - Sqrt[2]
]) + (Sqrt[(2 - Sqrt[2])/(2 + Sqrt[2])]*ArcTan[(Sqrt[2 - Sqrt[2]] + 2*x)/S
qrt[2 + Sqrt[2]]] + Log[1 + Sqrt[2 - Sqrt[2]]*x + x^2]/2)/(2*Sqrt[2 - Sqrt
[2]]))/2 + ((Sqrt[(2 + Sqrt[2])/(2 - Sqrt[2])]*ArcTan[(-Sqrt[2 + Sqrt[2]]
+ 2*x)/Sqrt[2 - Sqrt[2]]] - Log[1 - Sqrt[2 + Sqrt[2]]*x + x^2]/2)/(2*Sqrt[
2 + Sqrt[2]]) + (Sqrt[(2 + Sqrt[2])/(2 - Sqrt[2])]*ArcTan[(Sqrt[2 + Sqrt[2
]] + 2*x)/Sqrt[2 - Sqrt[2]]] + Log[1 + Sqrt[2 + Sqrt[2]]*x + x^2]/2)/(2*Sq
rt[2 + Sqrt[2]]))/2
```

3.13.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1407
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   Int[(r - x)/(q - r*
x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(r + x)/(q + r*x + x^2), x], x]]]
 /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

rule 1743
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> With[{
q = Rt[2*d*e, 2]}, Simp[e^2/(2*c)   Int[1/(d + q*x^(n/2) + e*x^n), x], x] +
 Simp[e^2/(2*c)   Int[1/(d - q*x^(n/2) + e*x^n), x], x]] /; FreeQ[{a, c, d,
 e}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && PosQ[d*
e]
```

3.13.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.62 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.08

method	result	size
default	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{8}+1\right)}\frac{\left(\_R^{4}+1\right)\ln\left(x-\_R\right)}{\_R^{7}}\right)}{8}$	27
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{8}+1\right)}\frac{\left(\_R^{4}+1\right)\ln\left(x-\_R\right)}{\_R^{7}}\right)}{8}$	27
meijerg	Expression too large to display	566

input
```
int((x^4+1)/(x^8+1),x,method=_RETURNVERBOSE)
```

output
```
1/8*sum((_R^4+1)/_R^7*ln(x-_R),_R=RootOf(_Z^8+1))
```

3.13.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.54

$$\begin{aligned}\int \frac{1+x^4}{1+x^8}\,dx =& \frac{1}{8}\sqrt{2}(-1)^{\frac{1}{8}}\log\left(8\sqrt{2}\left((-1)^{\frac{5}{8}}+(-1)^{\frac{1}{8}}\right)+16\,x\right)\\ &-\frac{1}{8}\sqrt{2}(-1)^{\frac{1}{8}}\log\left(-8\sqrt{2}\left((-1)^{\frac{5}{8}}+(-1)^{\frac{1}{8}}\right)+16\,x\right)\\ &-\frac{1}{8}i\sqrt{2}(-1)^{\frac{1}{8}}\log\left(-8\sqrt{2}\left(i\,(-1)^{\frac{5}{8}}+i\,(-1)^{\frac{1}{8}}\right)+16\,x\right)\\ &+\frac{1}{8}i\sqrt{2}(-1)^{\frac{1}{8}}\log\left(-8\sqrt{2}\left(-i\,(-1)^{\frac{5}{8}}-i\,(-1)^{\frac{1}{8}}\right)+16\,x\right)\\ &-\left(\frac{1}{8}i+\frac{1}{8}\right)(-1)^{\frac{1}{8}}\log\left(32\,x+(16i+16)\,(-1)^{\frac{5}{8}}-(16i+16)\,(-1)^{\frac{1}{8}}\right)\\ &+\left(\frac{1}{8}i-\frac{1}{8}\right)(-1)^{\frac{1}{8}}\log\left(32\,x-(16i-16)\,(-1)^{\frac{5}{8}}+(16i-16)\,(-1)^{\frac{1}{8}}\right)\\ &-\left(\frac{1}{8}i-\frac{1}{8}\right)(-1)^{\frac{1}{8}}\log\left(32\,x+(16i-16)\,(-1)^{\frac{5}{8}}-(16i-16)\,(-1)^{\frac{1}{8}}\right)\\ &+\left(\frac{1}{8}i+\frac{1}{8}\right)(-1)^{\frac{1}{8}}\log\left(32\,x-(16i+16)\,(-1)^{\frac{5}{8}}+(16i+16)\,(-1)^{\frac{1}{8}}\right)\end{aligned}$$

input
```
integrate((x^4+1)/(x^8+1),x, algorithm="fricas")
```

output
```
1/8*sqrt(2)*(-1)^(1/8)*log(8*sqrt(2)*((-1)^(5/8) + (-1)^(1/8)) + 16*x) - 1
/8*sqrt(2)*(-1)^(1/8)*log(-8*sqrt(2)*((-1)^(5/8) + (-1)^(1/8)) + 16*x) - 1
/8*I*sqrt(2)*(-1)^(1/8)*log(-8*sqrt(2)*(I*(-1)^(5/8) + I*(-1)^(1/8)) + 16*
x) + 1/8*I*sqrt(2)*(-1)^(1/8)*log(-8*sqrt(2)*(-I*(-1)^(5/8) - I*(-1)^(1/8)
) + 16*x) - (1/8*I + 1/8)*(-1)^(1/8)*log(32*x + (16*I + 16)*(-1)^(5/8) - (
16*I + 16)*(-1)^(1/8)) + (1/8*I - 1/8)*(-1)^(1/8)*log(32*x - (16*I - 16)*(
-1)^(5/8) + (16*I - 16)*(-1)^(1/8)) - (1/8*I - 1/8)*(-1)^(1/8)*log(32*x +
(16*I - 16)*(-1)^(5/8) - (16*I - 16)*(-1)^(1/8)) + (1/8*I + 1/8)*(-1)^(1/8
)*log(32*x - (16*I + 16)*(-1)^(5/8) + (16*I + 16)*(-1)^(1/8))
```

3.13.6 Sympy [A] (verification not implemented)

Time = 1.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.05

$$\int \frac{1+x^4}{1+x^8}\,dx = \text{RootSum}\left(1048576t^8+1,\left(t\mapsto t\log\left(4096t^5+4t+x\right)\right)\right)$$

input
```
integrate((x**4+1)/(x**8+1),x)
```

output
```
RootSum(1048576*_t**8 + 1, Lambda(_t, _t*log(4096*_t**5 + 4*_t + x)))
```

3.13.7 Maxima [F]

$$\int \frac{1+x^4}{1+x^8}\,dx = \int \frac{x^4+1}{x^8+1}\,dx$$

input
```
integrate((x^4+1)/(x^8+1),x, algorithm="maxima")
```

output
```
integrate((x^4 + 1)/(x^8 + 1), x)
```

3.13.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.71

$$
\begin{aligned}
\int \frac{1+x^4}{1+x^8}\,dx = {} & \frac{1}{8}\sqrt{-2\sqrt{2}+4}\arctan\left(\frac{2x+\sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}}\right) \\
& + \frac{1}{8}\sqrt{-2\sqrt{2}+4}\arctan\left(\frac{2x-\sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}}\right) \\
& + \frac{1}{8}\sqrt{2\sqrt{2}+4}\arctan\left(\frac{2x+\sqrt{\sqrt{2}+2}}{\sqrt{-\sqrt{2}+2}}\right) \\
& + \frac{1}{8}\sqrt{2\sqrt{2}+4}\arctan\left(\frac{2x-\sqrt{\sqrt{2}+2}}{\sqrt{-\sqrt{2}+2}}\right) \\
& + \frac{1}{16}\sqrt{-2\sqrt{2}+4}\log\left(x^2+x\sqrt{\sqrt{2}+2}+1\right) \\
& - \frac{1}{16}\sqrt{-2\sqrt{2}+4}\log\left(x^2-x\sqrt{\sqrt{2}+2}+1\right) \\
& + \frac{1}{16}\sqrt{2\sqrt{2}+4}\log\left(x^2+x\sqrt{-\sqrt{2}+2}+1\right) \\
& - \frac{1}{16}\sqrt{2\sqrt{2}+4}\log\left(x^2-x\sqrt{-\sqrt{2}+2}+1\right)
\end{aligned}
$$

input
```
integrate((x^4+1)/(x^8+1),x, algorithm="giac")
```

output
```
1/8*sqrt(-2*sqrt(2) + 4)*arctan((2*x + sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) +
2)) + 1/8*sqrt(-2*sqrt(2) + 4)*arctan((2*x - sqrt(-sqrt(2) + 2))/sqrt(sqrt
(2) + 2)) + 1/8*sqrt(2*sqrt(2) + 4)*arctan((2*x + sqrt(sqrt(2) + 2))/sqrt(
-sqrt(2) + 2)) + 1/8*sqrt(2*sqrt(2) + 4)*arctan((2*x - sqrt(sqrt(2) + 2))/
sqrt(-sqrt(2) + 2)) + 1/16*sqrt(-2*sqrt(2) + 4)*log(x^2 + x*sqrt(sqrt(2) +
 2) + 1) - 1/16*sqrt(-2*sqrt(2) + 4)*log(x^2 - x*sqrt(sqrt(2) + 2) + 1) +
1/16*sqrt(2*sqrt(2) + 4)*log(x^2 + x*sqrt(-sqrt(2) + 2) + 1) - 1/16*sqrt(2
*sqrt(2) + 4)*log(x^2 - x*sqrt(-sqrt(2) + 2) + 1)
```

3.13.9 Mupad [B] (verification not implemented)

Time = 8.75 (sec) , antiderivative size = 311, normalized size of antiderivative = 0.90

$$\int \frac{1+x^4}{1+x^8}\,dx = -\ln\left(\left(\left(\frac{\sqrt{-2\sqrt{2}-4}}{16}-\frac{\sqrt{4-2\sqrt{2}}}{16}\right)^3\left(65536\,x-16384\sqrt{-2\sqrt{2}-4}+16384\sqrt{4-2\sqrt{2}}\right)+256\right)\left(\frac{\sqrt{-2\sqrt{2}-4}}{16}-\frac{\sqrt{4-2\sqrt{2}}}{16}\right)\right.$$
$$+\operatorname{atan}\left(\frac{x\sqrt{\sqrt{2}-2}\,1\mathrm{i}}{2}+\frac{x\sqrt{\sqrt{2}+2}\,1\mathrm{i}}{2}+\frac{\sqrt{2}\,x\sqrt{\sqrt{2}-2}\,1\mathrm{i}}{2}-\frac{\sqrt{2}\,x\sqrt{\sqrt{2}+2}\,1\mathrm{i}}{2}\right)\left(\frac{\sqrt{2}\sqrt{\sqrt{2}-2}\,1\mathrm{i}}{8}+\frac{\sqrt{2}\sqrt{\sqrt{2}+2}\,1\mathrm{i}}{8}\right)$$
$$-\frac{\operatorname{atan}\left(x\left(\sqrt{2}+2\right)^{3/2}\left(1-\tfrac{1}{2}\mathrm{i}\right)+\sqrt{2}\,x\left(\sqrt{2}+2\right)^{3/2}\left(-\tfrac{3}{4}+\tfrac{1}{4}\mathrm{i}\right)\right)\left(-2+\sqrt{2}\left(1-\mathrm{i}\right)\right)\sqrt{\sqrt{2}+2}\,1\mathrm{i}}{8}$$
$$+\frac{\operatorname{atan}\left(x\left(\sqrt{2}+2\right)^{3/2}\left(\tfrac{1}{2}+1\mathrm{i}\right)+\sqrt{2}\,x\left(\sqrt{2}+2\right)^{3/2}\left(-\tfrac{1}{4}-\tfrac{3}{4}\mathrm{i}\right)\right)\left(\sqrt{2}\left(1+1\mathrm{i}\right)-2\mathrm{i}\right)\sqrt{\sqrt{2}+2}\,1\mathrm{i}}{8}$$
$$+\sqrt{2}\,\ln\left(x+\left(\sqrt{2}+2\right)^{3/2}\left(-\frac{1}{2}-\mathrm{i}\right)+\sqrt{2}\left(\sqrt{2}+2\right)^{3/2}\left(\frac{1}{4}+\frac{3}{4}\mathrm{i}\right)\right)\left(\frac{\sqrt{\sqrt{2}-2}}{16}+\frac{\sqrt{\sqrt{2}+2}}{16}\right)1\mathrm{i}$$

input
```
int((x^4 + 1)/(x^8 + 1),x)
```

output
```
atan((x*(2^(1/2) - 2)^(1/2)*1i)/2 + (x*(2^(1/2) + 2)^(1/2)*1i)/2 + (2^(1/2
)*x*(2^(1/2) - 2)^(1/2)*1i)/2 - (2^(1/2)*x*(2^(1/2) + 2)^(1/2)*1i)/2)*((2^
(1/2)*(2^(1/2) - 2)^(1/2)*1i)/8 + (2^(1/2)*(2^(1/2) + 2)^(1/2)*1i)/8) - lo
g(((- 2*2^(1/2) - 4)^(1/2)/16 - (4 - 2*2^(1/2))^(1/2)/16)^3*(65536*x - 163
84*(- 2*2^(1/2) - 4)^(1/2) + 16384*(4 - 2*2^(1/2))^(1/2)) + 256)*((- 2*2^(
1/2) - 4)^(1/2)/16 - (4 - 2*2^(1/2))^(1/2)/16) - (atan(x*(2^(1/2) + 2)^(3/
2)*(1 - 1i/2) - 2^(1/2)*x*(2^(1/2) + 2)^(3/2)*(3/4 - 1i/4))*(2^(1/2)*(1 -
1i) - 2)*(2^(1/2) + 2)^(1/2)*1i)/8 + (atan(x*(2^(1/2) + 2)^(3/2)*(1/2 + 1i
) - 2^(1/2)*x*(2^(1/2) + 2)^(3/2)*(1/4 + 3i/4))*(2^(1/2)*(1 + 1i) - 2i)*(2
^(1/2) + 2)^(1/2)*1i)/8 + 2^(1/2)*log(x - (2^(1/2) + 2)^(3/2)*(1/2 + 1i) +
 2^(1/2)*(2^(1/2) + 2)^(3/2)*(1/4 + 3i/4))*((2^(1/2) - 2)^(1/2)/16 + (2^(1
/2) + 2)^(1/2)/16)*1i
```

3.14 $\int \frac{1+x^4}{1-x^4+x^8}\,dx$

3.14.1 Optimal result

Integrand size = 18, antiderivative size = 331

$$\begin{aligned}\int \frac{1+x^4}{1-x^4+x^8}\,dx = &-\frac{1}{4}\sqrt{2-\sqrt{3}}\arctan\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)\\
&-\frac{1}{4}\sqrt{2+\sqrt{3}}\arctan\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)\\
&+\frac{1}{4}\sqrt{2-\sqrt{3}}\arctan\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)\\
&+\frac{1}{4}\sqrt{2+\sqrt{3}}\arctan\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)\\
&-\frac{\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)}{8\sqrt{2-\sqrt{3}}}+\frac{\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)}{8\sqrt{2-\sqrt{3}}}\\
&-\frac{\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)}{8\sqrt{2+\sqrt{3}}}+\frac{\log\left(1+\sqrt{2+\sqrt{3}}x+x^2\right)}{8\sqrt{2+\sqrt{3}}}\end{aligned}$$

output
```
-1/8*ln(1+x^2-x*(1/2*6^(1/2)-1/2*2^(1/2)))/(1/2*6^(1/2)-1/2*2^(1/2))+1/8*l
n(1+x^2+x*(1/2*6^(1/2)-1/2*2^(1/2)))/(1/2*6^(1/2)-1/2*2^(1/2))-1/4*arctan(
(-2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))*(1/2*6^(1/2)-1/2
*2^(1/2))+1/4*arctan((2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2
)))*(1/2*6^(1/2)-1/2*2^(1/2))-1/8*ln(1+x^2-x*(1/2*6^(1/2)+1/2*2^(1/2)))/(1
/2*6^(1/2)+1/2*2^(1/2))+1/8*ln(1+x^2+x*(1/2*6^(1/2)+1/2*2^(1/2)))/(1/2*6^(
1/2)+1/2*2^(1/2))-1/4*arctan((-2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1
/2*2^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))+1/4*arctan((2*x+1/2*6^(1/2)+1/2*2^(
1/2))/(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))
```

3.14.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.17

$$\int \frac{1+x^4}{1-x^4+x^8}\,dx = \frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{\log(x-\#1)+\log(x-\#1)\#1^4}{-\#1^3+2\#1^7}\&\right]$$

input
```
Integrate[(1 + x^4)/(1 - x^4 + x^8),x]
```

output
```
RootSum[1 - #1^4 + #1^8 & , (Log[x - #1] + Log[x - #1]*#1^4)/(-#1^3 + 2*#1
^7) & ]/4
```

3.14.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.20, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {1749, 1407, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4+1}{x^8-x^4+1}\,dx$$

$$\Big\downarrow \text{1749}$$

$$\frac{1}{2}\int \frac{1}{x^4-\sqrt{3}x^2+1}dx+\frac{1}{2}\int \frac{1}{x^4+\sqrt{3}x^2+1}dx$$

$$\Big\downarrow \text{1407}$$

$$\frac{1}{2}\left(\frac{\int \frac{\sqrt{2-\sqrt{3}}-x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\int \frac{x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}\right)+\frac{1}{2}\left(\frac{\int \frac{\sqrt{2+\sqrt{3}}-x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{\int \frac{x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}\right)$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{2}\left(\frac{\frac{1}{2}\sqrt{2-\sqrt{3}}\int \frac{1}{x^2-\sqrt{2-\sqrt{3}}x+1}dx-\frac{1}{2}\int -\frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\frac{1}{2}\sqrt{2-\sqrt{3}}\int \frac{1}{x^2+\sqrt{2-\sqrt{3}}x+1}dx+\frac{1}{2}\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}\right.$$
$$\frac{1}{2}\left(\frac{\frac{1}{2}\sqrt{2+\sqrt{3}}\int \frac{1}{x^2-\sqrt{2+\sqrt{3}}x+1}dx-\frac{1}{2}\int -\frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\sqrt{2+\sqrt{3}}\int \frac{1}{x^2+\sqrt{2+\sqrt{3}}x+1}dx+\frac{1}{2}\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}\right.$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{\frac{1}{2}\sqrt{2-\sqrt{3}}\int \frac{1}{x^2-\sqrt{2-\sqrt{3}}x+1}dx+\frac{1}{2}\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\frac{1}{2}\sqrt{2-\sqrt{3}}\int \frac{1}{x^2+\sqrt{2-\sqrt{3}}x+1}dx+\frac{1}{2}\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}\right.$$
$$\frac{1}{2}\left(\frac{\frac{1}{2}\sqrt{2+\sqrt{3}}\int \frac{1}{x^2-\sqrt{2+\sqrt{3}}x+1}dx+\frac{1}{2}\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\sqrt{2+\sqrt{3}}\int \frac{1}{x^2+\sqrt{2+\sqrt{3}}x+1}dx+\frac{1}{2}\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}\right.$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(\frac{\frac{1}{2}\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx-\sqrt{2-\sqrt{3}}\int \frac{1}{-\left(2x-\sqrt{2-\sqrt{3}}\right)^2-\sqrt{3}-2}d\left(2x-\sqrt{2-\sqrt{3}}\right)}{2\sqrt{2-\sqrt{3}}}+\frac{\frac{1}{2}\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx-\sqrt{2-\cdots}}{\cdots}\right.$$
$$\frac{1}{2}\left(\frac{\frac{1}{2}\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx-\sqrt{2+\sqrt{3}}\int \frac{1}{-\left(2x-\sqrt{2+\sqrt{3}}\right)^2+\sqrt{3}-2}d\left(2x-\sqrt{2+\sqrt{3}}\right)}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx-\sqrt{2+\cdots}}{\cdots}\right.$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(\frac{\frac{1}{2}\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx+\sqrt{\frac{2-\sqrt{3}}{2+\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{2-\sqrt{3}}}+\frac{\frac{1}{2}\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx+\sqrt{\frac{2-\sqrt{3}}{2+\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{2-\sqrt{3}}}\right)+$$
$$\frac{1}{2}\left(\frac{\frac{1}{2}\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx+\sqrt{\frac{2+\sqrt{3}}{2-\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx+\sqrt{\frac{2+\sqrt{3}}{2-\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{2+\sqrt{3}}}\right)$$

3.14. $\int \frac{1+x^4}{1-x^4+x^8}\,dx$

$$\Big\downarrow 1103$$

$$\frac{1}{2}\left(\frac{\sqrt{\frac{2-\sqrt{3}}{2+\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{2}\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{2\sqrt{2-\sqrt{3}}}+\frac{\sqrt{\frac{2-\sqrt{3}}{2+\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)+\frac{1}{2}\log\left(x^2+\sqrt{2\right.}\right.$$

$$\frac{1}{2}\left(\frac{\sqrt{\frac{2+\sqrt{3}}{2-\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)-\frac{1}{2}\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)}{2\sqrt{2+\sqrt{3}}}+\frac{\sqrt{\frac{2+\sqrt{3}}{2-\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)+\frac{1}{2}\log\left(x^2+\sqrt{2\right.}\right.$$

input
```
Int[(1 + x^4)/(1 - x^4 + x^8),x]
```

output
```
((Sqrt[(2 - Sqrt[3])/(2 + Sqrt[3])]*ArcTan[(-Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt
[2 + Sqrt[3]]] - Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2]/2)/(2*Sqrt[2 - Sqrt[3]
]) + (Sqrt[(2 - Sqrt[3])/(2 + Sqrt[3])]*ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/S
qrt[2 + Sqrt[3]]] + Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2]/2)/(2*Sqrt[2 - Sqrt
[3]]))/2 + ((Sqrt[(2 + Sqrt[3])/(2 - Sqrt[3])]*ArcTan[(-Sqrt[2 + Sqrt[3]]
+ 2*x)/Sqrt[2 - Sqrt[3]]] - Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2]/2)/(2*Sqrt[
2 + Sqrt[3]]) + (Sqrt[(2 + Sqrt[3])/(2 - Sqrt[3])]*ArcTan[(Sqrt[2 + Sqrt[3
]] + 2*x)/Sqrt[2 - Sqrt[3]]] + Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2]/2)/(2*Sq
rt[2 + Sqrt[3]]))/2
```

3.14.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1407
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   Int[(r - x)/(q - r*
x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(r + x)/(q + r*x + x^2), x], x]]]
 /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

rule 1749
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e
+ q*x^(n/2) + x^n, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x^(n/2) +
 x^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[2*(d/e) - b/c,
0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

3.14.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.13

method	result	size
default	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{8}-\_Z^{4}+1\right)}\frac{\left(\_R^{4}+1\right)\ln\left(x-\_R\right)}{2\_R^{7}-\_R^{3}}\right)}{4}$	42
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{8}-\_Z^{4}+1\right)}\frac{\left(\_R^{4}+1\right)\ln\left(x-\_R\right)}{2\_R^{7}-\_R^{3}}\right)}{4}$	42

input
```
int((x^4+1)/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum((_R^4+1)/(2*_R^7-_R^3)*ln(x-_R),_R=RootOf(_Z^8-_Z^4+1))
```

3.14.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.18

$$\begin{aligned}
\int \frac{1+x^4}{1-x^4+x^8}\,dx = {} & \frac{1}{8}\sqrt{2}\sqrt{\sqrt{2}\sqrt{\sqrt{-3}+1}}\log\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{\sqrt{-3}+1}}(\sqrt{-3}+1)+4x\right) \\
& - \frac{1}{8}\sqrt{2}\sqrt{\sqrt{2}\sqrt{\sqrt{-3}+1}}\log\left(-\sqrt{2}\sqrt{\sqrt{2}\sqrt{\sqrt{-3}+1}}(\sqrt{-3}+1)+4x\right) \\
& + \frac{1}{8}\sqrt{2}\sqrt{-\sqrt{2}\sqrt{\sqrt{-3}+1}}\log\left(\sqrt{2}\sqrt{-\sqrt{2}\sqrt{\sqrt{-3}+1}}(\sqrt{-3}+1)\right. \\
& \left. + 4x\right) \\
& - \frac{1}{8}\sqrt{2}\sqrt{-\sqrt{2}\sqrt{\sqrt{-3}+1}}\log\left(-\sqrt{2}\sqrt{-\sqrt{2}\sqrt{\sqrt{-3}+1}}(\sqrt{-3}+1)\right. \\
& \left. + 4x\right) \\
& - \frac{1}{8}\sqrt{2}\sqrt{\sqrt{2}\sqrt{-\sqrt{-3}+1}}\log\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{-\sqrt{-3}+1}}(\sqrt{-3}-1)\right. \\
& \left. + 4x\right) \\
& + \frac{1}{8}\sqrt{2}\sqrt{\sqrt{2}\sqrt{-\sqrt{-3}+1}}\log\left(-\sqrt{2}\sqrt{\sqrt{2}\sqrt{-\sqrt{-3}+1}}(\sqrt{-3}-1)\right. \\
& \left. + 4x\right) \\
& - \frac{1}{8}\sqrt{2}\sqrt{-\sqrt{2}\sqrt{-\sqrt{-3}+1}}\log\left(\sqrt{2}\sqrt{-\sqrt{2}\sqrt{-\sqrt{-3}+1}}(\sqrt{-3}-1)\right. \\
& \left. + 4x\right) \\
& + \frac{1}{8}\sqrt{2}\sqrt{-\sqrt{2}\sqrt{-\sqrt{-3}+1}}\log\left(-\sqrt{2}\sqrt{-\sqrt{2}\sqrt{-\sqrt{-3}+1}}(\sqrt{-3}-1)\right. \\
& \left. + 4x\right)
\end{aligned}$$

input
```
integrate((x^4+1)/(x^8-x^4+1),x, algorithm="fricas")
```

output
```
1/8*sqrt(2)*sqrt(sqrt(2)*sqrt(sqrt(-3) + 1))*log(sqrt(2)*sqrt(sqrt(2)*sqrt
(sqrt(-3) + 1))*(sqrt(-3) + 1) + 4*x) - 1/8*sqrt(2)*sqrt(sqrt(2)*sqrt(sqrt
(-3) + 1))*log(-sqrt(2)*sqrt(sqrt(2)*sqrt(sqrt(-3) + 1))*(sqrt(-3) + 1) +
4*x) + 1/8*sqrt(2)*sqrt(-sqrt(2)*sqrt(sqrt(-3) + 1))*log(sqrt(2)*sqrt(-sqr
t(2)*sqrt(sqrt(-3) + 1))*(sqrt(-3) + 1) + 4*x) - 1/8*sqrt(2)*sqrt(-sqrt(2)
*sqrt(sqrt(-3) + 1))*log(-sqrt(2)*sqrt(-sqrt(2)*sqrt(sqrt(-3) + 1))*(sqrt(
-3) + 1) + 4*x) - 1/8*sqrt(2)*sqrt(sqrt(2)*sqrt(-sqrt(-3) + 1))*log(sqrt(2
)*sqrt(sqrt(2)*sqrt(-sqrt(-3) + 1))*(sqrt(-3) - 1) + 4*x) + 1/8*sqrt(2)*sq
rt(sqrt(2)*sqrt(-sqrt(-3) + 1))*log(-sqrt(2)*sqrt(sqrt(2)*sqrt(-sqrt(-3) +
 1))*(sqrt(-3) - 1) + 4*x) - 1/8*sqrt(2)*sqrt(-sqrt(2)*sqrt(-sqrt(-3) + 1)
)*log(sqrt(2)*sqrt(-sqrt(2)*sqrt(-sqrt(-3) + 1))*(sqrt(-3) - 1) + 4*x) + 1
/8*sqrt(2)*sqrt(-sqrt(2)*sqrt(-sqrt(-3) + 1))*log(-sqrt(2)*sqrt(-sqrt(2)*s
qrt(-sqrt(-3) + 1))*(sqrt(-3) - 1) + 4*x)
```

3.14.6 Sympy [A] (verification not implemented)

Time = 1.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.06

$$\int \frac{1+x^4}{1-x^4+x^8}\,dx = \operatorname{RootSum}\left(65536t^8-256t^4+1,\left(t\mapsto t\log\left(1024t^5+x\right)\right)\right)$$

input
```
integrate((x**4+1)/(x**8-x**4+1),x)
```

output
```
RootSum(65536*_t**8 - 256*_t**4 + 1, Lambda(_t, _t*log(1024*_t**5 + x)))
```

3.14.7 Maxima [F]

$$\int \frac{1+x^4}{1-x^4+x^8}\,dx = \int \frac{x^4+1}{x^8-x^4+1}\,dx$$

input
```
integrate((x^4+1)/(x^8-x^4+1),x, algorithm="maxima")
```

output
```
integrate((x^4 + 1)/(x^8 - x^4 + 1), x)
```

3.14.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.74

$$
\begin{aligned}
\int \frac{1+x^4}{1-x^4+x^8}\,dx = {} & \frac{1}{8}\left(\sqrt{6}-\sqrt{2}\right)\arctan\left(\frac{4\,x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right) \\
& + \frac{1}{8}\left(\sqrt{6}-\sqrt{2}\right)\arctan\left(\frac{4\,x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right) \\
& + \frac{1}{8}\left(\sqrt{6}+\sqrt{2}\right)\arctan\left(\frac{4\,x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right) \\
& + \frac{1}{8}\left(\sqrt{6}+\sqrt{2}\right)\arctan\left(\frac{4\,x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right) \\
& + \frac{1}{16}\left(\sqrt{6}-\sqrt{2}\right)\log\left(x^2+\frac{1}{2}\,x\left(\sqrt{6}+\sqrt{2}\right)+1\right) \\
& - \frac{1}{16}\left(\sqrt{6}-\sqrt{2}\right)\log\left(x^2-\frac{1}{2}\,x\left(\sqrt{6}+\sqrt{2}\right)+1\right) \\
& + \frac{1}{16}\left(\sqrt{6}+\sqrt{2}\right)\log\left(x^2+\frac{1}{2}\,x\left(\sqrt{6}-\sqrt{2}\right)+1\right) \\
& - \frac{1}{16}\left(\sqrt{6}+\sqrt{2}\right)\log\left(x^2-\frac{1}{2}\,x\left(\sqrt{6}-\sqrt{2}\right)+1\right)
\end{aligned}
$$

input
```
integrate((x^4+1)/(x^8-x^4+1),x, algorithm="giac")
```

output
```
1/8*(sqrt(6) - sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt(2
))) + 1/8*(sqrt(6) - sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6) +
sqrt(2))) + 1/8*(sqrt(6) + sqrt(2))*arctan((4*x + sqrt(6) + sqrt(2))/(sqrt
(6) - sqrt(2))) + 1/8*(sqrt(6) + sqrt(2))*arctan((4*x - sqrt(6) - sqrt(2))
/(sqrt(6) - sqrt(2))) + 1/16*(sqrt(6) - sqrt(2))*log(x^2 + 1/2*x*(sqrt(6)
+ sqrt(2)) + 1) - 1/16*(sqrt(6) - sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) + sqrt
(2)) + 1) + 1/16*(sqrt(6) + sqrt(2))*log(x^2 + 1/2*x*(sqrt(6) - sqrt(2)) +
 1) - 1/16*(sqrt(6) + sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) - sqrt(2)) + 1)
```

3.14.9 Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.44

$$\int \frac{1+x^4}{1-x^4+x^8}\,dx = -\mathrm{atan}\left(\frac{\sqrt{6}\,x\,(27-27\mathrm{i})}{27\sqrt{3}-81\mathrm{i}}\right)\left(\sqrt{2}\left(\frac{1}{8}+\frac{1}{8}\mathrm{i}\right)+\sqrt{6}\left(-\frac{1}{8}+\frac{1}{8}\mathrm{i}\right)\right) - \mathrm{atan}\left(\frac{\sqrt{6}\,x\,(27+27\mathrm{i})}{27\sqrt{3}-81\mathrm{i}}\right)\left(\sqrt{2}\left(\frac{1}{8}-\frac{1}{8}\mathrm{i}\right)+\sqrt{6}\left(\frac{1}{8}+\frac{1}{8}\mathrm{i}\right)\right) - \mathrm{atan}\left(\frac{\sqrt{6}\,x\,(27-27\mathrm{i})}{27\sqrt{3}+81\mathrm{i}}\right)\left(\sqrt{2}\left(\frac{1}{8}+\frac{1}{8}\mathrm{i}\right)+\sqrt{6}\left(\frac{1}{8}-\frac{1}{8}\mathrm{i}\right)\right) - \mathrm{atan}\left(\frac{\sqrt{6}\,x\,(27+27\mathrm{i})}{27\sqrt{3}+81\mathrm{i}}\right)\left(\sqrt{2}\left(\frac{1}{8}-\frac{1}{8}\mathrm{i}\right)+\sqrt{6}\left(-\frac{1}{8}-\frac{1}{8}\mathrm{i}\right)\right)$$

input
```
int((x^4 + 1)/(x^8 - x^4 + 1),x)
```

output
```
- atan((6^(1/2)*x*(27 - 27i))/(27*3^(1/2) - 81i))*(2^(1/2)*(1/8 + 1i/8) -
6^(1/2)*(1/8 - 1i/8)) - atan((6^(1/2)*x*(27 + 27i))/(27*3^(1/2) - 81i))*(2
^(1/2)*(1/8 - 1i/8) + 6^(1/2)*(1/8 + 1i/8)) - atan((6^(1/2)*x*(27 - 27i))/
(27*3^(1/2) + 81i))*(2^(1/2)*(1/8 + 1i/8) + 6^(1/2)*(1/8 - 1i/8)) - atan((
6^(1/2)*x*(27 + 27i))/(27*3^(1/2) + 81i))*(2^(1/2)*(1/8 - 1i/8) - 6^(1/2)*
(1/8 + 1i/8))
```

3.15 $\int \frac{1+x^4}{1-2x^4+x^8} dx$

3.15.1 Optimal result

Integrand size = 18, antiderivative size = 27

$$\int \frac{1+x^4}{1-2x^4+x^8} dx = \frac{x}{2(1-x^4)} + \frac{\arctan(x)}{4} + \frac{\operatorname{arctanh}(x)}{4}$$

output
```
1/2*x/(-x^4+1)+1/4*arctan(x)+1/4*arctanh(x)
```

3.15.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int \frac{1+x^4}{1-2x^4+x^8} dx = \frac{1}{8}\left(-\frac{4x}{-1+x^4} + 2\arctan(x) - \log(1-x) + \log(1+x)\right)$$

input
```
Integrate[(1 + x^4)/(1 - 2*x^4 + x^8),x]
```

output
```
((-4*x)/(-1 + x^4) + 2*ArcTan[x] - Log[1 - x] + Log[1 + x])/8
```

3.15.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1380, 910, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4+1}{x^8-2x^4+1} \, dx$$

$$\Big\downarrow \text{1380}$$

$$\int \frac{x^4+1}{(1-x^4)^2}dx$$

$$\Big\downarrow \text{910}$$

$$\frac{1}{2}\int \frac{1}{1-x^4}dx+\frac{x}{2(1-x^4)}$$

$$\Big\downarrow \text{756}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{1-x^2}dx+\frac{1}{2}\int \frac{1}{x^2+1}dx\right)+\frac{x}{2(1-x^4)}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{1-x^2}dx+\frac{\arctan(x)}{2}\right)+\frac{x}{2(1-x^4)}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{\arctan(x)}{2}+\frac{\text{arctanh}(x)}{2}\right)+\frac{x}{2(1-x^4)}$$

input
```
Int[(1 + x^4)/(1 - 2*x^4 + x^8),x]
```

output
```
x/(2*(1 - x^4)) + (ArcTan[x]/2 + ArcTanh[x]/2)/2
```

3.15.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

3.15.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

method	result	size
risch	$-\frac{x}{2(x^4-1)}+\frac{\arctan(x)}{4}-\frac{\ln(x-1)}{8}+\frac{\ln(x+1)}{8}$	28
default	$-\frac{1}{8(x+1)}+\frac{\ln(x+1)}{8}+\frac{x}{4x^2+4}+\frac{\arctan(x)}{4}-\frac{1}{8(x-1)}-\frac{\ln(x-1)}{8}$	42
parallelrisch	$-\frac{i\ln(x-i)x^4-i\ln(x+i)x^4+\ln(x-1)x^4-\ln(x+1)x^4-i\ln(x-i)+i\ln(x+i)-\ln(x-1)+\ln(x+1)+4x}{8(x^4-1)}$	79

input
```
int((x^4+1)/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x/(x^4-1)+1/4*arctan(x)-1/8*ln(x-1)+1/8*ln(x+1)
```

3.15.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(19) = 38$.

Time = 0.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.59

$$\int \frac{1+x^4}{1-2x^4+x^8}\,dx = \frac{2\,(x^4-1)\arctan{(x)}+(x^4-1)\log{(x+1)}-(x^4-1)\log{(x-1)}-4\,x}{8\,(x^4-1)}$$

input
```
integrate((x^4+1)/(x^8-2*x^4+1),x, algorithm="fricas")
```

output
```
1/8*(2*(x^4 - 1)*arctan(x) + (x^4 - 1)*log(x + 1) - (x^4 - 1)*log(x - 1) -
 4*x)/(x^4 - 1)
```

3.15.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{1+x^4}{1-2x^4+x^8}\,dx = -\frac{x}{2x^4-2}-\frac{\log{(x-1)}}{8}+\frac{\log{(x+1)}}{8}+\frac{\operatorname{atan}{(x)}}{4}$$

input
```
integrate((x**4+1)/(x**8-2*x**4+1),x)
```

output
```
-x/(2*x**4 - 2) - log(x - 1)/8 + log(x + 1)/8 + atan(x)/4
```

3.15.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1+x^4}{1-2x^4+x^8}\,dx = -\frac{x}{2\,(x^4-1)}+\frac{1}{4}\arctan{(x)}+\frac{1}{8}\log{(x+1)}-\frac{1}{8}\log{(x-1)}$$

input
```
integrate((x^4+1)/(x^8-2*x^4+1),x, algorithm="maxima")
```

output
```
-1/2*x/(x^4 - 1) + 1/4*arctan(x) + 1/8*log(x + 1) - 1/8*log(x - 1)
```

3.15.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{1+x^4}{1-2x^4+x^8}\,dx = -\frac{x}{2\,(x^4-1)} + \frac{1}{4}\,\arctan\left(x\right) + \frac{1}{8}\,\log\left(|x+1|\right) - \frac{1}{8}\,\log\left(|x-1|\right)$$

input
```
integrate((x^4+1)/(x^8-2*x^4+1),x, algorithm="giac")
```

output
```
-1/2*x/(x^4 - 1) + 1/4*arctan(x) + 1/8*log(abs(x + 1)) - 1/8*log(abs(x - 1
))
```

3.15.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{1+x^4}{1-2x^4+x^8}\,dx = \frac{\mathrm{atan}(x)}{4} + \frac{\mathrm{atanh}(x)}{4} - \frac{x}{2\,\left(x^4-1\right)}$$

input
```
int((x^4 + 1)/(x^8 - 2*x^4 + 1),x)
```

output
```
atan(x)/4 + atanh(x)/4 - x/(2*(x^4 - 1))
```

3.16 $\int \frac{1+x^4}{1-3x^4+x^8}\,dx$

3.16.1 Optimal result

Integrand size = 18, antiderivative size = 131

$$\int \frac{1+x^4}{1-3x^4+x^8}\,dx=\frac{\arctan\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)}{\sqrt{2\left(-1+\sqrt{5}\right)}}-\frac{\arctan\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{2\left(1+\sqrt{5}\right)}}+\frac{\operatorname{arctanh}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)}{\sqrt{2\left(-1+\sqrt{5}\right)}}-\frac{\operatorname{arctanh}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{2\left(1+\sqrt{5}\right)}}$$

output
```
arctan(x*2^(1/2)/(5^(1/2)-1)^(1/2))/(-2+2*5^(1/2))^(1/2)+arctanh(x*2^(1/2)
/(5^(1/2)-1)^(1/2))/(-2+2*5^(1/2))^(1/2)-arctan(x*2^(1/2)/(5^(1/2)+1)^(1/2
))/(2+2*5^(1/2))^(1/2)-arctanh(x*2^(1/2)/(5^(1/2)+1)^(1/2))/(2+2*5^(1/2))^
(1/2)
```

3.16.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00

$$\int \frac{1+x^4}{1-3x^4+x^8}\,dx=\frac{\arctan\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)}{\sqrt{2\left(-1+\sqrt{5}\right)}}-\frac{\arctan\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{2\left(1+\sqrt{5}\right)}}+\frac{\operatorname{arctanh}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)}{\sqrt{2\left(-1+\sqrt{5}\right)}}-\frac{\operatorname{arctanh}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{2\left(1+\sqrt{5}\right)}}$$

input
```
Integrate[(1 + x^4)/(1 - 3*x^4 + x^8),x]
```

output
```
ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x]/Sqrt[2*(-1 + Sqrt[5])] - ArcTan[Sqrt[2/(1
 + Sqrt[5])]*x]/Sqrt[2*(1 + Sqrt[5])] + ArcTanh[Sqrt[2/(-1 + Sqrt[5])]*x]/
Sqrt[2*(-1 + Sqrt[5])] - ArcTanh[Sqrt[2/(1 + Sqrt[5])]*x]/Sqrt[2*(1 + Sqrt
[5])]
```

3.16.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1749, 1406, 216, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4+1}{x^8-3x^4+1}dx$$

$$\Big\downarrow \; 1749$$

$$\frac{1}{2}\int \frac{1}{x^4-\sqrt{5}x^2+1}dx+\frac{1}{2}\int \frac{1}{x^4+\sqrt{5}x^2+1}dx$$

$$\Big\downarrow \; 1406$$

$$\frac{1}{2}\left(\int \frac{1}{x^2+\frac{1}{2}\left(-1-\sqrt{5}\right)}dx-\int \frac{1}{x^2+\frac{1}{2}\left(1-\sqrt{5}\right)}dx\right)+\frac{1}{2}\left(\int \frac{1}{x^2+\frac{1}{2}\left(-1+\sqrt{5}\right)}dx-\int \frac{1}{x^2+\frac{1}{2}\left(1+\sqrt{5}\right)}dx\right)$$

$$\Big\downarrow \; 216$$

$$\frac{1}{2}\left(\int \frac{1}{x^2+\frac{1}{2}\left(-1-\sqrt{5}\right)}dx-\int \frac{1}{x^2+\frac{1}{2}\left(1-\sqrt{5}\right)}dx\right)+\frac{1}{2}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\arctan\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)-\sqrt{\frac{2}{1+\sqrt{5}}}\arctan\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)\right)$$

$$\Big\downarrow \; 220$$

$$\frac{1}{2}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\arctan\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)-\sqrt{\frac{2}{1+\sqrt{5}}}\arctan\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)\right)+\frac{1}{2}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\operatorname{arctanh}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)-\sqrt{\frac{2}{1+\sqrt{5}}}\operatorname{arctanh}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)\right)$$

input
```
Int[(1 + x^4)/(1 - 3*x^4 + x^8),x]
```

output
```
(Sqrt[2/(-1 + Sqrt[5])]*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x] - Sqrt[2/(1 + Sqr
t[5])]*ArcTan[Sqrt[2/(1 + Sqrt[5])]*x])/2 + (Sqrt[2/(-1 + Sqrt[5])]*ArcTan
h[Sqrt[2/(-1 + Sqrt[5])]*x] - Sqrt[2/(1 + Sqrt[5])]*ArcTanh[Sqrt[2/(1 + Sq
rt[5])]*x])/2
```

3.16.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

rule 1749
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e
+ q*x^(n/2) + x^n, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x^(n/2) +
 x^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[2*(d/e) - b/c,
0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

3.16.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.43

method	result	size
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-\_Z^2-1\right)}\_R\ln\left(\_R^3-\_R+x\right)\right)}{4}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+\_Z^2-1\right)}\_R\ln\left(-\_R^3-\_R+x\right)\right)}{4}$	56
default	$-\frac{\operatorname{arctanh}\left(\frac{2x}{\sqrt{2\sqrt{5}+2}}\right)}{\sqrt{2\sqrt{5}+2}}+\frac{\arctan\left(\frac{2x}{\sqrt{2\sqrt{5}-2}}\right)}{\sqrt{2\sqrt{5}-2}}-\frac{\arctan\left(\frac{2x}{\sqrt{2\sqrt{5}+2}}\right)}{\sqrt{2\sqrt{5}+2}}+\frac{\operatorname{arctanh}\left(\frac{2x}{\sqrt{2\sqrt{5}-2}}\right)}{\sqrt{2\sqrt{5}-2}}$	96

input
```
int((x^4+1)/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(_R*ln(_R^3-_R+x),_R=RootOf(_Z^4-_Z^2-1))+1/4*sum(_R*ln(-_R^3-_R+x)
,_R=RootOf(_Z^4+_Z^2-1))
```

3.16.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 317 vs. 2(95) = 190.

Time = 0.28 (sec) , antiderivative size = 317, normalized size of antiderivative = 2.42

$$\begin{aligned}\int \frac{1+x^4}{1-3x^4+x^8}\,dx &= \frac{1}{8}\sqrt{2}\sqrt{\sqrt{5}+1}\log\left(\left(\sqrt{5}\sqrt{2}-\sqrt{2}\right)\sqrt{\sqrt{5}+1}+4x\right)\\ &-\frac{1}{8}\sqrt{2}\sqrt{\sqrt{5}+1}\log\left(-\left(\sqrt{5}\sqrt{2}-\sqrt{2}\right)\sqrt{\sqrt{5}+1}+4x\right)\\ &-\frac{1}{8}\sqrt{2}\sqrt{\sqrt{5}-1}\log\left(\left(\sqrt{5}\sqrt{2}+\sqrt{2}\right)\sqrt{\sqrt{5}-1}+4x\right)\\ &+\frac{1}{8}\sqrt{2}\sqrt{\sqrt{5}-1}\log\left(-\left(\sqrt{5}\sqrt{2}+\sqrt{2}\right)\sqrt{\sqrt{5}-1}+4x\right)\\ &-\frac{1}{8}\sqrt{2}\sqrt{-\sqrt{5}+1}\log\left(\left(\sqrt{5}\sqrt{2}+\sqrt{2}\right)\sqrt{-\sqrt{5}+1}+4x\right)\\ &+\frac{1}{8}\sqrt{2}\sqrt{-\sqrt{5}+1}\log\left(-\left(\sqrt{5}\sqrt{2}+\sqrt{2}\right)\sqrt{-\sqrt{5}+1}+4x\right)\\ &+\frac{1}{8}\sqrt{2}\sqrt{-\sqrt{5}-1}\log\left(\left(\sqrt{5}\sqrt{2}-\sqrt{2}\right)\sqrt{-\sqrt{5}-1}+4x\right)\\ &-\frac{1}{8}\sqrt{2}\sqrt{-\sqrt{5}-1}\log\left(-\left(\sqrt{5}\sqrt{2}-\sqrt{2}\right)\sqrt{-\sqrt{5}-1}+4x\right)\end{aligned}$$

input
```
integrate((x^4+1)/(x^8-3*x^4+1),x, algorithm="fricas")
```

output
```
1/8*sqrt(2)*sqrt(sqrt(5) + 1)*log((sqrt(5)*sqrt(2) - sqrt(2))*sqrt(sqrt(5)
 + 1) + 4*x) - 1/8*sqrt(2)*sqrt(sqrt(5) + 1)*log(-(sqrt(5)*sqrt(2) - sqrt(
2))*sqrt(sqrt(5) + 1) + 4*x) - 1/8*sqrt(2)*sqrt(sqrt(5) - 1)*log((sqrt(5)*
sqrt(2) + sqrt(2))*sqrt(sqrt(5) - 1) + 4*x) + 1/8*sqrt(2)*sqrt(sqrt(5) - 1
)*log(-(sqrt(5)*sqrt(2) + sqrt(2))*sqrt(sqrt(5) - 1) + 4*x) - 1/8*sqrt(2)*
sqrt(-sqrt(5) + 1)*log((sqrt(5)*sqrt(2) + sqrt(2))*sqrt(-sqrt(5) + 1) + 4*
x) + 1/8*sqrt(2)*sqrt(-sqrt(5) + 1)*log(-(sqrt(5)*sqrt(2) + sqrt(2))*sqrt(
-sqrt(5) + 1) + 4*x) + 1/8*sqrt(2)*sqrt(-sqrt(5) - 1)*log((sqrt(5)*sqrt(2)
 - sqrt(2))*sqrt(-sqrt(5) - 1) + 4*x) - 1/8*sqrt(2)*sqrt(-sqrt(5) - 1)*log
(-(sqrt(5)*sqrt(2) - sqrt(2))*sqrt(-sqrt(5) - 1) + 4*x)
```

3.16.6 Sympy [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.37

$$\int \frac{1+x^4}{1-3x^4+x^8}\,dx = \operatorname{RootSum}\left(256t^4-16t^2-1,\left(t\mapsto t\log\left(1024t^5-8t+x\right)\right)\right) + \operatorname{RootSum}\left(256t^4+16t^2-1,\left(t\mapsto t\log\left(1024t^5-8t+x\right)\right)\right)$$

input
```
integrate((x**4+1)/(x**8-3*x**4+1),x)
```

output
```
RootSum(256*_t**4 - 16*_t**2 - 1, Lambda(_t, _t*log(1024*_t**5 - 8*_t + x)
)) + RootSum(256*_t**4 + 16*_t**2 - 1, Lambda(_t, _t*log(1024*_t**5 - 8*_t
 + x)))
```

3.16.7 Maxima [F]

$$\int \frac{1+x^4}{1-3x^4+x^8}\,dx = \int \frac{x^4+1}{x^8-3\,x^4+1}\,dx$$

input
```
integrate((x^4+1)/(x^8-3*x^4+1),x, algorithm="maxima")
```

output
```
integrate((x^4 + 1)/(x^8 - 3*x^4 + 1), x)
```

3.16.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.12

$$\int \frac{1+x^4}{1-3x^4+x^8}\,dx = -\frac{1}{4}\sqrt{2\sqrt{5}-2}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right) + \frac{1}{4}\sqrt{2\sqrt{5}+2}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right) - \frac{1}{8}\sqrt{2\sqrt{5}-2}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right) + \frac{1}{8}\sqrt{2\sqrt{5}-2}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right) + \frac{1}{8}\sqrt{2\sqrt{5}+2}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right) - \frac{1}{8}\sqrt{2\sqrt{5}+2}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)$$

input
```
integrate((x^4+1)/(x^8-3*x^4+1),x, algorithm="giac")
```

output
```
-1/4*sqrt(2*sqrt(5) - 2)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) + 1/4*sqrt(2*sq
rt(5) + 2)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/8*sqrt(2*sqrt(5) - 2)*log
(abs(x + sqrt(1/2*sqrt(5) + 1/2))) + 1/8*sqrt(2*sqrt(5) - 2)*log(abs(x - s
qrt(1/2*sqrt(5) + 1/2))) + 1/8*sqrt(2*sqrt(5) + 2)*log(abs(x + sqrt(1/2*sq
rt(5) - 1/2))) - 1/8*sqrt(2*sqrt(5) + 2)*log(abs(x - sqrt(1/2*sqrt(5) - 1/
2)))
```

3.16.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 269, normalized size of antiderivative = 2.05

$$\int \frac{1+x^4}{1-3x^4+x^8}\,dx = -\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x\,\sqrt{\sqrt{5}-1}\,1875\mathrm{i}}{2\left(875\,\sqrt{5}-1875\right)}-\frac{\sqrt{2}\,\sqrt{5}\,x\,\sqrt{\sqrt{5}-1}\,875\mathrm{i}}{2\left(875\,\sqrt{5}-1875\right)}\right)\,\sqrt{\sqrt{5}-1}\,1\mathrm{i}}{4} - \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x\,\sqrt{\sqrt{5}+1}\,1875\mathrm{i}}{2\left(875\,\sqrt{5}+1875\right)}+\frac{\sqrt{2}\,\sqrt{5}\,x\,\sqrt{\sqrt{5}+1}\,875\mathrm{i}}{2\left(875\,\sqrt{5}+1875\right)}\right)\,\sqrt{\sqrt{5}+1}\,1\mathrm{i}}{4} + \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x\,\sqrt{1-\sqrt{5}}\,1875\mathrm{i}}{2\left(875\,\sqrt{5}-1875\right)}-\frac{\sqrt{2}\,\sqrt{5}\,x\,\sqrt{1-\sqrt{5}}\,875\mathrm{i}}{2\left(875\,\sqrt{5}-1875\right)}\right)\,\sqrt{1-\sqrt{5}}\,1\mathrm{i}}{4} + \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x\,\sqrt{-\sqrt{5}-1}\,1875\mathrm{i}}{2\left(875\,\sqrt{5}+1875\right)}+\frac{\sqrt{2}\,\sqrt{5}\,x\,\sqrt{-\sqrt{5}-1}\,875\mathrm{i}}{2\left(875\,\sqrt{5}+1875\right)}\right)\,\sqrt{-\sqrt{5}-1}\,1\mathrm{i}}{4}$$

input
```
int((x^4 + 1)/(x^8 - 3*x^4 + 1),x)
```

output
```
(2^(1/2)*atan((2^(1/2)*x*(1 - 5^(1/2))^(1/2)*1875i)/(2*(875*5^(1/2) - 1875
)) - (2^(1/2)*5^(1/2)*x*(1 - 5^(1/2))^(1/2)*875i)/(2*(875*5^(1/2) - 1875))
)*(1 - 5^(1/2))^(1/2)*1i)/4 - (2^(1/2)*atan((2^(1/2)*x*(5^(1/2) + 1)^(1/2)
*1875i)/(2*(875*5^(1/2) + 1875)) + (2^(1/2)*5^(1/2)*x*(5^(1/2) + 1)^(1/2)*
875i)/(2*(875*5^(1/2) + 1875)))*(5^(1/2) + 1)^(1/2)*1i)/4 - (2^(1/2)*atan(
(2^(1/2)*x*(5^(1/2) - 1)^(1/2)*1875i)/(2*(875*5^(1/2) - 1875)) - (2^(1/2)*
5^(1/2)*x*(5^(1/2) - 1)^(1/2)*875i)/(2*(875*5^(1/2) - 1875)))*(5^(1/2) - 1
)^(1/2)*1i)/4 + (2^(1/2)*atan((2^(1/2)*x*(- 5^(1/2) - 1)^(1/2)*1875i)/(2*(
875*5^(1/2) + 1875)) + (2^(1/2)*5^(1/2)*x*(- 5^(1/2) - 1)^(1/2)*875i)/(2*(
875*5^(1/2) + 1875)))*(- 5^(1/2) - 1)^(1/2)*1i)/4
```

3.17 $\int \frac{1+x^4}{1-4x^4+x^8} dx$

3.17.1 Optimal result

Integrand size = 18, antiderivative size = 157

$$\int \frac{1+x^4}{1-4x^4+x^8} dx = \frac{\arctan\left(\frac{\sqrt[4]{2}x}{\sqrt{-1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{-1+\sqrt{3}}} - \frac{\arctan\left(\frac{\sqrt[4]{2}x}{\sqrt{1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{1+\sqrt{3}}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{2}x}{\sqrt{-1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{-1+\sqrt{3}}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{2}x}{\sqrt{1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{1+\sqrt{3}}}$$

output
```
1/4*arctan(2^(1/4)*x/(3^(1/2)-1)^(1/2))*2^(3/4)/(3^(1/2)-1)^(1/2)+1/4*arct
anh(2^(1/4)*x/(3^(1/2)-1)^(1/2))*2^(3/4)/(3^(1/2)-1)^(1/2)-1/4*arctan(2^(1
/4)*x/(1+3^(1/2))^(1/2))*2^(3/4)/(1+3^(1/2))^(1/2)-1/4*arctanh(2^(1/4)*x/(
1+3^(1/2))^(1/2))*2^(3/4)/(1+3^(1/2))^(1/2)
```

3.17.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.34

$$\int \frac{1+x^4}{1-4x^4+x^8} dx = \frac{1}{8}\text{RootSum}\left[1-4\#1^4+\#1^8\&, \frac{\log(x-\#1)+\log(x-\#1)\#1^4}{-2\#1^3+\#1^7}\&\right]$$

input
```
Integrate[(1 + x^4)/(1 - 4*x^4 + x^8),x]
```

output
```
RootSum[1 - 4*#1^4 + #1^8 & , (Log[x - #1] + Log[x - #1]*#1^4)/(-2*#1^3 +
#1^7) & ]/8
```

3.17.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1749, 1406, 216, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4+1}{x^8-4x^4+1}\,dx$$
$$\Big\downarrow \ 1749$$
$$\frac{1}{2}\int \frac{1}{x^4-\sqrt{6}x^2+1}dx+\frac{1}{2}\int \frac{1}{x^4+\sqrt{6}x^2+1}dx$$
$$\Big\downarrow \ 1406$$
$$\frac{1}{2}\left(\frac{\int \frac{1}{x^2-\frac{1+\sqrt{3}}{\sqrt{2}}}dx}{\sqrt{2}}-\frac{\int \frac{1}{x^2+\frac{1-\sqrt{3}}{\sqrt{2}}}dx}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\int \frac{1}{x^2-\frac{1-\sqrt{3}}{\sqrt{2}}}dx}{\sqrt{2}}-\frac{\int \frac{1}{x^2+\frac{1+\sqrt{3}}{\sqrt{2}}}dx}{\sqrt{2}}\right)$$
$$\Big\downarrow \ 216$$
$$\frac{1}{2}\left(\frac{\int \frac{1}{x^2-\frac{1+\sqrt{3}}{\sqrt{2}}}dx}{\sqrt{2}}-\frac{\int \frac{1}{x^2+\frac{1-\sqrt{3}}{\sqrt{2}}}dx}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt[4]{2}x}{\sqrt{\sqrt{3}-1}}\right)}{\sqrt[4]{2}\sqrt{\sqrt{3}-1}}-\frac{\arctan\left(\frac{\sqrt[4]{2}x}{\sqrt{1+\sqrt{3}}}\right)}{\sqrt[4]{2}\sqrt{1+\sqrt{3}}}\right)$$
$$\Big\downarrow \ 220$$
$$\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt[4]{2}x}{\sqrt{\sqrt{3}-1}}\right)}{\sqrt[4]{2}\sqrt{\sqrt{3}-1}}-\frac{\arctan\left(\frac{\sqrt[4]{2}x}{\sqrt{1+\sqrt{3}}}\right)}{\sqrt[4]{2}\sqrt{1+\sqrt{3}}}\right)+\frac{1}{2}\left(\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}x}{\sqrt{\sqrt{3}-1}}\right)}{\sqrt[4]{2}\sqrt{\sqrt{3}-1}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}x}{\sqrt{1+\sqrt{3}}}\right)}{\sqrt[4]{2}\sqrt{1+\sqrt{3}}}\right)$$

input
```
Int[(1 + x^4)/(1 - 4*x^4 + x^8),x]
```

output
```
(ArcTan[(2^(1/4)*x)/Sqrt[-1 + Sqrt[3]]]/(2^(1/4)*Sqrt[-1 + Sqrt[3]]) - Arc
Tan[(2^(1/4)*x)/Sqrt[1 + Sqrt[3]]]/(2^(1/4)*Sqrt[1 + Sqrt[3]]))/2 + (ArcTa
nh[(2^(1/4)*x)/Sqrt[-1 + Sqrt[3]]]/(2^(1/4)*Sqrt[-1 + Sqrt[3]]) - ArcTanh[
(2^(1/4)*x)/Sqrt[1 + Sqrt[3]]]/(2^(1/4)*Sqrt[1 + Sqrt[3]]))/2
```

3.17.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

rule 1749
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e
+ q*x^(n/2) + x^n, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x^(n/2) +
 x^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[2*(d/e) - b/c,
0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

3.17.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.25

method	result	size
default	$\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^8-4\_Z^4+1\right)}\frac{\left(\_R^4+1\right)\ln\left(x-\_R\right)}{\_R^7-2\_R^3}\right)}{8}$	40
risch	$\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^8-4\_Z^4+1\right)}\frac{\left(\_R^4+1\right)\ln\left(x-\_R\right)}{\_R^7-2\_R^3}\right)}{8}$	40

input
```
int((x^4+1)/(x^8-4*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/8*sum((_R^4+1)/(_R^7-2*_R^3)*ln(x-_R),_R=RootOf(_Z^8-4*_Z^4+1))
```

3.17.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 349 vs. $2(101) = 202$.

Time = 0.29 (sec) , antiderivative size = 349, normalized size of antiderivative = 2.22

$$\int \frac{1+x^4}{1-4x^4+x^8}\,dx = \frac{1}{8}\sqrt{2}\sqrt{-\sqrt{\sqrt{3}+2}}\log\left(\left(\sqrt{3}\sqrt{2}-\sqrt{2}\right)\sqrt{-\sqrt{\sqrt{3}+2}}+2\,x\right) - \frac{1}{8}\sqrt{2}\sqrt{-\sqrt{\sqrt{3}+2}}\log\left(-\left(\sqrt{3}\sqrt{2}-\sqrt{2}\right)\sqrt{-\sqrt{\sqrt{3}+2}}+2\,x\right) - \frac{1}{8}\sqrt{2}\sqrt{-\sqrt{-\sqrt{3}+2}}\log\left(\left(\sqrt{3}\sqrt{2}+\sqrt{2}\right)\sqrt{-\sqrt{-\sqrt{3}+2}}+2\,x\right) + \frac{1}{8}\sqrt{2}\sqrt{-\sqrt{-\sqrt{3}+2}}\log\left(-\left(\sqrt{3}\sqrt{2}+\sqrt{2}\right)\sqrt{-\sqrt{-\sqrt{3}+2}}+2\,x\right) + \frac{1}{8}\sqrt{2}\left(\sqrt{3}+2\right)^{\frac{1}{4}}\log\left(\left(\sqrt{3}\sqrt{2}-\sqrt{2}\right)\left(\sqrt{3}+2\right)^{\frac{1}{4}}+2\,x\right) - \frac{1}{8}\sqrt{2}\left(\sqrt{3}+2\right)^{\frac{1}{4}}\log\left(-\left(\sqrt{3}\sqrt{2}-\sqrt{2}\right)\left(\sqrt{3}+2\right)^{\frac{1}{4}}+2\,x\right) - \frac{1}{8}\sqrt{2}\left(-\sqrt{3}+2\right)^{\frac{1}{4}}\log\left(\left(\sqrt{3}\sqrt{2}+\sqrt{2}\right)\left(-\sqrt{3}+2\right)^{\frac{1}{4}}+2\,x\right) + \frac{1}{8}\sqrt{2}\left(-\sqrt{3}+2\right)^{\frac{1}{4}}\log\left(-\left(\sqrt{3}\sqrt{2}+\sqrt{2}\right)\left(-\sqrt{3}+2\right)^{\frac{1}{4}}+2\,x\right)$$

input
```
integrate((x^4+1)/(x^8-4*x^4+1),x, algorithm="fricas")
```

output
```
1/8*sqrt(2)*sqrt(-sqrt(sqrt(3) + 2))*log((sqrt(3)*sqrt(2) - sqrt(2))*sqrt(
-sqrt(sqrt(3) + 2)) + 2*x) - 1/8*sqrt(2)*sqrt(-sqrt(sqrt(3) + 2))*log(-(sq
rt(3)*sqrt(2) - sqrt(2))*sqrt(-sqrt(sqrt(3) + 2)) + 2*x) - 1/8*sqrt(2)*sqr
t(-sqrt(-sqrt(3) + 2))*log((sqrt(3)*sqrt(2) + sqrt(2))*sqrt(-sqrt(-sqrt(3)
 + 2)) + 2*x) + 1/8*sqrt(2)*sqrt(-sqrt(-sqrt(3) + 2))*log(-(sqrt(3)*sqrt(2
) + sqrt(2))*sqrt(-sqrt(-sqrt(3) + 2)) + 2*x) + 1/8*sqrt(2)*(sqrt(3) + 2)^
(1/4)*log((sqrt(3)*sqrt(2) - sqrt(2))*(sqrt(3) + 2)^(1/4) + 2*x) - 1/8*sqr
t(2)*(sqrt(3) + 2)^(1/4)*log(-(sqrt(3)*sqrt(2) - sqrt(2))*(sqrt(3) + 2)^(1
/4) + 2*x) - 1/8*sqrt(2)*(-sqrt(3) + 2)^(1/4)*log((sqrt(3)*sqrt(2) + sqrt(
2))*(-sqrt(3) + 2)^(1/4) + 2*x) + 1/8*sqrt(2)*(-sqrt(3) + 2)^(1/4)*log(-(s
qrt(3)*sqrt(2) + sqrt(2))*(-sqrt(3) + 2)^(1/4) + 2*x)
```

3.17. $\int \frac{1+x^4}{1-4x^4+x^8}\,dx$

3.17.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.15

$$\int \frac{1+x^4}{1-4x^4+x^8}\,dx = \text{RootSum}\left(1048576t^8 - 4096t^4 + 1, \left(t \mapsto t\log\left(4096t^5 - 12t + x\right)\right)\right)$$

input
```
integrate((x**4+1)/(x**8-4*x**4+1),x)
```

output
```
RootSum(1048576*_t**8 - 4096*_t**4 + 1, Lambda(_t, _t*log(4096*_t**5 - 12*
_t + x)))
```

3.17.7 Maxima [F]

$$\int \frac{1+x^4}{1-4x^4+x^8}\,dx = \int \frac{x^4+1}{x^8-4\,x^4+1}\,dx$$

input
```
integrate((x^4+1)/(x^8-4*x^4+1),x, algorithm="maxima")
```

output
```
integrate((x^4 + 1)/(x^8 - 4*x^4 + 1), x)
```

3.17.8 Giac [F]

$$\int \frac{1+x^4}{1-4x^4+x^8}\,dx = \int \frac{x^4+1}{x^8-4\,x^4+1}\,dx$$

input
```
integrate((x^4+1)/(x^8-4*x^4+1),x, algorithm="giac")
```

output
```
integrate((x^4 + 1)/(x^8 - 4*x^4 + 1), x)
```

3.17.9 Mupad [B] (verification not implemented)

Time = 8.42 (sec) , antiderivative size = 399, normalized size of antiderivative = 2.54

$$\int \frac{1+x^4}{1-4x^4+x^8}\,dx$$
$$= \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{5184\sqrt{2}\,x\left(\sqrt{3}+2\right)^{1/4}}{3888\sqrt{\sqrt{3}+2}+2160\sqrt{3}\sqrt{\sqrt{3}+2}}+\frac{3024\sqrt{2}\sqrt{3}\,x\left(\sqrt{3}+2\right)^{1/4}}{3888\sqrt{\sqrt{3}+2}+2160\sqrt{3}\sqrt{\sqrt{3}+2}}\right)\left(\sqrt{3}+2\right)^{1/4}}{4}$$
$$+ \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x\left(2-\sqrt{3}\right)^{1/4}5184\mathrm{i}}{2160\sqrt{3}\sqrt{2-\sqrt{3}}-3888\sqrt{2-\sqrt{3}}}-\frac{\sqrt{2}\sqrt{3}\,x\left(2-\sqrt{3}\right)^{1/4}3024\mathrm{i}}{2160\sqrt{3}\sqrt{2-\sqrt{3}}-3888\sqrt{2-\sqrt{3}}}\right)\left(2-\sqrt{3}\right)^{1/4}1\mathrm{i}}{4}$$
$$- \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{5184\sqrt{2}\,x\left(2-\sqrt{3}\right)^{1/4}}{2160\sqrt{3}\sqrt{2-\sqrt{3}}-3888\sqrt{2-\sqrt{3}}}-\frac{3024\sqrt{2}\sqrt{3}\,x\left(2-\sqrt{3}\right)^{1/4}}{2160\sqrt{3}\sqrt{2-\sqrt{3}}-3888\sqrt{2-\sqrt{3}}}\right)\left(2-\sqrt{3}\right)^{1/4}}{4}$$
$$- \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x\left(\sqrt{3}+2\right)^{1/4}5184\mathrm{i}}{3888\sqrt{\sqrt{3}+2}+2160\sqrt{3}\sqrt{\sqrt{3}+2}}+\frac{\sqrt{2}\sqrt{3}\,x\left(\sqrt{3}+2\right)^{1/4}3024\mathrm{i}}{3888\sqrt{\sqrt{3}+2}+2160\sqrt{3}\sqrt{\sqrt{3}+2}}\right)\left(\sqrt{3}+2\right)^{1/4}1\mathrm{i}}{4}$$

input
```
int((x^4 + 1)/(x^8 - 4*x^4 + 1),x)
```

output
```
(2^(1/2)*atan((2^(1/2)*x*(2 - 3^(1/2))^(1/4)*5184i)/(2160*3^(1/2)*(2 - 3^(
1/2))^(1/2) - 3888*(2 - 3^(1/2))^(1/2)) - (2^(1/2)*3^(1/2)*x*(2 - 3^(1/2))
^(1/4)*3024i)/(2160*3^(1/2)*(2 - 3^(1/2))^(1/2) - 3888*(2 - 3^(1/2))^(1/2)
))*(2 - 3^(1/2))^(1/4)*1i)/4 - (2^(1/2)*atan((5184*2^(1/2)*x*(2 - 3^(1/2))
^(1/4))/(2160*3^(1/2)*(2 - 3^(1/2))^(1/2) - 3888*(2 - 3^(1/2))^(1/2)) - (3
024*2^(1/2)*3^(1/2)*x*(2 - 3^(1/2))^(1/4))/(2160*3^(1/2)*(2 - 3^(1/2))^(1/
2) - 3888*(2 - 3^(1/2))^(1/2)))*(2 - 3^(1/2))^(1/4))/4 + (2^(1/2)*atan((51
84*2^(1/2)*x*(3^(1/2) + 2)^(1/4))/(3888*(3^(1/2) + 2)^(1/2) + 2160*3^(1/2)
*(3^(1/2) + 2)^(1/2)) + (3024*2^(1/2)*3^(1/2)*x*(3^(1/2) + 2)^(1/4))/(3888
*(3^(1/2) + 2)^(1/2) + 2160*3^(1/2)*(3^(1/2) + 2)^(1/2)))*(3^(1/2) + 2)^(1
/4))/4 - (2^(1/2)*atan((2^(1/2)*x*(3^(1/2) + 2)^(1/4)*5184i)/(3888*(3^(1/2
) + 2)^(1/2) + 2160*3^(1/2)*(3^(1/2) + 2)^(1/2)) + (2^(1/2)*3^(1/2)*x*(3^(
1/2) + 2)^(1/4)*3024i)/(3888*(3^(1/2) + 2)^(1/2) + 2160*3^(1/2)*(3^(1/2) +
 2)^(1/2)))*(3^(1/2) + 2)^(1/4)*1i)/4
```

3.18 $\int \frac{1+x^4}{1-5x^4+x^8}\,dx$

3.18.1 Optimal result

Integrand size = 18, antiderivative size = 171

$$\int \frac{1+x^4}{1-5x^4+x^8}\,dx = \frac{\arctan\left(\sqrt{\frac{2}{-\sqrt{3}+\sqrt{7}}}x\right)}{\sqrt{6\left(-\sqrt{3}+\sqrt{7}\right)}} - \frac{\arctan\left(\sqrt{\frac{2}{\sqrt{3}+\sqrt{7}}}x\right)}{\sqrt{6\left(\sqrt{3}+\sqrt{7}\right)}} + \frac{\operatorname{arctanh}\left(\sqrt{\frac{2}{-\sqrt{3}+\sqrt{7}}}x\right)}{\sqrt{6\left(-\sqrt{3}+\sqrt{7}\right)}} - \frac{\operatorname{arctanh}\left(\sqrt{\frac{2}{\sqrt{3}+\sqrt{7}}}x\right)}{\sqrt{6\left(\sqrt{3}+\sqrt{7}\right)}}$$

output
```
arctan(x*2^(1/2)/(7^(1/2)-3^(1/2))^(1/2))/(-6*3^(1/2)+6*7^(1/2))^(1/2)+arc
tanh(x*2^(1/2)/(7^(1/2)-3^(1/2))^(1/2))/(-6*3^(1/2)+6*7^(1/2))^(1/2)-arcta
n(x*2^(1/2)/(7^(1/2)+3^(1/2))^(1/2))/(6*3^(1/2)+6*7^(1/2))^(1/2)-arctanh(x
*2^(1/2)/(7^(1/2)+3^(1/2))^(1/2))/(6*3^(1/2)+6*7^(1/2))^(1/2)
```

3.18.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.32

$$\int \frac{1+x^4}{1-5x^4+x^8}\,dx = \frac{1}{4}\text{RootSum}\left[1-5\#1^4+\#1^8\&,\frac{\log(x-\#1)+\log(x-\#1)\#1^4}{-5\#1^3+2\#1^7}\&\right]$$

input
```
Integrate[(1 + x^4)/(1 - 5*x^4 + x^8),x]
```

output
```
RootSum[1 - 5*#1^4 + #1^8 & , (Log[x - #1] + Log[x - #1]*#1^4)/(-5*#1^3 +
2*#1^7) & ]/4
```

3.18.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.15, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1749, 1406, 216, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4+1}{x^8-5x^4+1}\,dx$$

$$\Big\downarrow \text{1749}$$

$$\frac{1}{2}\int \frac{1}{x^4-\sqrt{7}x^2+1}dx+\frac{1}{2}\int \frac{1}{x^4+\sqrt{7}x^2+1}dx$$

$$\Big\downarrow \text{1406}$$

$$\frac{1}{2}\left(\frac{\int \frac{1}{x^2+\frac{1}{2}\left(-\sqrt{3}-\sqrt{7}\right)}dx}{\sqrt{3}}-\frac{\int \frac{1}{x^2+\frac{1}{2}\left(\sqrt{3}-\sqrt{7}\right)}dx}{\sqrt{3}}\right)+\frac{1}{2}\left(\frac{\int \frac{1}{x^2+\frac{1}{2}\left(-\sqrt{3}+\sqrt{7}\right)}dx}{\sqrt{3}}-\frac{\int \frac{1}{x^2+\frac{1}{2}\left(\sqrt{3}+\sqrt{7}\right)}dx}{\sqrt{3}}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\left(\frac{\int \frac{1}{x^2+\frac{1}{2}\left(-\sqrt{3}-\sqrt{7}\right)}dx}{\sqrt{3}}-\frac{\int \frac{1}{x^2+\frac{1}{2}\left(\sqrt{3}-\sqrt{7}\right)}dx}{\sqrt{3}}\right)+\frac{1}{2}\left(\sqrt{\frac{2}{3\left(\sqrt{7}-\sqrt{3}\right)}}\arctan\left(\sqrt{\frac{2}{\sqrt{7}-\sqrt{3}}}x\right)-\sqrt{\frac{2}{3\left(\sqrt{3}+\sqrt{7}\right)}}\arctan\left(\sqrt{\frac{2}{\sqrt{3}+\sqrt{7}}}x\right)\right)$$

$$\Big\downarrow \text{220}$$

$$\frac{1}{2}\left(\sqrt{\frac{2}{3\left(\sqrt{7}-\sqrt{3}\right)}}\arctan\left(\sqrt{\frac{2}{\sqrt{7}-\sqrt{3}}}x\right)-\sqrt{\frac{2}{3\left(\sqrt{3}+\sqrt{7}\right)}}\arctan\left(\sqrt{\frac{2}{\sqrt{3}+\sqrt{7}}}x\right)\right)+\frac{1}{2}\left(\sqrt{\frac{2}{3\left(\sqrt{7}-\sqrt{3}\right)}}\text{arctanh}\left(\sqrt{\frac{2}{\sqrt{7}-\sqrt{3}}}x\right)-\sqrt{\frac{2}{3\left(\sqrt{3}+\sqrt{7}\right)}}\text{arctanh}\left(\sqrt{\frac{2}{\sqrt{3}+\sqrt{7}}}x\right)\right)$$

input
```
Int[(1 + x^4)/(1 - 5*x^4 + x^8),x]
```

output
```
(Sqrt[2/(3*(-Sqrt[3] + Sqrt[7]))]*ArcTan[Sqrt[2/(-Sqrt[3] + Sqrt[7])]*x] -
 Sqrt[2/(3*(Sqrt[3] + Sqrt[7]))]*ArcTan[Sqrt[2/(Sqrt[3] + Sqrt[7])]*x])/2
+ (Sqrt[2/(3*(-Sqrt[3] + Sqrt[7]))]*ArcTanh[Sqrt[2/(-Sqrt[3] + Sqrt[7])]*x
] - Sqrt[2/(3*(Sqrt[3] + Sqrt[7]))]*ArcTanh[Sqrt[2/(Sqrt[3] + Sqrt[7])]*x]
)/2
```

3.18.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

rule 1749
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e
+ q*x^(n/2) + x^n, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x^(n/2) +
 x^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[2*(d/e) - b/c,
0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

3.18.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.25

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}-5\_Z^{4}+1\right)}\frac{\left(\_R^{4}+1\right)\ln\left(x-\_R\right)}{2\_R^{7}-5\_R^{3}}\right)}{4}$	42
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}-5\_Z^{4}+1\right)}\frac{\left(\_R^{4}+1\right)\ln\left(x-\_R\right)}{2\_R^{7}-5\_R^{3}}\right)}{4}$	42

input
```
int((x^4+1)/(x^8-5*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum((_R^4+1)/(2*_R^7-5*_R^3)*ln(x-_R),_R=RootOf(_Z^8-5*_Z^4+1))
```

3.18.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 509 vs. $2(123) = 246$.

Time = 0.30 (sec) , antiderivative size = 509, normalized size of antiderivative = 2.98

$$
\begin{aligned}
&\int \frac{1+x^4}{1-5x^4+x^8}\,dx \\
&= \frac{1}{24}\sqrt{6}\sqrt{\sqrt{2}\sqrt{\sqrt{7}\sqrt{3}+5}}\log\left(\left(\sqrt{7}\sqrt{6}\sqrt{3}-3\sqrt{6}\right)\sqrt{\sqrt{2}\sqrt{\sqrt{7}\sqrt{3}+5}}+12\,x\right) \\
&\quad-\frac{1}{24}\sqrt{6}\sqrt{\sqrt{2}\sqrt{\sqrt{7}\sqrt{3}+5}}\log\left(-\left(\sqrt{7}\sqrt{6}\sqrt{3}-3\sqrt{6}\right)\sqrt{\sqrt{2}\sqrt{\sqrt{7}\sqrt{3}+5}}+12\,x\right) \\
&\quad+\frac{1}{24}\sqrt{6}\sqrt{-\sqrt{2}\sqrt{\sqrt{7}\sqrt{3}+5}}\log\left(\left(\sqrt{7}\sqrt{6}\sqrt{3}-3\sqrt{6}\right)\sqrt{-\sqrt{2}\sqrt{\sqrt{7}\sqrt{3}+5}}+12\,x\right) \\
&\quad-\frac{1}{24}\sqrt{6}\sqrt{-\sqrt{2}\sqrt{\sqrt{7}\sqrt{3}+5}}\log\left(-\left(\sqrt{7}\sqrt{6}\sqrt{3}-3\sqrt{6}\right)\sqrt{-\sqrt{2}\sqrt{\sqrt{7}\sqrt{3}+5}}+12\,x\right) \\
&\quad-\frac{1}{24}\sqrt{6}\sqrt{\sqrt{2}\sqrt{-\sqrt{7}\sqrt{3}+5}}\log\left(\left(\sqrt{7}\sqrt{6}\sqrt{3}+3\sqrt{6}\right)\sqrt{\sqrt{2}\sqrt{-\sqrt{7}\sqrt{3}+5}}+12\,x\right) \\
&\quad+\frac{1}{24}\sqrt{6}\sqrt{\sqrt{2}\sqrt{-\sqrt{7}\sqrt{3}+5}}\log\left(-\left(\sqrt{7}\sqrt{6}\sqrt{3}+3\sqrt{6}\right)\sqrt{\sqrt{2}\sqrt{-\sqrt{7}\sqrt{3}+5}}+12\,x\right) \\
&\quad-\frac{1}{24}\sqrt{6}\sqrt{-\sqrt{2}\sqrt{-\sqrt{7}\sqrt{3}+5}}\log\left(\left(\sqrt{7}\sqrt{6}\sqrt{3}+3\sqrt{6}\right)\sqrt{-\sqrt{2}\sqrt{-\sqrt{7}\sqrt{3}+5}}+12\,x\right) \\
&\quad+\frac{1}{24}\sqrt{6}\sqrt{-\sqrt{2}\sqrt{-\sqrt{7}\sqrt{3}+5}}\log\left(-\left(\sqrt{7}\sqrt{6}\sqrt{3}+3\sqrt{6}\right)\sqrt{-\sqrt{2}\sqrt{-\sqrt{7}\sqrt{3}+5}}+12\,x\right)
\end{aligned}
$$

input
```
integrate((x^4+1)/(x^8-5*x^4+1),x, algorithm="fricas")
```

output
```
1/24*sqrt(6)*sqrt(sqrt(2)*sqrt(sqrt(7)*sqrt(3) + 5))*log((sqrt(7)*sqrt(6)*
sqrt(3) - 3*sqrt(6))*sqrt(sqrt(2)*sqrt(sqrt(7)*sqrt(3) + 5)) + 12*x) - 1/2
4*sqrt(6)*sqrt(sqrt(2)*sqrt(sqrt(7)*sqrt(3) + 5))*log(-(sqrt(7)*sqrt(6)*sq
rt(3) - 3*sqrt(6))*sqrt(sqrt(2)*sqrt(sqrt(7)*sqrt(3) + 5)) + 12*x) + 1/24*
sqrt(6)*sqrt(-sqrt(2)*sqrt(sqrt(7)*sqrt(3) + 5))*log((sqrt(7)*sqrt(6)*sqrt
(3) - 3*sqrt(6))*sqrt(-sqrt(2)*sqrt(sqrt(7)*sqrt(3) + 5)) + 12*x) - 1/24*s
qrt(6)*sqrt(-sqrt(2)*sqrt(sqrt(7)*sqrt(3) + 5))*log(-(sqrt(7)*sqrt(6)*sqrt
(3) - 3*sqrt(6))*sqrt(-sqrt(2)*sqrt(sqrt(7)*sqrt(3) + 5)) + 12*x) - 1/24*s
qrt(6)*sqrt(sqrt(2)*sqrt(-sqrt(7)*sqrt(3) + 5))*log((sqrt(7)*sqrt(6)*sqrt(
3) + 3*sqrt(6))*sqrt(sqrt(2)*sqrt(-sqrt(7)*sqrt(3) + 5)) + 12*x) + 1/24*sq
rt(6)*sqrt(sqrt(2)*sqrt(-sqrt(7)*sqrt(3) + 5))*log(-(sqrt(7)*sqrt(6)*sqrt(
3) + 3*sqrt(6))*sqrt(sqrt(2)*sqrt(-sqrt(7)*sqrt(3) + 5)) + 12*x) - 1/24*sq
rt(6)*sqrt(-sqrt(2)*sqrt(-sqrt(7)*sqrt(3) + 5))*log((sqrt(7)*sqrt(6)*sqrt(
3) + 3*sqrt(6))*sqrt(-sqrt(2)*sqrt(-sqrt(7)*sqrt(3) + 5)) + 12*x) + 1/24*s
qrt(6)*sqrt(-sqrt(2)*sqrt(-sqrt(7)*sqrt(3) + 5))*log(-(sqrt(7)*sqrt(6)*sqr
t(3) + 3*sqrt(6))*sqrt(-sqrt(2)*sqrt(-sqrt(7)*sqrt(3) + 5)) + 12*x)
```

3.18.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.14

$$\int \frac{1+x^4}{1-5x^4+x^8}\,dx = \operatorname{RootSum}\left(5308416t^8 - 11520t^4 + 1, \left(t \mapsto t\log\left(9216t^5 - 16t + x\right)\right)\right)$$

input
```
integrate((x**4+1)/(x**8-5*x**4+1),x)
```

output
```
RootSum(5308416*_t**8 - 11520*_t**4 + 1, Lambda(_t, _t*log(9216*_t**5 - 16
*_t + x)))
```

3.18.7 Maxima [F]

$$\int \frac{1+x^4}{1-5x^4+x^8}\,dx = \int \frac{x^4+1}{x^8-5\,x^4+1}\,dx$$

input
```
integrate((x^4+1)/(x^8-5*x^4+1),x, algorithm="maxima")
```

output
```
integrate((x^4 + 1)/(x^8 - 5*x^4 + 1), x)
```

3.18.8 Giac [F]

$$\int \frac{1+x^4}{1-5x^4+x^8}\,dx = \int \frac{x^4+1}{x^8-5\,x^4+1}\,dx$$

input
```
integrate((x^4+1)/(x^8-5*x^4+1),x, algorithm="giac")
```

output
```
integrate((x^4 + 1)/(x^8 - 5*x^4 + 1), x)
```

3.18.9 Mupad [B] (verification not implemented)

Time = 8.46 (sec) , antiderivative size = 483, normalized size of antiderivative = 2.82

$$\begin{aligned}
&\int \frac{1+x^4}{1-5x^4+x^8}\,dx \\
&= \frac{2^{3/4}\sqrt{3}\,\mathrm{atan}\left(\frac{12005\,2^{3/4}\sqrt{3}\,x\left(5-\sqrt{21}\right)^{1/4}}{2\left(4802\sqrt{2}\sqrt{5-\sqrt{21}}-1029\sqrt{2}\sqrt{21}\sqrt{5-\sqrt{21}}\right)}-\frac{7889\,2^{3/4}\sqrt{3}\sqrt{21}\,x\left(5-\sqrt{21}\right)^{1/4}}{6\left(4802\sqrt{2}\sqrt{5-\sqrt{21}}-1029\sqrt{2}\sqrt{21}\sqrt{5-\sqrt{21}}\right)}\right)\left(5-\sqrt{21}\right)^{1/4}}{12} \\
&\quad - \frac{2^{3/4}\sqrt{3}\,\mathrm{atan}\left(\frac{2^{3/4}\sqrt{3}\,x\left(5-\sqrt{21}\right)^{1/4}12005i}{2\left(4802\sqrt{2}\sqrt{5-\sqrt{21}}-1029\sqrt{2}\sqrt{21}\sqrt{5-\sqrt{21}}\right)}-\frac{2^{3/4}\sqrt{3}\sqrt{21}\,x\left(5-\sqrt{21}\right)^{1/4}7889i}{6\left(4802\sqrt{2}\sqrt{5-\sqrt{21}}-1029\sqrt{2}\sqrt{21}\sqrt{5-\sqrt{21}}\right)}\right)\left(5-\sqrt{21}\right)^{1/4}}{12} \\
&\quad + \frac{2^{3/4}\sqrt{3}\,\mathrm{atan}\left(\frac{12005\,2^{3/4}\sqrt{3}\,x\left(\sqrt{21}+5\right)^{1/4}}{2\left(4802\sqrt{2}\sqrt{\sqrt{21}+5}+1029\sqrt{2}\sqrt{21}\sqrt{\sqrt{21}+5}\right)}+\frac{7889\,2^{3/4}\sqrt{3}\sqrt{21}\,x\left(\sqrt{21}+5\right)^{1/4}}{6\left(4802\sqrt{2}\sqrt{\sqrt{21}+5}+1029\sqrt{2}\sqrt{21}\sqrt{\sqrt{21}+5}\right)}\right)\left(\sqrt{21}+5\right)^{1/4}}{12} \\
&\quad - \frac{2^{3/4}\sqrt{3}\,\mathrm{atan}\left(\frac{2^{3/4}\sqrt{3}\,x\left(\sqrt{21}+5\right)^{1/4}12005i}{2\left(4802\sqrt{2}\sqrt{\sqrt{21}+5}+1029\sqrt{2}\sqrt{21}\sqrt{\sqrt{21}+5}\right)}+\frac{2^{3/4}\sqrt{3}\sqrt{21}\,x\left(\sqrt{21}+5\right)^{1/4}7889i}{6\left(4802\sqrt{2}\sqrt{\sqrt{21}+5}+1029\sqrt{2}\sqrt{21}\sqrt{\sqrt{21}+5}\right)}\right)\left(\sqrt{21}+5\right)^{1/4}}{12}
\end{aligned}$$

input
```
int((x^4 + 1)/(x^8 - 5*x^4 + 1),x)
```

output
```
(2^(3/4)*3^(1/2)*atan((12005*2^(3/4)*3^(1/2)*x*(5 - 21^(1/2))^(1/4))/(2*(4
802*2^(1/2)*(5 - 21^(1/2))^(1/2) - 1029*2^(1/2)*21^(1/2)*(5 - 21^(1/2))^(1
/2))) - (7889*2^(3/4)*3^(1/2)*21^(1/2)*x*(5 - 21^(1/2))^(1/4))/(6*(4802*2^
(1/2)*(5 - 21^(1/2))^(1/2) - 1029*2^(1/2)*21^(1/2)*(5 - 21^(1/2))^(1/2))))
*(5 - 21^(1/2))^(1/4))/12 - (2^(3/4)*3^(1/2)*atan((2^(3/4)*3^(1/2)*x*(5 -
21^(1/2))^(1/4)*12005i)/(2*(4802*2^(1/2)*(5 - 21^(1/2))^(1/2) - 1029*2^(1/
2)*21^(1/2)*(5 - 21^(1/2))^(1/2))) - (2^(3/4)*3^(1/2)*21^(1/2)*x*(5 - 21^(
1/2))^(1/4)*7889i)/(6*(4802*2^(1/2)*(5 - 21^(1/2))^(1/2) - 1029*2^(1/2)*21
^(1/2)*(5 - 21^(1/2))^(1/2))))*(5 - 21^(1/2))^(1/4)*1i)/12 + (2^(3/4)*3^(1
/2)*atan((12005*2^(3/4)*3^(1/2)*x*(21^(1/2) + 5)^(1/4))/(2*(4802*2^(1/2)*(
21^(1/2) + 5)^(1/2) + 1029*2^(1/2)*21^(1/2)*(21^(1/2) + 5)^(1/2))) + (7889
*2^(3/4)*3^(1/2)*21^(1/2)*x*(21^(1/2) + 5)^(1/4))/(6*(4802*2^(1/2)*(21^(1/
2) + 5)^(1/2) + 1029*2^(1/2)*21^(1/2)*(21^(1/2) + 5)^(1/2))))*(21^(1/2) +
5)^(1/4))/12 - (2^(3/4)*3^(1/2)*atan((2^(3/4)*3^(1/2)*x*(21^(1/2) + 5)^(1/
4)*12005i)/(2*(4802*2^(1/2)*(21^(1/2) + 5)^(1/2) + 1029*2^(1/2)*21^(1/2)*(
21^(1/2) + 5)^(1/2))) + (2^(3/4)*3^(1/2)*21^(1/2)*x*(21^(1/2) + 5)^(1/4)*7
889i)/(6*(4802*2^(1/2)*(21^(1/2) + 5)^(1/2) + 1029*2^(1/2)*21^(1/2)*(21^(1
/2) + 5)^(1/2))))*(21^(1/2) + 5)^(1/4)*1i)/12
```

3.18. $\int \frac{1+x^4}{1-5x^4+x^8} \, dx$

3.19 $\int \frac{1+x^4}{1-6x^4+x^8} \, dx$

3.19.1 Optimal result

Integrand size = 18, antiderivative size = 117

$$\int \frac{1+x^4}{1-6x^4+x^8} \, dx = \frac{\arctan\left(\frac{x}{\sqrt{-1+\sqrt{2}}}\right)}{4\sqrt{-1+\sqrt{2}}} - \frac{\arctan\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{1+\sqrt{2}}} + \frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{-1+\sqrt{2}}}\right)}{4\sqrt{-1+\sqrt{2}}} - \frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{1+\sqrt{2}}}$$

output
```
1/4*arctan(x/(2^(1/2)-1)^(1/2))/(2^(1/2)-1)^(1/2)+1/4*arctanh(x/(2^(1/2)-1
)^(1/2))/(2^(1/2)-1)^(1/2)-1/4*arctan(x/(1+2^(1/2))^(1/2))/(1+2^(1/2))^(1/
2)-1/4*arctanh(x/(1+2^(1/2))^(1/2))/(1+2^(1/2))^(1/2)
```

3.19.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.95

$$\int \frac{1+x^4}{1-6x^4+x^8} \, dx = \frac{1}{4}\left(\sqrt{1+\sqrt{2}}\arctan\left(\frac{x}{\sqrt{-1+\sqrt{2}}}\right) - \sqrt{-1+\sqrt{2}}\arctan\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right) + \sqrt{1+\sqrt{2}}\operatorname{arctanh}\left(\frac{x}{\sqrt{-1+\sqrt{2}}}\right) - \sqrt{-1+\sqrt{2}}\operatorname{arctanh}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)\right)$$

input
```
Integrate[(1 + x^4)/(1 - 6*x^4 + x^8),x]
```

output
```
(Sqrt[1 + Sqrt[2]]*ArcTan[x/Sqrt[-1 + Sqrt[2]]] - Sqrt[-1 + Sqrt[2]]*ArcTa
n[x/Sqrt[1 + Sqrt[2]]] + Sqrt[1 + Sqrt[2]]*ArcTanh[x/Sqrt[-1 + Sqrt[2]]] -
 Sqrt[-1 + Sqrt[2]]*ArcTanh[x/Sqrt[1 + Sqrt[2]]])/4
```

3.19.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1749, 1406, 216, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4+1}{x^8-6x^4+1}dx$$

$$\Big\downarrow \text{1749}$$

$$\frac{1}{2}\int \frac{1}{x^4-2\sqrt{2}x^2+1}dx+\frac{1}{2}\int \frac{1}{x^4+2\sqrt{2}x^2+1}dx$$

$$\Big\downarrow \text{1406}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2-\sqrt{2}-1}dx-\frac{1}{2}\int \frac{1}{x^2-\sqrt{2}+1}dx\right)+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2+\sqrt{2}-1}dx-\frac{1}{2}\int \frac{1}{x^2+\sqrt{2}+1}dx\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2-\sqrt{2}-1}dx-\frac{1}{2}\int \frac{1}{x^2-\sqrt{2}+1}dx\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)}{2\sqrt{\sqrt{2}-1}}-\frac{\arctan\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{2\sqrt{1+\sqrt{2}}}\right)$$

$$\Big\downarrow \text{220}$$

$$\frac{1}{2}\left(\frac{\arctan\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)}{2\sqrt{\sqrt{2}-1}}-\frac{\arctan\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{2\sqrt{1+\sqrt{2}}}\right)+\frac{1}{2}\left(\frac{\text{arctanh}\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)}{2\sqrt{\sqrt{2}-1}}-\frac{\text{arctanh}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{2\sqrt{1+\sqrt{2}}}\right)$$

input
```
Int[(1 + x^4)/(1 - 6*x^4 + x^8),x]
```

output
```
(ArcTan[x/Sqrt[-1 + Sqrt[2]]]/(2*Sqrt[-1 + Sqrt[2]]) - ArcTan[x/Sqrt[1 + S
qrt[2]]]/(2*Sqrt[1 + Sqrt[2]]))/2 + (ArcTanh[x/Sqrt[-1 + Sqrt[2]]]/(2*Sqrt
[-1 + Sqrt[2]]) - ArcTanh[x/Sqrt[1 + Sqrt[2]]]/(2*Sqrt[1 + Sqrt[2]]))/2
```

3.19.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

rule 1749
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e
+ q*x^(n/2) + x^n, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x^(n/2) +
 x^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[2*(d/e) - b/c,
0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

3.19.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.16 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.50

method	result	siz
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+2\_Z^2-1\right)}\_R\ln\left(-\_R^3-2\_R+x\right)\right)}{8}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-2\_Z^2-1\right)}\_R\ln\left(\_R^3-2\_R+x\right)\right)}{8}$	58
default	$\frac{\arctan\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)}{4\sqrt{\sqrt{2}-1}}+\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)}{4\sqrt{\sqrt{2}-1}}-\frac{\arctan\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{1+\sqrt{2}}}-\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{1+\sqrt{2}}}$	78

input
```
int((x^4+1)/(x^8-6*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/8*sum(_R*ln(-_R^3-2*_R+x),_R=RootOf(_Z^4+2*_Z^2-1))+1/8*sum(_R*ln(_R^3-2
*_R+x),_R=RootOf(_Z^4-2*_Z^2-1))
```

3.19.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. 2(77) = 154.

Time = 0.29 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.89

$$
\begin{aligned}
\int \frac{1+x^4}{1-6x^4+x^8}\,dx = &-\frac{1}{8}\sqrt{\sqrt{2}-1}\log\left(\left(\sqrt{2}+1\right)\sqrt{\sqrt{2}-1}+x\right)\\
&+\frac{1}{8}\sqrt{\sqrt{2}-1}\log\left(-\left(\sqrt{2}+1\right)\sqrt{\sqrt{2}-1}+x\right)\\
&+\frac{1}{8}\sqrt{\sqrt{2}+1}\log\left(\sqrt{\sqrt{2}+1}\left(\sqrt{2}-1\right)+x\right)\\
&-\frac{1}{8}\sqrt{\sqrt{2}+1}\log\left(-\sqrt{\sqrt{2}+1}\left(\sqrt{2}-1\right)+x\right)\\
&-\frac{1}{8}\sqrt{-\sqrt{2}+1}\log\left(\left(\sqrt{2}+1\right)\sqrt{-\sqrt{2}+1}+x\right)\\
&+\frac{1}{8}\sqrt{-\sqrt{2}+1}\log\left(-\left(\sqrt{2}+1\right)\sqrt{-\sqrt{2}+1}+x\right)\\
&+\frac{1}{8}\sqrt{-\sqrt{2}-1}\log\left(\left(\sqrt{2}-1\right)\sqrt{-\sqrt{2}-1}+x\right)\\
&-\frac{1}{8}\sqrt{-\sqrt{2}-1}\log\left(-\left(\sqrt{2}-1\right)\sqrt{-\sqrt{2}-1}+x\right)
\end{aligned}
$$

input
```
integrate((x^4+1)/(x^8-6*x^4+1),x, algorithm="fricas")
```

output
```
-1/8*sqrt(sqrt(2) - 1)*log((sqrt(2) + 1)*sqrt(sqrt(2) - 1) + x) + 1/8*sqrt
(sqrt(2) - 1)*log(-(sqrt(2) + 1)*sqrt(sqrt(2) - 1) + x) + 1/8*sqrt(sqrt(2)
 + 1)*log(sqrt(sqrt(2) + 1)*(sqrt(2) - 1) + x) - 1/8*sqrt(sqrt(2) + 1)*log
(-sqrt(sqrt(2) + 1)*(sqrt(2) - 1) + x) - 1/8*sqrt(-sqrt(2) + 1)*log((sqrt(
2) + 1)*sqrt(-sqrt(2) + 1) + x) + 1/8*sqrt(-sqrt(2) + 1)*log(-(sqrt(2) + 1
)*sqrt(-sqrt(2) + 1) + x) + 1/8*sqrt(-sqrt(2) - 1)*log((sqrt(2) - 1)*sqrt(
-sqrt(2) - 1) + x) - 1/8*sqrt(-sqrt(2) - 1)*log(-(sqrt(2) - 1)*sqrt(-sqrt(
2) - 1) + x)
```

3.19.6 Sympy [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.42

$$\int \frac{1+x^4}{1-6x^4+x^8}\,dx = \operatorname{RootSum}\left(4096t^4-128t^2-1,\left(t\mapsto t\log\left(16384t^5-20t+x\right)\right)\right) + \operatorname{RootSum}\left(4096t^4+128t^2-1,\left(t\mapsto t\log\left(16384t^5-20t+x\right)\right)\right)$$

input
```
integrate((x**4+1)/(x**8-6*x**4+1),x)
```

output
```
RootSum(4096*_t**4 - 128*_t**2 - 1, Lambda(_t, _t*log(16384*_t**5 - 20*_t
+ x))) + RootSum(4096*_t**4 + 128*_t**2 - 1, Lambda(_t, _t*log(16384*_t**5
 - 20*_t + x)))
```

3.19.7 Maxima [F]

$$\int \frac{1+x^4}{1-6x^4+x^8}\,dx = \int \frac{x^4+1}{x^8-6\,x^4+1}\,dx$$

input
```
integrate((x^4+1)/(x^8-6*x^4+1),x, algorithm="maxima")
```

output
```
integrate((x^4 + 1)/(x^8 - 6*x^4 + 1), x)
```

3.19.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.05

$$\int \frac{1+x^4}{1-6x^4+x^8}\,dx = -\frac{1}{4}\sqrt{\sqrt{2}-1}\arctan\left(\frac{x}{\sqrt{\sqrt{2}+1}}\right)+\frac{1}{4}\sqrt{\sqrt{2}+1}\arctan\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right) - \frac{1}{8}\sqrt{\sqrt{2}-1}\log\left(\left|x+\sqrt{\sqrt{2}+1}\right|\right) + \frac{1}{8}\sqrt{\sqrt{2}-1}\log\left(\left|x-\sqrt{\sqrt{2}+1}\right|\right) + \frac{1}{8}\sqrt{\sqrt{2}+1}\log\left(\left|x+\sqrt{\sqrt{2}-1}\right|\right) - \frac{1}{8}\sqrt{\sqrt{2}+1}\log\left(\left|x-\sqrt{\sqrt{2}-1}\right|\right)$$

input
```
integrate((x^4+1)/(x^8-6*x^4+1),x, algorithm="giac")
```

output
```
-1/4*sqrt(sqrt(2) - 1)*arctan(x/sqrt(sqrt(2) + 1)) + 1/4*sqrt(sqrt(2) + 1)
*arctan(x/sqrt(sqrt(2) - 1)) - 1/8*sqrt(sqrt(2) - 1)*log(abs(x + sqrt(sqrt
(2) + 1))) + 1/8*sqrt(sqrt(2) - 1)*log(abs(x - sqrt(sqrt(2) + 1))) + 1/8*s
qrt(sqrt(2) + 1)*log(abs(x + sqrt(sqrt(2) - 1))) - 1/8*sqrt(sqrt(2) + 1)*l
og(abs(x - sqrt(sqrt(2) - 1)))
```

3.19.9 Mupad [B] (verification not implemented)

Time = 8.43 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.99

$$\int \frac{1+x^4}{1-6x^4+x^8}\,dx = -\frac{\mathrm{atan}\left(\frac{x\sqrt{\sqrt{2}-1}\,49152\mathrm{i}}{34816\sqrt{2}-49152}-\frac{\sqrt{2}\,x\sqrt{\sqrt{2}-1}\,34816\mathrm{i}}{34816\sqrt{2}-49152}\right)\sqrt{\sqrt{2}-1}\,1\mathrm{i}}{4} - \frac{\mathrm{atan}\left(\frac{x\sqrt{\sqrt{2}+1}\,49152\mathrm{i}}{34816\sqrt{2}+49152}+\frac{\sqrt{2}\,x\sqrt{\sqrt{2}+1}\,34816\mathrm{i}}{34816\sqrt{2}+49152}\right)\sqrt{\sqrt{2}+1}\,1\mathrm{i}}{4} + \frac{\mathrm{atan}\left(\frac{x\sqrt{1-\sqrt{2}}\,49152\mathrm{i}}{34816\sqrt{2}-49152}-\frac{\sqrt{2}\,x\sqrt{1-\sqrt{2}}\,34816\mathrm{i}}{34816\sqrt{2}-49152}\right)\sqrt{1-\sqrt{2}}\,1\mathrm{i}}{4} + \frac{\mathrm{atan}\left(\frac{x\sqrt{-\sqrt{2}-1}\,49152\mathrm{i}}{34816\sqrt{2}+49152}+\frac{\sqrt{2}\,x\sqrt{-\sqrt{2}-1}\,34816\mathrm{i}}{34816\sqrt{2}+49152}\right)\sqrt{-\sqrt{2}-1}\,1\mathrm{i}}{4}$$

input
```
int((x^4 + 1)/(x^8 - 6*x^4 + 1),x)
```

output
```
(atan((x*(1 - 2^(1/2))^(1/2)*49152i)/(34816*2^(1/2) - 49152) - (2^(1/2)*x*
(1 - 2^(1/2))^(1/2)*34816i)/(34816*2^(1/2) - 49152))*(1 - 2^(1/2))^(1/2)*1
i)/4 - (atan((x*(2^(1/2) + 1)^(1/2)*49152i)/(34816*2^(1/2) + 49152) + (2^(
1/2)*x*(2^(1/2) + 1)^(1/2)*34816i)/(34816*2^(1/2) + 49152))*(2^(1/2) + 1)^
(1/2)*1i)/4 - (atan((x*(2^(1/2) - 1)^(1/2)*49152i)/(34816*2^(1/2) - 49152)
 - (2^(1/2)*x*(2^(1/2) - 1)^(1/2)*34816i)/(34816*2^(1/2) - 49152))*(2^(1/2
) - 1)^(1/2)*1i)/4 + (atan((x*(- 2^(1/2) - 1)^(1/2)*49152i)/(34816*2^(1/2)
 + 49152) + (2^(1/2)*x*(- 2^(1/2) - 1)^(1/2)*34816i)/(34816*2^(1/2) + 4915
2))*(- 2^(1/2) - 1)^(1/2)*1i)/4
```

3.19. $\int \frac{1+x^4}{1-6x^4+x^8}\, dx$

3.20 $\int \frac{1-x^4}{1+bx^4+x^8}\,dx$

3.20.1 Optimal result

Integrand size = 20, antiderivative size = 511

$$\begin{aligned}\int \frac{1-x^4}{1+bx^4+x^8}\,dx = &-\frac{\sqrt{2+b}\arctan\left(\frac{\sqrt{2-\sqrt{2-b}}-2x}{\sqrt{2+\sqrt{2-b}}}\right)}{4\sqrt{2-\sqrt{2-b}}\sqrt{2-b}}+\frac{\sqrt{2+b}\arctan\left(\frac{\sqrt{2+\sqrt{2-b}}-2x}{\sqrt{2-\sqrt{2-b}}}\right)}{4\sqrt{2+\sqrt{2-b}}\sqrt{2-b}}\\
&+\frac{\sqrt{2+b}\arctan\left(\frac{\sqrt{2-\sqrt{2-b}}+2x}{\sqrt{2+\sqrt{2-b}}}\right)}{4\sqrt{2-\sqrt{2-b}}\sqrt{2-b}}-\frac{\sqrt{2+b}\arctan\left(\frac{\sqrt{2+\sqrt{2-b}}+2x}{\sqrt{2-\sqrt{2-b}}}\right)}{4\sqrt{2+\sqrt{2-b}}\sqrt{2-b}}\\
&+\frac{\sqrt{2-\sqrt{2-b}}\log\left(1-\sqrt{2-\sqrt{2-b}}x+x^2\right)}{8\sqrt{2-b}}\\
&-\frac{\sqrt{2-\sqrt{2-b}}\log\left(1+\sqrt{2-\sqrt{2-b}}x+x^2\right)}{8\sqrt{2-b}}\\
&-\frac{\sqrt{2+\sqrt{2-b}}\log\left(1-\sqrt{2+\sqrt{2-b}}x+x^2\right)}{8\sqrt{2-b}}\\
&+\frac{\sqrt{2+\sqrt{2-b}}\log\left(1+\sqrt{2+\sqrt{2-b}}x+x^2\right)}{8\sqrt{2-b}}\end{aligned}$$

output
```
-1/4*arctan((-2*x+(2-(2-b)^(1/2))^(1/2))/(2+(2-b)^(1/2))^(1/2))*(2+b)^(1/2
)/(2-b)^(1/2)/(2-(2-b)^(1/2))^(1/2)+1/4*arctan((2*x+(2-(2-b)^(1/2))^(1/2))
/(2+(2-b)^(1/2))^(1/2))*(2+b)^(1/2)/(2-b)^(1/2)/(2-(2-b)^(1/2))^(1/2)+1/8*
ln(1+x^2-x*(2-(2-b)^(1/2))^(1/2))*(2-(2-b)^(1/2))^(1/2)/(2-b)^(1/2)-1/8*ln
(1+x^2+x*(2-(2-b)^(1/2))^(1/2))*(2-(2-b)^(1/2))^(1/2)/(2-b)^(1/2)+1/4*arct
an((-2*x+(2+(2-b)^(1/2))^(1/2))/(2-(2-b)^(1/2))^(1/2))*(2+b)^(1/2)/(2-b)^(
1/2)/(2+(2-b)^(1/2))^(1/2)-1/4*arctan((2*x+(2+(2-b)^(1/2))^(1/2))/(2-(2-b)
^(1/2))^(1/2))*(2+b)^(1/2)/(2-b)^(1/2)/(2+(2-b)^(1/2))^(1/2)-1/8*ln(1+x^2-
x*(2+(2-b)^(1/2))^(1/2))*(2+(2-b)^(1/2))^(1/2)/(2-b)^(1/2)+1/8*ln(1+x^2+x*
(2+(2-b)^(1/2))^(1/2))*(2+(2-b)^(1/2))^(1/2)/(2-b)^(1/2)
```

3.20.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.11

$$\int \frac{1-x^4}{1+bx^4+x^8}\,dx = -\frac{1}{4}\text{RootSum}\left[1+b\#1^4+\#1^8\&,\frac{-\log(x-\#1)+\log(x-\#1)\#1^4}{b\#1^3+2\#1^7}\&\right]$$

input
```
Integrate[(1 - x^4)/(1 + b*x^4 + x^8),x]
```

output
```
-1/4*RootSum[1 + b*#1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*#1^4)/(b*#1
^3 + 2*#1^7) & ]
```

3.20.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 509, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {1751, 25, 1483, 27, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x^4}{bx^4+x^8+1}\,dx$$

$$\Big\downarrow \text{1751}$$

$$-\frac{\int -\frac{\sqrt{2-b}-2x^2}{x^4-\sqrt{2-b}x^2+1}dx}{2\sqrt{2-b}}-\frac{\int -\frac{2x^2+\sqrt{2-b}}{x^4+\sqrt{2-b}x^2+1}dx}{2\sqrt{2-b}}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{\sqrt{2-b}-2x^2}{x^4-\sqrt{2-b}x^2+1}dx}{2\sqrt{2-b}}+\frac{\int \frac{2x^2+\sqrt{2-b}}{x^4+\sqrt{2-b}x^2+1}dx}{2\sqrt{2-b}}$$

$$\Big\downarrow \ 1483$$

$$\frac{\frac{\int \frac{\sqrt{2-\sqrt{2-b}}\left(\sqrt{2-\sqrt{2-b}}x+\sqrt{2-b}\right)}{x^2-\sqrt{2-\sqrt{2-b}}x+1}dx}{2\sqrt{2-\sqrt{2-b}}}+\frac{\int \frac{\sqrt{2-\sqrt{2-b}}\sqrt{2-b}-\left(2-\sqrt{2-b}\right)x}{x^2+\sqrt{2-\sqrt{2-b}}x+1}dx}{2\sqrt{2-\sqrt{2-b}}}}{2\sqrt{2-b}}+\frac{\frac{\int \frac{\sqrt{\sqrt{2-b}+2}\sqrt{2-b}-\left(\sqrt{2-b}+2\right)x}{x^2-\sqrt{\sqrt{2-b}+2}x+1}dx}{2\sqrt{\sqrt{2-b}+2}}+\frac{\int \frac{\sqrt{\sqrt{2-b}+2}\left(\sqrt{\sqrt{2-b}+2}x+\sqrt{2-b}\right)}{x^2+\sqrt{\sqrt{2-b}+2}x+1}dx}{2\sqrt{\sqrt{2-b}+2}}}{2\sqrt{2-b}}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{1}{2}\int \frac{\sqrt{2-\sqrt{2-b}}x+\sqrt{2-b}}{x^2-\sqrt{2-\sqrt{2-b}}x+1}dx+\frac{\int \frac{\sqrt{2-\sqrt{2-b}}\sqrt{2-b}-\left(2-\sqrt{2-b}\right)x}{x^2+\sqrt{2-\sqrt{2-b}}x+1}dx}{2\sqrt{2-\sqrt{2-b}}}}{2\sqrt{2-b}}+\frac{\frac{\int \frac{\sqrt{\sqrt{2-b}+2}\sqrt{2-b}-\left(\sqrt{2-b}+2\right)x}{x^2-\sqrt{\sqrt{2-b}+2}x+1}dx}{2\sqrt{\sqrt{2-b}+2}}+\frac{1}{2}\int \frac{\sqrt{\sqrt{2-b}+2}x+\sqrt{2-b}}{x^2+\sqrt{\sqrt{2-b}+2}x+1}dx}{2\sqrt{2-b}}$$

$$\Big\downarrow \ 1142$$

$$\frac{\frac{1}{2}\left(\frac{1}{2}\left(\sqrt{2-b}+2\right)\int \frac{1}{x^2-\sqrt{2-\sqrt{2-b}}x+1}dx+\frac{1}{2}\sqrt{2-\sqrt{2-b}}\int -\frac{\sqrt{2-\sqrt{2-b}}-2x}{x^2-\sqrt{2-\sqrt{2-b}}x+1}dx\right)+\frac{\frac{1}{2}\sqrt{2-\sqrt{2-b}}\left(\sqrt{2-b}+2\right)\int \frac{1}{x^2+\sqrt{2-\sqrt{2-\cdots}}}\cdots}{2\sqrt{\cdots}}}{2\sqrt{2-b}}$$
$$\frac{\frac{-\frac{1}{2}\left(2-\sqrt{2-b}\right)\sqrt{\sqrt{2-b}+2}\int \frac{1}{x^2-\sqrt{\sqrt{2-b}+2}x+1}dx-\frac{1}{2}\left(\sqrt{2-b}+2\right)\int -\frac{\sqrt{\sqrt{2-b}+2}-2x}{x^2-\sqrt{\sqrt{2-b}+2}x+1}dx}{2\sqrt{\sqrt{2-b}+2}}+\frac{1}{2}\left(\frac{1}{2}\sqrt{\sqrt{2-b}+2}\int \frac{2x+\sqrt{\sqrt{2-b}+2}}{x^2+\sqrt{\sqrt{2-b}+2}x+1}dx-\cdots\right.}{2\sqrt{2-b}}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{1}{2}\left(\frac{1}{2}\left(\sqrt{2-b}+2\right)\int \frac{1}{x^2-\sqrt{2-\sqrt{2-b}}x+1}dx-\frac{1}{2}\sqrt{2-\sqrt{2-b}}\int \frac{\sqrt{2-\sqrt{2-b}}-2x}{x^2-\sqrt{2-\sqrt{2-b}}x+1}dx\right)+\frac{\frac{1}{2}\sqrt{2-\sqrt{2-b}}\left(\sqrt{2-b}+2\right)\int \frac{1}{x^2+\sqrt{2-\sqrt{2-b}}\cdots}\cdots}{2\sqrt{2}\cdots}}{2\sqrt{2-b}}$$
$$\frac{\frac{\frac{1}{2}\left(\sqrt{2-b}+2\right)\int \frac{\sqrt{\sqrt{2-b}+2}-2x}{x^2-\sqrt{\sqrt{2-b}+2}x+1}dx-\frac{1}{2}\left(2-\sqrt{2-b}\right)\sqrt{\sqrt{2-b}+2}\int \frac{1}{x^2-\sqrt{\sqrt{2-b}+2}x+1}dx}{2\sqrt{\sqrt{2-b}+2}}+\frac{1}{2}\left(\frac{1}{2}\sqrt{\sqrt{2-b}+2}\int \frac{2x+\sqrt{\sqrt{2-b}+2}}{x^2+\sqrt{\sqrt{2-b}+2}x+1}dx-\frac{1}{2}\left(2\cdots\right.\right.}{2\sqrt{2-b}}$$

$$\Big\downarrow \ 1083$$

3.20. $\int \frac{1-x^4}{1+bx^4+x^8}\,dx$

$$\frac{\frac{1}{2}\left(-\frac{1}{2}\sqrt{2-\sqrt{2-b}}\int \frac{\sqrt{2-\sqrt{2-b}}-2x}{x^2-\sqrt{2-\sqrt{2-b}}x+1}dx-\left(\sqrt{2-b}+2\right)\int \frac{1}{-\left(2x-\sqrt{2-\sqrt{2-b}}\right)^2-\sqrt{2-b}-2}d\left(2x-\sqrt{2-\sqrt{2-b}}\right)\right)+}{2\sqrt{2-b}}$$

$$\frac{\frac{\frac{1}{2}(\sqrt{2-b}+2)\int \frac{\sqrt{\sqrt{2-b}+2}-2x}{x^2-\sqrt{\sqrt{2-b}+2}x+1}dx+(2-\sqrt{2-b})\sqrt{\sqrt{2-b}+2}\int \frac{1}{-\left(2x-\sqrt{\sqrt{2-b}+2}\right)^2+\sqrt{2-b}-2}d\left(2x-\sqrt{\sqrt{2-b}+2}\right)}{2\sqrt{\sqrt{2-b}+2}}+\frac{1}{2}\left(\frac{1}{2}\sqrt{\sqrt{2-b}+2}\int \frac{2x}{x^2+}\right.}{2\sqrt{2-b}}$$

$$\Big\downarrow \text{217}$$

$$\frac{\frac{1}{2}\left(\sqrt{\sqrt{2-b}+2}\arctan\left(\frac{2x-\sqrt{2-\sqrt{2-b}}}{\sqrt{\sqrt{2-b}+2}}\right)-\frac{1}{2}\sqrt{2-\sqrt{2-b}}\int \frac{\sqrt{2-\sqrt{2-b}}-2x}{x^2-\sqrt{2-\sqrt{2-b}}x+1}dx\right)+\frac{\sqrt{2-\sqrt{2-b}}\sqrt{\sqrt{2-b}+2}\arctan\left(\frac{\sqrt{2-\sqrt{2-}}}{\sqrt{\sqrt{2-}}}\right.}{2\sqrt{}}}{2\sqrt{2-b}}$$

$$\frac{\frac{\frac{1}{2}(\sqrt{2-b}+2)\int \frac{\sqrt{\sqrt{2-b}+2}-2x}{x^2-\sqrt{\sqrt{2-b}+2}x+1}dx-\sqrt{2-\sqrt{2-b}}\sqrt{\sqrt{2-b}+2}\arctan\left(\frac{2x-\sqrt{\sqrt{2-b}+2}}{\sqrt{2-\sqrt{2-b}}}\right)}{2\sqrt{\sqrt{2-b}+2}}+\frac{1}{2}\left(\frac{1}{2}\sqrt{\sqrt{2-b}+2}\int \frac{2x+\sqrt{\sqrt{2-b}+2}}{x^2+\sqrt{\sqrt{2-b}+2}x+1}dx-\sqrt{2}\right.}{2\sqrt{2-b}}$$

$$\Big\downarrow \text{1103}$$

$$\frac{\frac{1}{2}\left(\sqrt{\sqrt{2-b}+2}\arctan\left(\frac{2x-\sqrt{2-\sqrt{2-b}}}{\sqrt{\sqrt{2-b}+2}}\right)+\frac{1}{2}\sqrt{2-\sqrt{2-b}}\log\left(-\sqrt{2-\sqrt{2-b}}x+x^2+1\right)\right)+\frac{\sqrt{2-\sqrt{2-b}}\sqrt{\sqrt{2-b}+2}\,\mathrm{ar}}{}}{2\sqrt{2-b}}$$

$$\frac{\frac{-\sqrt{2-\sqrt{2-b}}\sqrt{\sqrt{2-b}+2}\arctan\left(\frac{2x-\sqrt{\sqrt{2-b}+2}}{\sqrt{2-\sqrt{2-b}}}\right)-\frac{1}{2}(\sqrt{2-b}+2)\log\left(-\sqrt{\sqrt{2-b}+2}x+x^2+1\right)}{2\sqrt{\sqrt{2-b}+2}}+\frac{1}{2}\left(\frac{1}{2}\sqrt{\sqrt{2-b}+2}\log\left(\sqrt{\sqrt{2-b}+2}x\right.\right.}{2\sqrt{2-b}}$$

input
```
Int[(1 - x^4)/(1 + b*x^4 + x^8),x]
```

output
```
((Sqrt[2 + Sqrt[2 - b]]*ArcTan[(-Sqrt[2 - Sqrt[2 - b]] + 2*x)/Sqrt[2 + Sqr
t[2 - b]]] + (Sqrt[2 - Sqrt[2 - b]]*Log[1 - Sqrt[2 - Sqrt[2 - b]]*x + x^2]
)/2)/2 + (Sqrt[2 - Sqrt[2 - b]]*Sqrt[2 + Sqrt[2 - b]]*ArcTan[(Sqrt[2 - Sqr
t[2 - b]] + 2*x)/Sqrt[2 + Sqrt[2 - b]]] - ((2 - Sqrt[2 - b])*Log[1 + Sqrt[
2 - Sqrt[2 - b]]*x + x^2])/2)/(2*Sqrt[2 - Sqrt[2 - b]]))/(2*Sqrt[2 - b]) +
 ((-(Sqrt[2 - Sqrt[2 - b]]*Sqrt[2 + Sqrt[2 - b]]*ArcTan[(-Sqrt[2 + Sqrt[2
- b]] + 2*x)/Sqrt[2 - Sqrt[2 - b]]]) - ((2 + Sqrt[2 - b])*Log[1 - Sqrt[2 +
 Sqrt[2 - b]]*x + x^2])/2)/(2*Sqrt[2 + Sqrt[2 - b]]) + (-(Sqrt[2 - Sqrt[2
- b]]*ArcTan[(Sqrt[2 + Sqrt[2 - b]] + 2*x)/Sqrt[2 - Sqrt[2 - b]]]) + (Sqrt
[2 + Sqrt[2 - b]]*Log[1 + Sqrt[2 + Sqrt[2 - b]]*x + x^2])/2)/2)/(2*Sqrt[2
- b])
```

3.20. $\int \frac{1-x^4}{1+bx^4+x^8}\,dx$

3.20.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 1751
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x
^(n/2))/Simp[d/e + q*x^(n/2) - x^n, x], x], x] + Simp[e/(2*c*q)   Int[(q +
2*x^(n/2))/Simp[d/e - q*x^(n/2) - x^n, x], x], x]] /; FreeQ[{a, b, c, d, e}
, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGt
Q[n/2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

3.20.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.09

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}+\_Z^{4}b+1\right)}\frac{\left(-\_R^{4}+1\right)\ln\left(x-\_R\right)}{2\_R^{7}+\_R^{3}b}\right)}{4}$	44
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}+\_Z^{4}b+1\right)}\frac{\left(-\_R^{4}+1\right)\ln\left(x-\_R\right)}{2\_R^{7}+\_R^{3}b}\right)}{4}$	44

input
```
int((-x^4+1)/(x^8+b*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum((-_R^4+1)/(2*_R^7+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8+_Z^4*b+1))
```

3.20.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1177 vs. 2(397) = 794.

Time = 0.28 (sec) , antiderivative size = 1177, normalized size of antiderivative = 2.30

$$\int \frac{1-x^4}{1+bx^4+x^8}\,dx = \text{Too large to display}$$

input
```
integrate((-x^4+1)/(x^8+b*x^4+1),x, algorithm="fricas")
```

output
```
1/4*sqrt(sqrt(1/2)*sqrt(-((b^2 - 4*b + 4)*sqrt((b + 2)/(b^3 - 6*b^2 + 12*b
 - 8)) + b)/(b^2 - 4*b + 4)))*log(1/2*((b^2 - 4*b + 4)*sqrt((b + 2)/(b^3 -
 6*b^2 + 12*b - 8)) - b + 2)*sqrt(sqrt(1/2)*sqrt(-((b^2 - 4*b + 4)*sqrt((b
 + 2)/(b^3 - 6*b^2 + 12*b - 8)) + b)/(b^2 - 4*b + 4))) + x) - 1/4*sqrt(sqr
t(1/2)*sqrt(-((b^2 - 4*b + 4)*sqrt((b + 2)/(b^3 - 6*b^2 + 12*b - 8)) + b)/
(b^2 - 4*b + 4)))*log(-1/2*((b^2 - 4*b + 4)*sqrt((b + 2)/(b^3 - 6*b^2 + 12
*b - 8)) - b + 2)*sqrt(sqrt(1/2)*sqrt(-((b^2 - 4*b + 4)*sqrt((b + 2)/(b^3
- 6*b^2 + 12*b - 8)) + b)/(b^2 - 4*b + 4))) + x) + 1/4*sqrt(-sqrt(1/2)*sqr
t(-((b^2 - 4*b + 4)*sqrt((b + 2)/(b^3 - 6*b^2 + 12*b - 8)) + b)/(b^2 - 4*b
 + 4)))*log(1/2*((b^2 - 4*b + 4)*sqrt((b + 2)/(b^3 - 6*b^2 + 12*b - 8)) -
b + 2)*sqrt(-sqrt(1/2)*sqrt(-((b^2 - 4*b + 4)*sqrt((b + 2)/(b^3 - 6*b^2 +
12*b - 8)) + b)/(b^2 - 4*b + 4))) + x) - 1/4*sqrt(-sqrt(1/2)*sqrt(-((b^2 -
 4*b + 4)*sqrt((b + 2)/(b^3 - 6*b^2 + 12*b - 8)) + b)/(b^2 - 4*b + 4)))*lo
g(-1/2*((b^2 - 4*b + 4)*sqrt((b + 2)/(b^3 - 6*b^2 + 12*b - 8)) - b + 2)*sq
rt(-sqrt(1/2)*sqrt(-((b^2 - 4*b + 4)*sqrt((b + 2)/(b^3 - 6*b^2 + 12*b - 8)
) + b)/(b^2 - 4*b + 4))) + x) - 1/4*sqrt(sqrt(1/2)*sqrt(((b^2 - 4*b + 4)*s
qrt((b + 2)/(b^3 - 6*b^2 + 12*b - 8)) - b)/(b^2 - 4*b + 4)))*log(1/2*((b^2
 - 4*b + 4)*sqrt((b + 2)/(b^3 - 6*b^2 + 12*b - 8)) + b - 2)*sqrt(sqrt(1/2)
*sqrt(((b^2 - 4*b + 4)*sqrt((b + 2)/(b^3 - 6*b^2 + 12*b - 8)) - b)/(b^2 -
4*b + 4))) + x) + 1/4*sqrt(sqrt(1/2)*sqrt(((b^2 - 4*b + 4)*sqrt((b + 2)...
```

3.20.6 Sympy [A] (verification not implemented)

Time = 1.87 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.15

$$\int \frac{1-x^4}{1+bx^4+x^8}\,dx = -\operatorname{RootSum}\left(t^8\cdot\left(65536b^4-524288b^3+1572864b^2-2097152b+1048576\right)+t^4\cdot\left(256b^3-1024b^2+102\right.\right.$$

input
```
integrate((-x**4+1)/(x**8+b*x**4+1),x)
```

output
```
-RootSum(_t**8*(65536*b**4 - 524288*b**3 + 1572864*b**2 - 2097152*b + 1048
576) + _t**4*(256*b**3 - 1024*b**2 + 1024*b) + 1, Lambda(_t, _t*log(1024*_
t**5*b**2 - 4096*_t**5*b + 4096*_t**5 + 4*_t*b - 4*_t + x)))
```

3.20.7 Maxima [F]

$$\int \frac{1-x^4}{1+bx^4+x^8}\,dx=\int -\frac{x^4-1}{x^8+bx^4+1}\,dx$$

input
```
integrate((-x^4+1)/(x^8+b*x^4+1),x, algorithm="maxima")
```

output
```
-integrate((x^4 - 1)/(x^8 + b*x^4 + 1), x)
```

3.20.8 Giac [F]

$$\int \frac{1-x^4}{1+bx^4+x^8}\,dx=\int -\frac{x^4-1}{x^8+bx^4+1}\,dx$$

input
```
integrate((-x^4+1)/(x^8+b*x^4+1),x, algorithm="giac")
```

output
```
integrate(-(x^4 - 1)/(x^8 + b*x^4 + 1), x)
```

3.20.9 Mupad [B] (verification not implemented)

Time = 9.53 (sec) , antiderivative size = 5341, normalized size of antiderivative = 10.45

$$\int \frac{1-x^4}{1+bx^4+x^8}\,dx=\text{Too large to display}$$

input
```
int(-(x^4 - 1)/(b*x^4 + x^8 + 1),x)
```

output
```
- atan((((-(4*b + ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 -
32*b - 8*b^3 + b^4 + 16)))^(1/4)*(256*b + ((-(4*b + ((b - 2)^5*(b + 2))^(1
/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(1/4)*(262144
*b - 196608*b^2 - 196608*b^3 + 49152*b^4 + 49152*b^5 - 4096*b^6 - 4096*b^7
 + 262144) + x*(32768*b - 65536*b^2 - 32768*b^3 + 20480*b^4 + 10240*b^5 -
2048*b^6 - 1024*b^7 + 65536))*(-(4*b + ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 +
 b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(3/4) - 64*b^3 - 16*b^4 +
256) - x*(32*b + 48*b^2 + 24*b^3 + 4*b^4))*(-(4*b + ((b - 2)^5*(b + 2))^(1
/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(1/4)*1i - ((
-(4*b + ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b
^3 + b^4 + 16)))^(1/4)*(256*b + ((-(4*b + ((b - 2)^5*(b + 2))^(1/2) - 4*b^
2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(1/4)*(262144*b - 19660
8*b^2 - 196608*b^3 + 49152*b^4 + 49152*b^5 - 4096*b^6 - 4096*b^7 + 262144)
 - x*(32768*b - 65536*b^2 - 32768*b^3 + 20480*b^4 + 10240*b^5 - 2048*b^6 -
 1024*b^7 + 65536))*(-(4*b + ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512
*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(3/4) - 64*b^3 - 16*b^4 + 256) + x*(
32*b + 48*b^2 + 24*b^3 + 4*b^4))*(-(4*b + ((b - 2)^5*(b + 2))^(1/2) - 4*b^
2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(1/4)*1i)/(((-(4*b + ((
b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 +
 16)))^(1/4)*(256*b + ((-(4*b + ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3...
```

3.20. $\int \frac{1-x^4}{1+bx^4+x^8}\,dx$

3.21 $\int \frac{1-x^4}{1+3x^4+x^8} \, dx$

3.21.1 Optimal result

Integrand size = 20, antiderivative size = 411

$$\int \frac{1-x^4}{1+3x^4+x^8}\,dx = -\frac{\sqrt[4]{3+\sqrt{5}}\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\ 2^{3/4}} + \frac{\sqrt[4]{3+\sqrt{5}}\arctan\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\ 2^{3/4}} + \frac{\sqrt[4]{3-\sqrt{5}}\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\ 2^{3/4}} - \frac{\sqrt[4]{3-\sqrt{5}}\arctan\left(1+\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\ 2^{3/4}} - \frac{\sqrt[4]{3+\sqrt{5}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{4\ 2^{3/4}} + \frac{\sqrt[4]{3+\sqrt{5}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{4\ 2^{3/4}} + \frac{\sqrt[4]{3-\sqrt{5}}\log\left(\sqrt{2\left(3+\sqrt{5}\right)}-2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+2x^2\right)}{4\ 2^{3/4}} - \frac{\sqrt[4]{3-\sqrt{5}}\log\left(\sqrt{2\left(3+\sqrt{5}\right)}+2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+2x^2\right)}{4\ 2^{3/4}}$$

output
```
-1/4*arctan(-1+2^(3/4)*x/(3+5^(1/2))^(1/4))*(3-5^(1/2))^(1/4)*2^(1/4)-1/4*
arctan(1+2^(3/4)*x/(3+5^(1/2))^(1/4))*(3-5^(1/2))^(1/4)*2^(1/4)+1/8*ln(2*x
^2-2*2^(1/4)*x*(3+5^(1/2))^(1/4)+5^(1/2)+1)*(3-5^(1/2))^(1/4)*2^(1/4)-1/8*
ln(2*x^2+2*2^(1/4)*x*(3+5^(1/2))^(1/4)+5^(1/2)+1)*(3-5^(1/2))^(1/4)*2^(1/4
)+1/4*arctan(-1+2^(3/4)*x/(3-5^(1/2))^(1/4))*(3+5^(1/2))^(1/4)*2^(1/4)+1/4
*arctan(1+2^(3/4)*x/(3-5^(1/2))^(1/4))*(3+5^(1/2))^(1/4)*2^(1/4)-1/8*ln(2*
x^2-2*2^(1/4)*x*(3-5^(1/2))^(1/4)+5^(1/2)-1)*(3+5^(1/2))^(1/4)*2^(1/4)+1/8
*ln(2*x^2+2*2^(1/4)*x*(3-5^(1/2))^(1/4)+5^(1/2)-1)*(3+5^(1/2))^(1/4)*2^(1/
4)
```

3.21.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.14

$$\int \frac{1-x^4}{1+3x^4+x^8}\,dx = -\frac{1}{4}\text{RootSum}\left[1+3\#1^4+\#1^8\&,\frac{-\log(x-\#1)+\log(x-\#1)\#1^4}{3\#1^3+2\#1^7}\&\right]$$

input
```
Integrate[(1 - x^4)/(1 + 3*x^4 + x^8),x]
```

output
```
-1/4*RootSum[1 + 3*#1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*#1^4)/(3*#1
^3 + 2*#1^7) & ]
```

3.21.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 477, normalized size of antiderivative = 1.16, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {1750, 755, 27, 1476, 1082, 217, 1479, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x^4}{x^8+3x^4+1}dx$$

$$\Big\downarrow \text{1750}$$

$$-\frac{1}{2}\left(1-\sqrt{5}\right)\int \frac{1}{x^4+\frac{1}{2}\left(3-\sqrt{5}\right)}dx-\frac{1}{2}\left(1+\sqrt{5}\right)\int \frac{1}{x^4+\frac{1}{2}\left(3+\sqrt{5}\right)}dx$$

$$\Big\downarrow \textcolor{blue}{755}$$

$$-\frac{1}{2}\left(1-\sqrt{5}\right)\left(\frac{\int \frac{2\left(\sqrt{3-\sqrt{5}}-\sqrt{2}x^2\right)}{2x^4-\sqrt{5}+3}dx}{2\sqrt{3-\sqrt{5}}}+\frac{\int \frac{2\left(\sqrt{2}x^2+\sqrt{3-\sqrt{5}}\right)}{2x^4-\sqrt{5}+3}dx}{2\sqrt{3-\sqrt{5}}}\right)-\frac{1}{2}\left(1+\sqrt{5}\right)\left(\frac{\int \frac{2\left(\sqrt{3+\sqrt{5}}-\sqrt{2}x^2\right)}{2x^4+\sqrt{5}+3}dx}{2\sqrt{3+\sqrt{5}}}+\frac{\int \frac{2\left(\sqrt{2}x^2+\sqrt{3+\sqrt{5}}\right)}{2x^4+\sqrt{5}+3}dx}{2\sqrt{3+\sqrt{5}}}\right)$$

$$\Big\downarrow \textcolor{blue}{27}$$

$$-\frac{1}{2}\left(1-\sqrt{5}\right)\left(\frac{\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{2x^4-\sqrt{5}+3}dx}{\sqrt{3-\sqrt{5}}}+\frac{\int \frac{\sqrt{2}x^2+\sqrt{3-\sqrt{5}}}{2x^4-\sqrt{5}+3}dx}{\sqrt{3-\sqrt{5}}}\right)-\frac{1}{2}\left(1+\sqrt{5}\right)\left(\frac{\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{2x^4+\sqrt{5}+3}dx}{\sqrt{3+\sqrt{5}}}+\frac{\int \frac{\sqrt{2}x^2+\sqrt{3+\sqrt{5}}}{2x^4+\sqrt{5}+3}dx}{\sqrt{3+\sqrt{5}}}\right)$$

$$\Big\downarrow \textcolor{blue}{1476}$$

$$-\frac{1}{2}\left(1-\sqrt{5}\right)\left(\frac{\frac{\int \frac{1}{x^2-\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}}dx}{2\sqrt{2}}+\frac{\int \frac{1}{x^2+\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}}dx}{2\sqrt{2}}}{\sqrt{3-\sqrt{5}}}+\frac{\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{2x^4-\sqrt{5}+3}dx}{\sqrt{3-\sqrt{5}}}\right)-\frac{1}{2}\left(1+\sqrt{5}\right)\left(\frac{\frac{\int \frac{1}{x^2-\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\sqrt{2}}+\frac{\int \frac{1}{x^2+\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\sqrt{2}}}{\sqrt{3+\sqrt{5}}}+\frac{\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{2x^4+\sqrt{5}+3}dx}{\sqrt{3+\sqrt{5}}}\right)$$

$$\Big\downarrow \textcolor{blue}{1082}$$

3.21. $\int \frac{1-x^4}{1+3x^4+x^8}\,dx$

$$-\frac{1}{2}\left(1-\sqrt{5}\right)\left(\frac{\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{2x^4-\sqrt{5}+3}dx}{\sqrt{3-\sqrt{5}}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)^2-1}d\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}-\frac{\int \frac{1}{-\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)^2-1}d\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+\cdots\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}\cdots}{\sqrt{3-\sqrt{5}}}\right.$$

$$\frac{1}{2}\left(1+\sqrt{5}\right)\left(\frac{\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{2x^4+\sqrt{5}+3}dx}{\sqrt{3+\sqrt{5}}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)^2-1}d\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\int \frac{1}{-\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)^2-1}d\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\cdots\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}\cdots}{\sqrt{3+\sqrt{5}}}\right.$$

$$\downarrow 217$$

$$-\frac{1}{2}\left(1-\sqrt{5}\right)\left(\frac{\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{2x^4-\sqrt{5}+3}dx}{\sqrt{3-\sqrt{5}}}+\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}}{\sqrt{3-\sqrt{5}}}\right)-$$

$$\frac{1}{2}\left(1+\sqrt{5}\right)\left(\frac{\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{2x^4+\sqrt{5}+3}dx}{\sqrt{3+\sqrt{5}}}+\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{\sqrt{3+\sqrt{5}}}\right)$$

3.21. $\int \frac{1-x^4}{1+3x^4+x^8}\,dx$

$$\downarrow \text{1479}$$

$$-\frac{1}{2}\left(1-\sqrt{5}\right)\left(\frac{-\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\int -\frac{\sqrt[4]{2\left(3-\sqrt{5}\right)}-2x}{x^2-\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}}dx-\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\int -\frac{2x+\sqrt[4]{2\left(3-\cdots\right.}}{x^2+\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\cdots}\cdots}{\sqrt{3-\sqrt{5}}}\right.$$

$$\frac{1}{2}\left(1+\sqrt{5}\right)\left(\frac{-\frac{\int -\frac{\sqrt[4]{2\left(3+\sqrt{5}\right)}-2x}{x^2-\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\int -\frac{2x+\sqrt[4]{2\left(3+\sqrt{5}\right)}}{x^2+\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{\sqrt{3+\sqrt{5}}}+\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\cdots}{\sqrt{3+\cdots}}\right.$$

$$\downarrow \text{25}$$

$$-\frac{1}{2}\left(1-\sqrt{5}\right)\left(\frac{\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\int \frac{\sqrt[4]{2\left(3-\sqrt{5}\right)}-2x}{x^2-\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}}dx+\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\int \frac{2x+\sqrt[4]{2\left(3-\sqrt{5}\right)}}{x^2+\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(\cdots\right.}}\cdots}{\sqrt{3-\sqrt{5}}}\right.$$

$$\frac{1}{2}\left(1+\sqrt{5}\right)\left(\frac{\frac{\int \frac{\sqrt[4]{2\left(3+\sqrt{5}\right)}-2x}{x^2-\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}+\frac{\int \frac{2x+\sqrt[4]{2\left(3+\sqrt{5}\right)}}{x^2+\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{\sqrt{3+\sqrt{5}}}+\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\text{arcta}\cdots}{\cdots}}{\sqrt{3+\sqrt{5}}}\right.$$

3.21. $\int \frac{1-x^4}{1+3x^4+x^8}\,dx$

$$\Big\downarrow \text{ 1103}$$

$$-\frac{1}{2}\left(1-\sqrt{5}\right)\left(\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}}{\sqrt{3-\sqrt{5}}}+\frac{\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\cdots\right.}{\cdots}\right.$$

$$\frac{1}{2}\left(1+\sqrt{5}\right)\left(\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{\sqrt{3+\sqrt{5}}}+\frac{\frac{\log\left(2x^2+2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\log\left(2x^2-2\cdots\right.}{\cdots}}{\sqrt{3+\sqrt{5}}}\right.$$

input
```
Int[(1 - x^4)/(1 + 3*x^4 + x^8),x]
```

output
```
-1/2*((1 - Sqrt[5])*((-(ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)]/(2^(3/
4)*(3 - Sqrt[5])^(1/4))) + ArcTan[1 + (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)]/(2^
(3/4)*(3 - Sqrt[5])^(1/4)))/Sqrt[3 - Sqrt[5]] + (-1/4*(((3 + Sqrt[5])/2)^(
1/4)*Log[Sqrt[2*(3 - Sqrt[5])] - 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2]) + (
((3 + Sqrt[5])/2)^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])] + 2*(2*(3 - Sqrt[5]))^(1
/4)*x + 2*x^2])/4)/Sqrt[3 - Sqrt[5]])) - ((1 + Sqrt[5])*((-(ArcTan[1 - (2^
(3/4)*x)/(3 + Sqrt[5])^(1/4)]/(2^(3/4)*(3 + Sqrt[5])^(1/4))) + ArcTan[1 +
(2^(3/4)*x)/(3 + Sqrt[5])^(1/4)]/(2^(3/4)*(3 + Sqrt[5])^(1/4)))/Sqrt[3 + S
qrt[5]] + (-1/2*Log[Sqrt[2*(3 + Sqrt[5])] - 2*(2*(3 + Sqrt[5]))^(1/4)*x +
2*x^2]/(2^(3/4)*(3 + Sqrt[5])^(1/4)) + Log[Sqrt[2*(3 + Sqrt[5])] + 2*(2*(3
 + Sqrt[5]))^(1/4)*x + 2*x^2]/(2*2^(3/4)*(3 + Sqrt[5])^(1/4)))/Sqrt[3 + Sq
rt[5]]))/2
```

3.21. $\int \frac{1-x^4}{1+3x^4+x^8}\,dx$

3.21.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1750
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && G
tQ[b^2 - 4*a*c, 0]
```

3.21.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.11

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}+3\_Z^{4}+1\right)}\frac{\left(-\_R^{4}+1\right)\ln\left(x-\_R\right)}{2\_R^{7}+3\_R^{3}}\right)}{4}$	44
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}+3\_Z^{4}+1\right)}\frac{\left(-\_R^{4}+1\right)\ln\left(x-\_R\right)}{2\_R^{7}+3\_R^{3}}\right)}{4}$	44

input
```
int((-x^4+1)/(x^8+3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum((-_R^4+1)/(2*_R^7+3*_R^3)*ln(x-_R),_R=RootOf(_Z^8+3*_Z^4+1))
```

3.21.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.02

$$\int \frac{1-x^4}{1+3x^4+x^8}\,dx = -\frac{1}{8}\sqrt{2}\sqrt{\sqrt{2}\sqrt{\sqrt{5}-3}}\log\left(\left(\sqrt{5}\sqrt{2}+\sqrt{2}\right)\sqrt{\sqrt{2}\sqrt{\sqrt{5}-3}}+4x\right) + \frac{1}{8}\sqrt{2}\sqrt{\sqrt{2}\sqrt{\sqrt{5}-3}}\log\left(-\left(\sqrt{5}\sqrt{2}+\sqrt{2}\right)\sqrt{\sqrt{2}\sqrt{\sqrt{5}-3}}+4x\right) - \frac{1}{8}\sqrt{2}\sqrt{-\sqrt{2}\sqrt{\sqrt{5}-3}}\log\left(\left(\sqrt{5}\sqrt{2}+\sqrt{2}\right)\sqrt{-\sqrt{2}\sqrt{\sqrt{5}-3}}+4x\right) + \frac{1}{8}\sqrt{2}\sqrt{-\sqrt{2}\sqrt{\sqrt{5}-3}}\log\left(-\left(\sqrt{5}\sqrt{2}+\sqrt{2}\right)\sqrt{-\sqrt{2}\sqrt{\sqrt{5}-3}}+4x\right) + \frac{1}{8}\sqrt{2}\sqrt{\sqrt{2}\sqrt{-\sqrt{5}-3}}\log\left(\left(\sqrt{5}\sqrt{2}-\sqrt{2}\right)\sqrt{\sqrt{2}\sqrt{-\sqrt{5}-3}}+4x\right) - \frac{1}{8}\sqrt{2}\sqrt{\sqrt{2}\sqrt{-\sqrt{5}-3}}\log\left(-\left(\sqrt{5}\sqrt{2}-\sqrt{2}\right)\sqrt{\sqrt{2}\sqrt{-\sqrt{5}-3}}+4x\right) + \frac{1}{8}\sqrt{2}\sqrt{-\sqrt{2}\sqrt{-\sqrt{5}-3}}\log\left(\left(\sqrt{5}\sqrt{2}-\sqrt{2}\right)\sqrt{-\sqrt{2}\sqrt{-\sqrt{5}-3}}+4x\right) - \frac{1}{8}\sqrt{2}\sqrt{-\sqrt{2}\sqrt{-\sqrt{5}-3}}\log\left(-\left(\sqrt{5}\sqrt{2}-\sqrt{2}\right)\sqrt{-\sqrt{2}\sqrt{-\sqrt{5}-3}}+4x\right)$$

input
```
integrate((-x^4+1)/(x^8+3*x^4+1),x, algorithm="fricas")
```

output
```
-1/8*sqrt(2)*sqrt(sqrt(2)*sqrt(sqrt(5) - 3))*log((sqrt(5)*sqrt(2) + sqrt(2
))*sqrt(sqrt(2)*sqrt(sqrt(5) - 3)) + 4*x) + 1/8*sqrt(2)*sqrt(sqrt(2)*sqrt(
sqrt(5) - 3))*log(-(sqrt(5)*sqrt(2) + sqrt(2))*sqrt(sqrt(2)*sqrt(sqrt(5) -
 3)) + 4*x) - 1/8*sqrt(2)*sqrt(-sqrt(2)*sqrt(sqrt(5) - 3))*log((sqrt(5)*sq
rt(2) + sqrt(2))*sqrt(-sqrt(2)*sqrt(sqrt(5) - 3)) + 4*x) + 1/8*sqrt(2)*sqr
t(-sqrt(2)*sqrt(sqrt(5) - 3))*log(-(sqrt(5)*sqrt(2) + sqrt(2))*sqrt(-sqrt(
2)*sqrt(sqrt(5) - 3)) + 4*x) + 1/8*sqrt(2)*sqrt(sqrt(2)*sqrt(-sqrt(5) - 3)
)*log((sqrt(5)*sqrt(2) - sqrt(2))*sqrt(sqrt(2)*sqrt(-sqrt(5) - 3)) + 4*x)
- 1/8*sqrt(2)*sqrt(sqrt(2)*sqrt(-sqrt(5) - 3))*log(-(sqrt(5)*sqrt(2) - sqr
t(2))*sqrt(sqrt(2)*sqrt(-sqrt(5) - 3)) + 4*x) + 1/8*sqrt(2)*sqrt(-sqrt(2)*
sqrt(-sqrt(5) - 3))*log((sqrt(5)*sqrt(2) - sqrt(2))*sqrt(-sqrt(2)*sqrt(-sq
rt(5) - 3)) + 4*x) - 1/8*sqrt(2)*sqrt(-sqrt(2)*sqrt(-sqrt(5) - 3))*log(-(s
qrt(5)*sqrt(2) - sqrt(2))*sqrt(-sqrt(2)*sqrt(-sqrt(5) - 3)) + 4*x)
```

3.21.6 Sympy [A] (verification not implemented)

Time = 0.88 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.06

$$\int \frac{1-x^4}{1+3x^4+x^8}\,dx = -\operatorname{RootSum}\left(65536t^8+768t^4+1,\left(t\mapsto t\log\left(1024t^5+8t+x\right)\right)\right)$$

input
```
integrate((-x**4+1)/(x**8+3*x**4+1),x)
```

output
```
-RootSum(65536*_t**8 + 768*_t**4 + 1, Lambda(_t, _t*log(1024*_t**5 + 8*_t
+ x)))
```

3.21.7 Maxima [F]

$$\int \frac{1-x^4}{1+3x^4+x^8}\,dx = \int -\frac{x^4-1}{x^8+3\,x^4+1}\,dx$$

input
```
integrate((-x^4+1)/(x^8+3*x^4+1),x, algorithm="maxima")
```

output
```
-integrate((x^4 - 1)/(x^8 + 3*x^4 + 1), x)
```

3.21.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.54

$$\int \frac{1-x^4}{1+3x^4+x^8}\,dx = \frac{1}{16}\left(\pi+4\arctan\left(x\sqrt{\sqrt{5}+1}+1\right)\right)\sqrt{\sqrt{5}+1} - \frac{1}{16}\left(\pi+4\arctan\left(-x\sqrt{\sqrt{5}+1}+1\right)\right)\sqrt{\sqrt{5}+1} - \frac{1}{16}\left(\pi+4\arctan\left(x\sqrt{\sqrt{5}-1}-1\right)\right)\sqrt{\sqrt{5}-1} + \frac{1}{16}\left(\pi+4\arctan\left(-x\sqrt{\sqrt{5}-1}-1\right)\right)\sqrt{\sqrt{5}-1} - \frac{1}{8}\sqrt{\sqrt{5}-1}\log\left(2500\left(x+\sqrt{\sqrt{5}+1}\right)^2+2500\,x^2\right) + \frac{1}{8}\sqrt{\sqrt{5}-1}\log\left(2500\left(x-\sqrt{\sqrt{5}+1}\right)^2+2500\,x^2\right) + \frac{1}{8}\sqrt{\sqrt{5}+1}\log\left(1156\left(x+\sqrt{\sqrt{5}-1}\right)^2+1156\,x^2\right) - \frac{1}{8}\sqrt{\sqrt{5}+1}\log\left(1156\left(x-\sqrt{\sqrt{5}-1}\right)^2+1156\,x^2\right)$$

input
```
integrate((-x^4+1)/(x^8+3*x^4+1),x, algorithm="giac")
```

output
```
1/16*(pi + 4*arctan(x*sqrt(sqrt(5) + 1) + 1))*sqrt(sqrt(5) + 1) - 1/16*(pi
 + 4*arctan(-x*sqrt(sqrt(5) + 1) + 1))*sqrt(sqrt(5) + 1) - 1/16*(pi + 4*ar
ctan(x*sqrt(sqrt(5) - 1) - 1))*sqrt(sqrt(5) - 1) + 1/16*(pi + 4*arctan(-x*
sqrt(sqrt(5) - 1) - 1))*sqrt(sqrt(5) - 1) - 1/8*sqrt(sqrt(5) - 1)*log(2500
*(x + sqrt(sqrt(5) + 1))^2 + 2500*x^2) + 1/8*sqrt(sqrt(5) - 1)*log(2500*(x
 - sqrt(sqrt(5) + 1))^2 + 2500*x^2) + 1/8*sqrt(sqrt(5) + 1)*log(1156*(x +
sqrt(sqrt(5) - 1))^2 + 1156*x^2) - 1/8*sqrt(sqrt(5) + 1)*log(1156*(x - sqr
t(sqrt(5) - 1))^2 + 1156*x^2)
```

3.21.9 Mupad [B] (verification not implemented)

Time = 8.37 (sec) , antiderivative size = 447, normalized size of antiderivative = 1.09

$$\int \frac{1-x^4}{1+3x^4+x^8}\,dx = \frac{2^{3/4}\operatorname{atan}\left(\frac{1875\,2^{3/4}\,x\left(\sqrt{5}-3\right)^{1/4}}{2\left(625\sqrt{2}\sqrt{\sqrt{5}-3}-250\sqrt{2}\sqrt{5}\sqrt{\sqrt{5}-3}\right)}-\frac{875\,2^{3/4}\sqrt{5}\,x\left(\sqrt{5}-3\right)^{1/4}}{2\left(625\sqrt{2}\sqrt{\sqrt{5}-3}-250\sqrt{2}\sqrt{5}\sqrt{\sqrt{5}-3}\right)}\right)\left(\sqrt{5}-3\right)^{1/4}}{4} - \frac{2^{3/4}\operatorname{atan}\left(\frac{2^{3/4}\,x\left(\sqrt{5}-3\right)^{1/4}1875\mathrm{i}}{2\left(625\sqrt{2}\sqrt{\sqrt{5}-3}-250\sqrt{2}\sqrt{5}\sqrt{\sqrt{5}-3}\right)}-\frac{2^{3/4}\sqrt{5}\,x\left(\sqrt{5}-3\right)^{1/4}875\mathrm{i}}{2\left(625\sqrt{2}\sqrt{\sqrt{5}-3}-250\sqrt{2}\sqrt{5}\sqrt{\sqrt{5}-3}\right)}\right)\left(\sqrt{5}-3\right)^{1/4}1\mathrm{i}}{4} + \frac{2^{3/4}\operatorname{atan}\left(\frac{1875\,2^{3/4}\,x\left(-\sqrt{5}-3\right)^{1/4}}{2\left(625\sqrt{2}\sqrt{-\sqrt{5}-3}+250\sqrt{2}\sqrt{5}\sqrt{-\sqrt{5}-3}\right)}+\frac{875\,2^{3/4}\sqrt{5}\,x\left(-\sqrt{5}-3\right)^{1/4}}{2\left(625\sqrt{2}\sqrt{-\sqrt{5}-3}+250\sqrt{2}\sqrt{5}\sqrt{-\sqrt{5}-3}\right)}\right)\left(-\sqrt{5}-3\right)^{1/4}}{4} - \frac{2^{3/4}\operatorname{atan}\left(\frac{2^{3/4}\,x\left(-\sqrt{5}-3\right)^{1/4}1875\mathrm{i}}{2\left(625\sqrt{2}\sqrt{-\sqrt{5}-3}+250\sqrt{2}\sqrt{5}\sqrt{-\sqrt{5}-3}\right)}+\frac{2^{3/4}\sqrt{5}\,x\left(-\sqrt{5}-3\right)^{1/4}875\mathrm{i}}{2\left(625\sqrt{2}\sqrt{-\sqrt{5}-3}+250\sqrt{2}\sqrt{5}\sqrt{-\sqrt{5}-3}\right)}\right)\left(-\sqrt{5}-3\right)^{1/4}1\mathrm{i}}{4}$$

input
```
int(-(x^4 - 1)/(3*x^4 + x^8 + 1),x)
```

output
```
(2^(3/4)*atan((1875*2^(3/4)*x*(5^(1/2) - 3)^(1/4))/(2*(625*2^(1/2)*(5^(1/2
) - 3)^(1/2) - 250*2^(1/2)*5^(1/2)*(5^(1/2) - 3)^(1/2))) - (875*2^(3/4)*5^
(1/2)*x*(5^(1/2) - 3)^(1/4))/(2*(625*2^(1/2)*(5^(1/2) - 3)^(1/2) - 250*2^(
1/2)*5^(1/2)*(5^(1/2) - 3)^(1/2))))*(5^(1/2) - 3)^(1/4))/4 - (2^(3/4)*atan
((2^(3/4)*x*(5^(1/2) - 3)^(1/4)*1875i)/(2*(625*2^(1/2)*(5^(1/2) - 3)^(1/2)
 - 250*2^(1/2)*5^(1/2)*(5^(1/2) - 3)^(1/2))) - (2^(3/4)*5^(1/2)*x*(5^(1/2)
 - 3)^(1/4)*875i)/(2*(625*2^(1/2)*(5^(1/2) - 3)^(1/2) - 250*2^(1/2)*5^(1/2
)*(5^(1/2) - 3)^(1/2))))*(5^(1/2) - 3)^(1/4)*1i)/4 + (2^(3/4)*atan((1875*2
^(3/4)*x*(- 5^(1/2) - 3)^(1/4))/(2*(625*2^(1/2)*(- 5^(1/2) - 3)^(1/2) + 25
0*2^(1/2)*5^(1/2)*(- 5^(1/2) - 3)^(1/2))) + (875*2^(3/4)*5^(1/2)*x*(- 5^(1
/2) - 3)^(1/4))/(2*(625*2^(1/2)*(- 5^(1/2) - 3)^(1/2) + 250*2^(1/2)*5^(1/2
)*(- 5^(1/2) - 3)^(1/2))))*(- 5^(1/2) - 3)^(1/4))/4 - (2^(3/4)*atan((2^(3/
4)*x*(- 5^(1/2) - 3)^(1/4)*1875i)/(2*(625*2^(1/2)*(- 5^(1/2) - 3)^(1/2) +
250*2^(1/2)*5^(1/2)*(- 5^(1/2) - 3)^(1/2))) + (2^(3/4)*5^(1/2)*x*(- 5^(1/2
) - 3)^(1/4)*875i)/(2*(625*2^(1/2)*(- 5^(1/2) - 3)^(1/2) + 250*2^(1/2)*5^(
1/2)*(- 5^(1/2) - 3)^(1/2))))*(- 5^(1/2) - 3)^(1/4)*1i)/4
```

3.22 $\int \frac{1-x^4}{1+2x^4+x^8}\,dx$

3.22.1 Optimal result

Integrand size = 20, antiderivative size = 97

$$\int \frac{1-x^4}{1+2x^4+x^8}\,dx = \frac{x}{2\left(1+x^4\right)} - \frac{\arctan\left(1-\sqrt{2}x\right)}{4\sqrt{2}} + \frac{\arctan\left(1+\sqrt{2}x\right)}{4\sqrt{2}} - \frac{\log\left(1-\sqrt{2}x+x^2\right)}{8\sqrt{2}} + \frac{\log\left(1+\sqrt{2}x+x^2\right)}{8\sqrt{2}}$$

output
```
1/2*x/(x^4+1)+1/8*arctan(-1+x*2^(1/2))*2^(1/2)+1/8*arctan(1+x*2^(1/2))*2^(
1/2)-1/16*ln(1+x^2-x*2^(1/2))*2^(1/2)+1/16*ln(1+x^2+x*2^(1/2))*2^(1/2)
```

3.22.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.93

$$\int \frac{1-x^4}{1+2x^4+x^8}\,dx = \frac{1}{16}\left(\frac{8x}{1+x^4} - 2\sqrt{2}\arctan\left(1-\sqrt{2}x\right) + 2\sqrt{2}\arctan\left(1+\sqrt{2}x\right) - \sqrt{2}\log\left(1-\sqrt{2}x+x^2\right) + \sqrt{2}\log\left(1+\sqrt{2}x+x^2\right)\right)$$

input
```
Integrate[(1 - x^4)/(1 + 2*x^4 + x^8),x]
```

output
```
((8*x)/(1 + x^4) - 2*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] + 2*Sqrt[2]*ArcTan[1 +
Sqrt[2]*x] - Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] + Sqrt[2]*Log[1 + Sqrt[2]*x
+ x^2])/16
```

3.22.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1380, 910, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x^4}{x^8+2x^4+1} \, dx$$

$$\Big\downarrow \text{1380}$$

$$\int \frac{1-x^4}{\left(x^4+1\right)^2}dx$$

$$\Big\downarrow \text{910}$$

$$\frac{1}{2}\int \frac{1}{x^4+1}dx+\frac{x}{2\left(x^4+1\right)}$$

$$\Big\downarrow \text{755}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx+\frac{1}{2}\int \frac{x^2+1}{x^4+1}dx\right)+\frac{x}{2\left(x^4+1\right)}$$

$$\Big\downarrow \text{1476}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2-\sqrt{2}x+1}dx+\frac{1}{2}\int \frac{1}{x^2+\sqrt{2}x+1}dx\right)+\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx\right)+\frac{x}{2\left(x^4+1\right)}$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx+\frac{1}{2}\left(\frac{\int \frac{1}{-\left(1-\sqrt{2}x\right)^2-1}d\left(1-\sqrt{2}x\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\left(\sqrt{2}x+1\right)^2-1}d\left(\sqrt{2}x+1\right)}{\sqrt{2}}\right)\right)+\frac{x}{2\left(x^4+1\right)}$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)\right)+\frac{x}{2\left(x^4+1\right)}$$

$$\Big\downarrow \text{1479}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}x+1\right)}{x^2+\sqrt{2}x+1}dx}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)\right)+\frac{x}{2\left(x^4+1\right)}$$

$$\Big\downarrow \ 25$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}x+1\right)}{x^2+\sqrt{2}x+1}dx}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)\right)+\frac{x}{2\left(x^4+1\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}x+1}{x^2+\sqrt{2}x+1}dx\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)\right)+\frac{x}{2\left(x^4+1\right)}$$

$$\Big\downarrow \ 1103$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(x^2+\sqrt{2}x+1\right)}{2\sqrt{2}}-\frac{\log\left(x^2-\sqrt{2}x+1\right)}{2\sqrt{2}}\right)\right)+\frac{x}{2\left(x^4+1\right)}$$

input
```
Int[(1 - x^4)/(1 + 2*x^4 + x^8),x]
```

output
```
x/(2*(1 + x^4)) + ((-(ArcTan[1 - Sqrt[2]*x]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*
x]/Sqrt[2])/2 + (-1/2*Log[1 - Sqrt[2]*x + x^2]/Sqrt[2] + Log[1 + Sqrt[2]*x
 + x^2]/(2*Sqrt[2]))/2)/2
```

3.22. $\int \frac{1-x^4}{1+2x^4+x^8}\,dx$

3.22.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.22.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.34

method	result	size
risch	$\frac{x}{2x^4+2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+1\right)}\frac{\ln\left(x-\_R\right)}{\_R^3}\right)}{8}$	33
default	$\frac{x}{2x^4+2}+\frac{\sqrt{2}\left(\ln\left(\frac{1+x^2+x\sqrt{2}}{1+x^2-x\sqrt{2}}\right)+2\arctan\left(x\sqrt{2}+1\right)+2\arctan\left(x\sqrt{2}-1\right)\right)}{16}$	63

input
```
int((-x^4+1)/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*x/(x^4+1)+1/8*sum(1/_R^3*ln(x-_R),_R=RootOf(_Z^4+1))
```

3.22.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00

$$\int \frac{1-x^4}{1+2x^4+x^8}\,dx$$
$$=\frac{\sqrt{2}((i+1)\;x^4+i+1)\log\left(2\,x+(i+1)\;\sqrt{2}\right)+\sqrt{2}(-(i-1)\;x^4-i+1)\log\left(2\,x-(i-1)\;\sqrt{2}\right)+\sqrt{2}((i}{16\left(x^4+1\right)}$$

input
```
integrate((-x^4+1)/(x^8+2*x^4+1),x, algorithm="fricas")
```

output
```
1/16*(sqrt(2)*((I + 1)*x^4 + I + 1)*log(2*x + (I + 1)*sqrt(2)) + sqrt(2)*(
-(I - 1)*x^4 - I + 1)*log(2*x - (I - 1)*sqrt(2)) + sqrt(2)*((I - 1)*x^4 +
I - 1)*log(2*x + (I - 1)*sqrt(2)) + sqrt(2)*(-(I + 1)*x^4 - I - 1)*log(2*x
 - (I + 1)*sqrt(2)) + 8*x)/(x^4 + 1)
```

3.22.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.85

$$\int \frac{1-x^4}{1+2x^4+x^8}\,dx = \frac{x}{2x^4+2} - \frac{\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{16} + \frac{\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{16} + \frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}x-1\right)}{8} + \frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}x+1\right)}{8}$$

input
```
integrate((-x**4+1)/(x**8+2*x**4+1),x)
```

output
```
x/(2*x**4 + 2) - sqrt(2)*log(x**2 - sqrt(2)*x + 1)/16 + sqrt(2)*log(x**2 +
 sqrt(2)*x + 1)/16 + sqrt(2)*atan(sqrt(2)*x - 1)/8 + sqrt(2)*atan(sqrt(2)*
x + 1)/8
```

3.22.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.85

$$\int \frac{1-x^4}{1+2x^4+x^8}\,dx = \frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x+\sqrt{2}\right)\right) + \frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x-\sqrt{2}\right)\right) + \frac{1}{16}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right) - \frac{1}{16}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right) + \frac{x}{2\,(x^4+1)}$$

input
```
integrate((-x^4+1)/(x^8+2*x^4+1),x, algorithm="maxima")
```

output
```
1/8*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/8*sqrt(2)*arctan(1/2*s
qrt(2)*(2*x - sqrt(2))) + 1/16*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/16*sqr
t(2)*log(x^2 - sqrt(2)*x + 1) + 1/2*x/(x^4 + 1)
```

3.22.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.85

$$\int \frac{1-x^4}{1+2x^4+x^8}\,dx = \frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right) + \frac{1}{16}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{1}{16}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)+\frac{x}{2\left(x^4+1\right)}$$

input
```
integrate((-x^4+1)/(x^8+2*x^4+1),x, algorithm="giac")
```

output
```
1/8*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/8*sqrt(2)*arctan(1/2*s
qrt(2)*(2*x - sqrt(2))) + 1/16*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/16*sqr
t(2)*log(x^2 - sqrt(2)*x + 1) + 1/2*x/(x^4 + 1)
```

3.22.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.45

$$\int \frac{1-x^4}{1+2x^4+x^8}\,dx = \frac{x}{2\left(x^4+1\right)}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{8}+\frac{1}{8}\mathrm{i}\right) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{8}-\frac{1}{8}\mathrm{i}\right)$$

input
```
int(-(x^4 - 1)/(2*x^4 + x^8 + 1),x)
```

output
```
2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(1/8 + 1i/8) + 2^(1/2)*atan(2^(1/2)*x
*(1/2 + 1i/2))*(1/8 - 1i/8) + x/(2*(x^4 + 1))
```

3.23 $\int \frac{1-x^4}{1+x^4+x^8} \, dx$

3.23.1 Optimal result

Integrand size = 18, antiderivative size = 140

$$\int \frac{1-x^4}{1+x^4+x^8} \, dx = -\frac{1}{4}\sqrt{3}\arctan\left(\frac{1-2x}{\sqrt{3}}\right)+\frac{1}{4}\arctan\left(\sqrt{3}-2x\right)+\frac{1}{4}\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right) -\frac{1}{4}\arctan\left(\sqrt{3}+2x\right)+\frac{1}{8}\log\left(1-x+x^2\right)-\frac{1}{8}\log\left(1+x+x^2\right) -\frac{1}{8}\sqrt{3}\log\left(1-\sqrt{3}x+x^2\right)+\frac{1}{8}\sqrt{3}\log\left(1+\sqrt{3}x+x^2\right)$$

output
```
-1/4*arctan(2*x-3^(1/2))-1/4*arctan(2*x+3^(1/2))+1/8*ln(x^2-x+1)-1/8*ln(x^
2+x+1)-1/4*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1/4*arctan(1/3*(1+2*x)*3^(1
/2))*3^(1/2)-1/8*ln(1+x^2-x*3^(1/2))*3^(1/2)+1/8*ln(1+x^2+x*3^(1/2))*3^(1/
2)
```

3.23.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.92

$$\int \frac{1-x^4}{1+x^4+x^8} \, dx = \frac{1}{8}\left(-2\sqrt{-2-2i\sqrt{3}}\arctan\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x\right) -2\sqrt{-2+2i\sqrt{3}}\arctan\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x\right)+2\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right) +2\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right)+\log\left(1-x+x^2\right)-\log\left(1+x+x^2\right)\right)$$

input
```
Integrate[(1 - x^4)/(1 + x^4 + x^8),x]
```

output
```
(-2*Sqrt[-2 - (2*I)*Sqrt[3]]*ArcTan[((1 - I*Sqrt[3])*x)/2] - 2*Sqrt[-2 + (
2*I)*Sqrt[3]]*ArcTan[((1 + I*Sqrt[3])*x)/2] + 2*Sqrt[3]*ArcTan[(-1 + 2*x)/
Sqrt[3]] + 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + Log[1 - x + x^2] - Log[1
+ x + x^2])/8
```

3.23.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.17, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1751, 25, 1483, 27, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x^4}{x^8+x^4+1}dx$$

$$\Big\downarrow \text{1751}$$

$$-\frac{1}{2}\int -\frac{1-2x^2}{x^4-x^2+1}dx-\frac{1}{2}\int -\frac{2x^2+1}{x^4+x^2+1}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\int \frac{1-2x^2}{x^4-x^2+1}dx+\frac{1}{2}\int \frac{2x^2+1}{x^4+x^2+1}dx$$

$$\Big\downarrow \text{1483}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{x+1}{x^2-x+1}dx+\frac{1}{2}\int \frac{1-x}{x^2+x+1}dx\right)+\frac{1}{2}\left(\frac{\int \frac{\sqrt{3}-3x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}+\frac{\int \frac{\sqrt{3}\left(\sqrt{3}x+1\right)}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{x+1}{x^2-x+1}dx+\frac{1}{2}\int \frac{1-x}{x^2+x+1}dx\right)+\frac{1}{2}\left(\frac{\int \frac{\sqrt{3}-3x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}+\frac{1}{2}\int \frac{\sqrt{3}x+1}{x^2+\sqrt{3}x+1}dx\right)$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{3}{2}\int \frac{1}{x^2-x+1}dx+\frac{1}{2}\int -\frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{2}\left(\frac{3}{2}\int \frac{1}{x^2+x+1}dx-\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx\right)\right)+ \frac{1}{2}\left(\frac{-\frac{1}{2}\sqrt{3}\int \frac{1}{x^2-\sqrt{3}x+1}dx-\frac{3}{2}\int -\frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}+\frac{1}{2}\left(\frac{1}{2}\sqrt{3}\int \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx-\frac{1}{2}\int \frac{1}{x^2+\sqrt{3}x+1}dx\right)\right)$$

$$\Big\downarrow \; 25$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{3}{2}\int \frac{1}{x^2-x+1}dx-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{2}\left(\frac{3}{2}\int \frac{1}{x^2+x+1}dx-\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx\right)\right)+ \frac{1}{2}\left(\frac{\frac{3}{2}\int \frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx-\frac{1}{2}\sqrt{3}\int \frac{1}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}+\frac{1}{2}\left(\frac{1}{2}\sqrt{3}\int \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx-\frac{1}{2}\int \frac{1}{x^2+\sqrt{3}x+1}dx\right)\right)$$

$$\Big\downarrow \; 1083$$

$$\frac{1}{2}\Bigg(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx-3\int \frac{1}{-(2x-1)^2-3}d(2x-1)\right)+\frac{1}{2}\Bigg(-\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx-3\int \frac{1}{-(2x+1)^2-3}d(\quad \frac{1}{2}\Bigg(\frac{\frac{3}{2}\int \frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx+\sqrt{3}\int \frac{1}{-\left(2x-\sqrt{3}\right)^2-1}d\left(2x-\sqrt{3}\right)}{2\sqrt{3}}+\frac{1}{2}\Bigg(\frac{1}{2}\sqrt{3}\int \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx+\int \frac{1}{-\left(2x+\sqrt{3}\right)^2-1}d\Big(2x$$

$$\Big\downarrow \; 217$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\sqrt{3}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{2}\left(\sqrt{3}\arctan\left(\frac{2x+1}{\sqrt{3}}\right)-\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx\right)\right)+ \frac{1}{2}\left(\frac{\frac{3}{2}\int \frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx+\sqrt{3}\arctan\left(\sqrt{3}-2x\right)}{2\sqrt{3}}+\frac{1}{2}\left(\frac{1}{2}\sqrt{3}\int \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx-\arctan\left(2x+\sqrt{3}\right)\right)\right)$$

$$\Big\downarrow \; 1103$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\sqrt{3}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)+\frac{1}{2}\log\left(x^2-x+1\right)\right)+\frac{1}{2}\left(\sqrt{3}\arctan\left(\frac{2x+1}{\sqrt{3}}\right)-\frac{1}{2}\log\left(x^2+x+1\right)\right)\right)+ \frac{1}{2}\left(\frac{\sqrt{3}\arctan\left(\sqrt{3}-2x\right)-\frac{3}{2}\log\left(x^2-\sqrt{3}x+1\right)}{2\sqrt{3}}+\frac{1}{2}\left(\frac{1}{2}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)-\arctan\left(2x+\sqrt{3}\right)\right)\right)$$

input
```
Int[(1 - x^4)/(1 + x^4 + x^8),x]
```

3.23. $\int \frac{1-x^4}{1+x^4+x^8}\,dx$

output
```
((Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + Log[1 - x + x^2]/2)/2 + (Sqrt[3]*Ar
cTan[(1 + 2*x)/Sqrt[3]] - Log[1 + x + x^2]/2)/2)/2 + ((Sqrt[3]*ArcTan[Sqrt
[3] - 2*x] - (3*Log[1 - Sqrt[3]*x + x^2])/2)/(2*Sqrt[3]) + (-ArcTan[Sqrt[3
] + 2*x] + (Sqrt[3]*Log[1 + Sqrt[3]*x + x^2])/2)/2)/2
```

3.23.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 1751
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x
^(n/2))/Simp[d/e + q*x^(n/2) - x^n, x], x], x] + Simp[e/(2*c*q)   Int[(q +
2*x^(n/2))/Simp[d/e - q*x^(n/2) - x^n, x], x], x]] /; FreeQ[{a, b, c, d, e}
, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGt
Q[n/2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

3.23.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.62

method	result
risch	$-\frac{\ln\left(4x^2+4x+4\right)}{8}+\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{4}+\frac{\ln\left(4x^2-4x+4\right)}{8}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{4}+\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^4-\_Z^2+1\right)}\_R\text{l}\right.}{4}$
default	$\frac{\ln\left(x^2-x+1\right)}{8}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{4}-\frac{\ln\left(x^2+x+1\right)}{8}+\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{4}-\frac{\ln\left(1+x^2-x\sqrt{3}\right)\sqrt{3}}{8}-\frac{\arctan\left(2x-\sqrt{3}\right)}{4}$

input
```
int((-x^4+1)/(x^8+x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/8*ln(4*x^2+4*x+4)+1/4*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+1/8*ln(4*x^2-
4*x+4)+1/4*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))+1/4*sum(_R*ln(-_R^3+_R+x),_
R=RootOf(_Z^4-_Z^2+1))
```

3.23.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.39

$$\int \frac{1-x^4}{1+x^4+x^8}\,dx = -\frac{1}{8}\sqrt{2}\sqrt{\sqrt{-3}+1}\log\left(\sqrt{2}\sqrt{\sqrt{-3}+1}(\sqrt{-3}-1)+4x\right) + \frac{1}{8}\sqrt{2}\sqrt{\sqrt{-3}+1}\log\left(-\sqrt{2}\sqrt{\sqrt{-3}+1}(\sqrt{-3}-1)+4x\right) + \frac{1}{8}\sqrt{2}\sqrt{-\sqrt{-3}+1}\log\left(\sqrt{2}(\sqrt{-3}+1)\sqrt{-\sqrt{-3}+1}+4x\right) - \frac{1}{8}\sqrt{2}\sqrt{-\sqrt{-3}+1}\log\left(-\sqrt{2}(\sqrt{-3}+1)\sqrt{-\sqrt{-3}+1}+4x\right) + \frac{1}{4}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{1}{4}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{1}{8}\log\left(x^2+x+1\right) + \frac{1}{8}\log\left(x^2-x+1\right)$$

input

```
integrate((-x^4+1)/(x^8+x^4+1),x, algorithm="fricas")
```

output

```
-1/8*sqrt(2)*sqrt(sqrt(-3) + 1)*log(sqrt(2)*sqrt(sqrt(-3) + 1)*(sqrt(-3) -
 1) + 4*x) + 1/8*sqrt(2)*sqrt(sqrt(-3) + 1)*log(-sqrt(2)*sqrt(sqrt(-3) + 1
)*(sqrt(-3) - 1) + 4*x) + 1/8*sqrt(2)*sqrt(-sqrt(-3) + 1)*log(sqrt(2)*(sqr
t(-3) + 1)*sqrt(-sqrt(-3) + 1) + 4*x) - 1/8*sqrt(2)*sqrt(-sqrt(-3) + 1)*lo
g(-sqrt(2)*(sqrt(-3) + 1)*sqrt(-sqrt(-3) + 1) + 4*x) + 1/4*sqrt(3)*arctan(
1/3*sqrt(3)*(2*x + 1)) + 1/4*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/8*l
og(x^2 + x + 1) + 1/8*log(x^2 - x + 1)
```

3.23.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.06

$$\int \frac{1-x^4}{1+x^4+x^8}\,dx = -\left(-\frac{1}{8}-\frac{\sqrt{3}i}{8}\right)\log\left(x+1024\left(-\frac{1}{8}-\frac{\sqrt{3}i}{8}\right)^5\right) - \left(-\frac{1}{8}+\frac{\sqrt{3}i}{8}\right)\log\left(x+1024\left(-\frac{1}{8}+\frac{\sqrt{3}i}{8}\right)^5\right) - \left(\frac{1}{8}-\frac{\sqrt{3}i}{8}\right)\log\left(x+1024\left(\frac{1}{8}-\frac{\sqrt{3}i}{8}\right)^5\right) - \left(\frac{1}{8}+\frac{\sqrt{3}i}{8}\right)\log\left(x+1024\left(\frac{1}{8}+\frac{\sqrt{3}i}{8}\right)^5\right) - \operatorname{RootSum}\left(256t^4-16t^2+1,\left(t\mapsto t\log\left(1024t^5+x\right)\right)\right)$$

input
```
integrate((-x**4+1)/(x**8+x**4+1),x)
```

output
```
-(-1/8 - sqrt(3)*I/8)*log(x + 1024*(-1/8 - sqrt(3)*I/8)**5) - (-1/8 + sqrt
(3)*I/8)*log(x + 1024*(-1/8 + sqrt(3)*I/8)**5) - (1/8 - sqrt(3)*I/8)*log(x
 + 1024*(1/8 - sqrt(3)*I/8)**5) - (1/8 + sqrt(3)*I/8)*log(x + 1024*(1/8 +
sqrt(3)*I/8)**5) - RootSum(256*_t**4 - 16*_t**2 + 1, Lambda(_t, _t*log(102
4*_t**5 + x)))
```

3.23.7 Maxima [F]

$$\int \frac{1-x^4}{1+x^4+x^8}\,dx = \int -\frac{x^4-1}{x^8+x^4+1}\,dx$$

input
```
integrate((-x^4+1)/(x^8+x^4+1),x, algorithm="maxima")
```

output
```
1/4*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/4*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) - 1/2*integrate((2*x^2 - 1)/(x^4 - x^2 + 1), x) - 1/8*log(x^2
+ x + 1) + 1/8*log(x^2 - x + 1)
```

3.23.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.77

$$\int \frac{1-x^4}{1+x^4+x^8}\,dx = \frac{1}{4}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{4}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{8}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)-\frac{1}{8}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)-\frac{1}{4}\arctan\left(2x+\sqrt{3}\right)-\frac{1}{4}\arctan\left(2x-\sqrt{3}\right)-\frac{1}{8}\log\left(x^2+x+1\right)+\frac{1}{8}\log\left(x^2-x+1\right)$$

input
```
integrate((-x^4+1)/(x^8+x^4+1),x, algorithm="giac")
```

output
```
1/4*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/4*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) + 1/8*sqrt(3)*log(x^2 + sqrt(3)*x + 1) - 1/8*sqrt(3)*log(x^2 -
 sqrt(3)*x + 1) - 1/4*arctan(2*x + sqrt(3)) - 1/4*arctan(2*x - sqrt(3)) -
1/8*log(x^2 + x + 1) + 1/8*log(x^2 - x + 1)
```

3.23.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.78

$$\int \frac{1-x^4}{1+x^4+x^8}\,dx = -\mathrm{atan}\left(\frac{54\sqrt{3}\,x}{-81+\sqrt{3}\,27\mathrm{i}}\right)\left(\frac{\sqrt{3}}{4}+\frac{1}{4}\mathrm{i}\right)+\mathrm{atan}\left(\frac{54\sqrt{3}\,x}{81+\sqrt{3}\,27\mathrm{i}}\right)\left(\frac{\sqrt{3}}{4}-\frac{1}{4}\mathrm{i}\right)+\mathrm{atan}\left(\frac{\sqrt{3}\,x\,54\mathrm{i}}{-81+\sqrt{3}\,27\mathrm{i}}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)-\mathrm{atan}\left(\frac{\sqrt{3}\,x\,54\mathrm{i}}{81+\sqrt{3}\,27\mathrm{i}}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)$$

input
```
int(-(x^4 - 1)/(x^4 + x^8 + 1),x)
```

output
```
atan((54*3^(1/2)*x)/(3^(1/2)*27i + 81))*(3^(1/2)/4 - 1i/4) - atan((54*3^(1
/2)*x)/(3^(1/2)*27i - 81))*(3^(1/2)/4 + 1i/4) + atan((3^(1/2)*x*54i)/(3^(1
/2)*27i - 81))*((3^(1/2)*1i)/4 - 1/4) - atan((3^(1/2)*x*54i)/(3^(1/2)*27i
+ 81))*((3^(1/2)*1i)/4 + 1/4)
```

3.24 $\int \frac{1-x^4}{1+x^8}\,dx$

3.24.1 Optimal result

Integrand size = 15, antiderivative size = 347

$$\begin{aligned}\int \frac{1-x^4}{1+x^8}\,dx =& -\frac{\arctan\left(\frac{\sqrt{2-\sqrt{2}}-2x}{\sqrt{2+\sqrt{2}}}\right)}{4\sqrt{2-\sqrt{2}}}+\frac{\arctan\left(\frac{\sqrt{2+\sqrt{2}}-2x}{\sqrt{2-\sqrt{2}}}\right)}{4\sqrt{2+\sqrt{2}}}+\frac{\arctan\left(\frac{\sqrt{2-\sqrt{2}}+2x}{\sqrt{2+\sqrt{2}}}\right)}{4\sqrt{2-\sqrt{2}}}\\ &-\frac{\arctan\left(\frac{\sqrt{2+\sqrt{2}}+2x}{\sqrt{2-\sqrt{2}}}\right)}{4\sqrt{2+\sqrt{2}}}+\frac{1}{8}\sqrt{\frac{1}{2}\left(2-\sqrt{2}\right)}\log\left(1-\sqrt{2-\sqrt{2}}x+x^2\right)\\ &-\frac{1}{8}\sqrt{\frac{1}{2}\left(2-\sqrt{2}\right)}\log\left(1+\sqrt{2-\sqrt{2}}x+x^2\right)\\ &-\frac{1}{8}\sqrt{\frac{1}{2}\left(2+\sqrt{2}\right)}\log\left(1-\sqrt{2+\sqrt{2}}x+x^2\right)\\ &+\frac{1}{8}\sqrt{\frac{1}{2}\left(2+\sqrt{2}\right)}\log\left(1+\sqrt{2+\sqrt{2}}x+x^2\right)\end{aligned}$$

output
```
1/16*ln(1+x^2-x*(2-2^(1/2))^(1/2))*(4-2*2^(1/2))^(1/2)-1/16*ln(1+x^2+x*(2-
2^(1/2))^(1/2))*(4-2*2^(1/2))^(1/2)-1/4*arctan((-2*x+(2-2^(1/2))^(1/2))/(2
+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2)+1/4*arctan((2*x+(2-2^(1/2))^(1/2))/(2+2
^(1/2))^(1/2))/(2-2^(1/2))^(1/2)-1/16*ln(1+x^2-x*(2+2^(1/2))^(1/2))*(4+2*2
^(1/2))^(1/2)+1/16*ln(1+x^2+x*(2+2^(1/2))^(1/2))*(4+2*2^(1/2))^(1/2)+1/4*a
rctan((-2*x+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)-1/4*ar
ctan((2*x+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)
```

3.24.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.74

$$\int \frac{1-x^4}{1+x^8} dx = \frac{1}{8}\left(2 \arctan\left(\cot\left(\frac{\pi}{8}\right) - x\csc\left(\frac{\pi}{8}\right)\right)\left(\cos\left(\frac{\pi}{8}\right) - \sin\left(\frac{\pi}{8}\right)\right) + \log\left(1 + x^2 - 2x\sin\left(\frac{\pi}{8}\right)\right)\left(\cos\left(\frac{\pi}{8}\right) - \sin\left(\frac{\pi}{8}\right)\right) + 2\arctan\left(\left(x + \cos\left(\frac{\pi}{8}\right)\right)\csc\left(\frac{\pi}{8}\right)\right)\left(-\cos\left(\frac{\pi}{8}\right) + \sin\left(\frac{\pi}{8}\right)\right) + \log\left(1 + x^2 + 2x\sin\left(\frac{\pi}{8}\right)\right)\left(-\cos\left(\frac{\pi}{8}\right) + \sin\left(\frac{\pi}{8}\right)\right) + 2\arctan\left(\sec\left(\frac{\pi}{8}\right)\left(x + \sin\left(\frac{\pi}{8}\right)\right)\right)\left(\cos\left(\frac{\pi}{8}\right) + \sin\left(\frac{\pi}{8}\right)\right) + 2\arctan\left(x\sec\left(\frac{\pi}{8}\right) - \tan\left(\frac{\pi}{8}\right)\right)\left(\cos\left(\frac{\pi}{8}\right) + \sin\left(\frac{\pi}{8}\right)\right) - \log\left(1 + x^2 - 2x\cos\left(\frac{\pi}{8}\right)\right)\left(\cos\left(\frac{\pi}{8}\right) + \sin\left(\frac{\pi}{8}\right)\right) + \log\left(1 + x^2 + 2x\cos\left(\frac{\pi}{8}\right)\right)\left(\cos\left(\frac{\pi}{8}\right) + \sin\left(\frac{\pi}{8}\right)\right)\right)$$

input
```
Integrate[(1 - x^4)/(1 + x^8),x]
```

output
```
(2*ArcTan[Cot[Pi/8] - x*Csc[Pi/8]]*(Cos[Pi/8] - Sin[Pi/8]) + Log[1 + x^2 -
 2*x*Sin[Pi/8]]*(Cos[Pi/8] - Sin[Pi/8]) + 2*ArcTan[(x + Cos[Pi/8])*Csc[Pi/
8]]*(-Cos[Pi/8] + Sin[Pi/8]) + Log[1 + x^2 + 2*x*Sin[Pi/8]]*(-Cos[Pi/8] +
Sin[Pi/8]) + 2*ArcTan[Sec[Pi/8]*(x + Sin[Pi/8])]*(Cos[Pi/8] + Sin[Pi/8]) +
 2*ArcTan[x*Sec[Pi/8] - Tan[Pi/8]]*(Cos[Pi/8] + Sin[Pi/8]) - Log[1 + x^2 -
 2*x*Cos[Pi/8]]*(Cos[Pi/8] + Sin[Pi/8]) + Log[1 + x^2 + 2*x*Cos[Pi/8]]*(Co
s[Pi/8] + Sin[Pi/8]))/8
```

3.24.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 337, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {1744, 1483, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x^4}{x^8+1} dx$$

$$\Big\downarrow \; 1744$$

$$\frac{1}{2}\int \frac{1-\sqrt{2}x^2}{x^4-\sqrt{2}x^2+1}dx+\frac{1}{2}\int \frac{\sqrt{2}x^2+1}{x^4+\sqrt{2}x^2+1}dx$$

$$\Big\downarrow \text{ 1483}$$

$$\frac{1}{2}\left(\frac{\int \frac{\sqrt{2-\sqrt{2}}-\left(1-\sqrt{2}\right)x}{x^2-\sqrt{2-\sqrt{2}}x+1}dx}{2\sqrt{2-\sqrt{2}}}+\frac{\int \frac{\left(1-\sqrt{2}\right)x+\sqrt{2-\sqrt{2}}}{x^2+\sqrt{2-\sqrt{2}}x+1}dx}{2\sqrt{2-\sqrt{2}}}\right)+\frac{1}{2}\left(\frac{\int \frac{\sqrt{2+\sqrt{2}}-\left(1+\sqrt{2}\right)x}{x^2-\sqrt{2+\sqrt{2}}x+1}dx}{2\sqrt{2+\sqrt{2}}}+\frac{\int \frac{\left(1+\sqrt{2}\right)x+\sqrt{2+\sqrt{2}}}{x^2+\sqrt{2+\sqrt{2}}x+1}dx}{2\sqrt{2+\sqrt{2}}}\right)$$

$$\Big\downarrow \text{ 1142}$$

$$\frac{1}{2}\left(\frac{\frac{1}{2}\sqrt{2+\sqrt{2}}\int \frac{1}{x^2-\sqrt{2-\sqrt{2}}x+1}dx-\frac{1}{2}\left(1-\sqrt{2}\right)\int -\frac{\sqrt{2-\sqrt{2}}-2x}{x^2-\sqrt{2-\sqrt{2}}x+1}dx}{2\sqrt{2-\sqrt{2}}}+\frac{\frac{1}{2}\sqrt{2+\sqrt{2}}\int \frac{1}{x^2+\sqrt{2-\sqrt{2}}x+1}dx+\frac{1}{2}\left(1-\sqrt{2}\right.}{2\sqrt{2-\sqrt{2}}}\right.$$
$$\frac{1}{2}\left(\frac{-\frac{1}{2}\sqrt{2-\sqrt{2}}\int \frac{1}{x^2-\sqrt{2+\sqrt{2}}x+1}dx-\frac{1}{2}\left(1+\sqrt{2}\right)\int -\frac{\sqrt{2+\sqrt{2}}-2x}{x^2-\sqrt{2+\sqrt{2}}x+1}dx}{2\sqrt{2+\sqrt{2}}}+\frac{\frac{1}{2}\left(1+\sqrt{2}\right)\int \frac{2x+\sqrt{2+\sqrt{2}}}{x^2+\sqrt{2+\sqrt{2}}x+1}dx-\frac{1}{2}\sqrt{2-}}{2\sqrt{2+\sqrt{2}}}\right.$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{2}\left(\frac{\frac{1}{2}\sqrt{2+\sqrt{2}}\int \frac{1}{x^2-\sqrt{2-\sqrt{2}}x+1}dx+\frac{1}{2}\left(1-\sqrt{2}\right)\int \frac{\sqrt{2-\sqrt{2}}-2x}{x^2-\sqrt{2-\sqrt{2}}x+1}dx}{2\sqrt{2-\sqrt{2}}}+\frac{\frac{1}{2}\sqrt{2+\sqrt{2}}\int \frac{1}{x^2+\sqrt{2-\sqrt{2}}x+1}dx+\frac{1}{2}\left(1-\sqrt{2}\right)}{2\sqrt{2-\sqrt{2}}}\right.$$
$$\frac{1}{2}\left(\frac{\frac{1}{2}\left(1+\sqrt{2}\right)\int \frac{\sqrt{2+\sqrt{2}}-2x}{x^2-\sqrt{2+\sqrt{2}}x+1}dx-\frac{1}{2}\sqrt{2-\sqrt{2}}\int \frac{1}{x^2-\sqrt{2+\sqrt{2}}x+1}dx}{2\sqrt{2+\sqrt{2}}}+\frac{\frac{1}{2}\left(1+\sqrt{2}\right)\int \frac{2x+\sqrt{2+\sqrt{2}}}{x^2+\sqrt{2+\sqrt{2}}x+1}dx-\frac{1}{2}\sqrt{2-\sqrt{2}}}{2\sqrt{2+\sqrt{2}}}\right.$$

$$\Big\downarrow \text{ 1083}$$

$$\frac{1}{2}\left(\frac{\frac{1}{2}\left(1-\sqrt{2}\right)\int \frac{\sqrt{2-\sqrt{2}}-2x}{x^2-\sqrt{2-\sqrt{2}}x+1}dx-\sqrt{2+\sqrt{2}}\int \frac{1}{-\left(2x-\sqrt{2-\sqrt{2}}\right)^2-\sqrt{2}-2}d\left(2x-\sqrt{2-\sqrt{2}}\right)}{2\sqrt{2-\sqrt{2}}}+\frac{\frac{1}{2}\left(1-\sqrt{2}\right)\int \frac{2x+\sqrt{2-}}{x^2+\sqrt{2-}}}{}\right.$$
$$\frac{1}{2}\left(\frac{\frac{1}{2}\left(1+\sqrt{2}\right)\int \frac{\sqrt{2+\sqrt{2}}-2x}{x^2-\sqrt{2+\sqrt{2}}x+1}dx+\sqrt{2-\sqrt{2}}\int \frac{1}{-\left(2x-\sqrt{2+\sqrt{2}}\right)^2+\sqrt{2}-2}d\left(2x-\sqrt{2+\sqrt{2}}\right)}{2\sqrt{2+\sqrt{2}}}+\frac{\frac{1}{2}\left(1+\sqrt{2}\right)\int \frac{2x+\sqrt{2-}}{x^2+\sqrt{2+}}}{}\right.$$

$$\Big\downarrow \text{ 217}$$

3.24. $\int \frac{1-x^4}{1+x^8}\, dx$

$$\frac{1}{2}\left(\frac{\frac{1}{2}(1-\sqrt{2})\int \frac{\sqrt{2-\sqrt{2}}-2x}{x^2-\sqrt{2-\sqrt{2}}x+1}dx+\arctan\left(\frac{2x-\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2-\sqrt{2}}}+\frac{\frac{1}{2}(1-\sqrt{2})\int \frac{2x+\sqrt{2-\sqrt{2}}}{x^2+\sqrt{2-\sqrt{2}}x+1}dx+\arctan\left(\frac{2x+\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2-\sqrt{2}}}\right.$$
$$\frac{1}{2}\left(\frac{\frac{1}{2}(1+\sqrt{2})\int \frac{\sqrt{2+\sqrt{2}}-2x}{x^2-\sqrt{2+\sqrt{2}}x+1}dx-\arctan\left(\frac{2x-\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2+\sqrt{2}}}+\frac{\frac{1}{2}(1+\sqrt{2})\int \frac{2x+\sqrt{2+\sqrt{2}}}{x^2+\sqrt{2+\sqrt{2}}x+1}dx-\arctan\left(\frac{2x+\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2+\sqrt{2}}}\right.$$

$\downarrow$ 1103

$$\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)-\frac{1}{2}(1-\sqrt{2})\log\left(x^2-\sqrt{2-\sqrt{2}}x+1\right)}{2\sqrt{2-\sqrt{2}}}+\frac{\arctan\left(\frac{2x+\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)+\frac{1}{2}(1-\sqrt{2})\log\left(x^2+\right.}{2\sqrt{2-\sqrt{2}}}\right.$$
$$\frac{1}{2}\left(\frac{-\arctan\left(\frac{2x-\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)-\frac{1}{2}(1+\sqrt{2})\log\left(x^2-\sqrt{2+\sqrt{2}}x+1\right)}{2\sqrt{2+\sqrt{2}}}+\frac{\frac{1}{2}(1+\sqrt{2})\log\left(x^2+\sqrt{2+\sqrt{2}}x+1\right)-\text{arc}}{2\sqrt{2+\sqrt{2}}}\right.$$

input
```
Int[(1 - x^4)/(1 + x^8),x]
```

output
```
((ArcTan[(-Sqrt[2 - Sqrt[2]] + 2*x)/Sqrt[2 + Sqrt[2]]] - ((1 - Sqrt[2])*Lo
g[1 - Sqrt[2 - Sqrt[2]]*x + x^2])/2)/(2*Sqrt[2 - Sqrt[2]]) + (ArcTan[(Sqrt
[2 - Sqrt[2]] + 2*x)/Sqrt[2 + Sqrt[2]]] + ((1 - Sqrt[2])*Log[1 + Sqrt[2 -
Sqrt[2]]*x + x^2])/2)/(2*Sqrt[2 - Sqrt[2]]))/2 + ((-ArcTan[(-Sqrt[2 + Sqrt
[2]] + 2*x)/Sqrt[2 - Sqrt[2]]] - ((1 + Sqrt[2])*Log[1 - Sqrt[2 + Sqrt[2]]*
x + x^2])/2)/(2*Sqrt[2 + Sqrt[2]]) + (-ArcTan[(Sqrt[2 + Sqrt[2]] + 2*x)/Sq
rt[2 - Sqrt[2]]] + ((1 + Sqrt[2])*Log[1 + Sqrt[2 + Sqrt[2]]*x + x^2])/2)/(
2*Sqrt[2 + Sqrt[2]]))/2
```

3.24.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 1744
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> With[{
q = Rt[-2*d*e, 2]}, Simp[d/(2*a)   Int[(d - q*x^(n/2))/(d - q*x^(n/2) - e*x
^n), x], x] + Simp[d/(2*a)   Int[(d + q*x^(n/2))/(d + q*x^(n/2) - e*x^n), x
], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 - a*e^2, 0] &
& IGtQ[n/2, 0] && NegQ[d*e]
```

3.24.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.59 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.08

method	result	size
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}+1\right)}\frac{\left(-\_R^{4}+1\right)\ln\left(x-\_R\right)}{\_R^{7}}}{8}$	29
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}+1\right)}\frac{\left(-\_R^{4}+1\right)\ln\left(x-\_R\right)}{\_R^{7}}}{8}$	29
meijerg	Expression too large to display	566

input
```
int((-x^4+1)/(x^8+1),x,method=_RETURNVERBOSE)
```

output
```
1/8*sum((-_R^4+1)/_R^7*ln(x-_R),_R=RootOf(_Z^8+1))
```

3.24.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.56

$$\begin{aligned}
\int \frac{1-x^4}{1+x^8}\,dx = & -\frac{1}{8}\sqrt{2}(-1)^{\frac{1}{8}}\log\left(8\sqrt{2}\left((-1)^{\frac{5}{8}}-(-1)^{\frac{1}{8}}\right)+16\,x\right) \\
& +\frac{1}{8}\sqrt{2}(-1)^{\frac{1}{8}}\log\left(-8\sqrt{2}\left((-1)^{\frac{5}{8}}-(-1)^{\frac{1}{8}}\right)+16\,x\right) \\
& +\frac{1}{8}i\sqrt{2}(-1)^{\frac{1}{8}}\log\left(-8\sqrt{2}\left(i\,(-1)^{\frac{5}{8}}-i\,(-1)^{\frac{1}{8}}\right)+16\,x\right) \\
& -\frac{1}{8}i\sqrt{2}(-1)^{\frac{1}{8}}\log\left(-8\sqrt{2}\left(-i\,(-1)^{\frac{5}{8}}+i\,(-1)^{\frac{1}{8}}\right)+16\,x\right) \\
& +\left(\frac{1}{8}i+\frac{1}{8}\right)(-1)^{\frac{1}{8}}\log\left(32\,x+(16i+16)\,(-1)^{\frac{5}{8}}+(16i+16)\,(-1)^{\frac{1}{8}}\right) \\
& -\left(\frac{1}{8}i-\frac{1}{8}\right)(-1)^{\frac{1}{8}}\log\left(32\,x-(16i-16)\,(-1)^{\frac{5}{8}}-(16i-16)\,(-1)^{\frac{1}{8}}\right) \\
& +\left(\frac{1}{8}i-\frac{1}{8}\right)(-1)^{\frac{1}{8}}\log\left(32\,x+(16i-16)\,(-1)^{\frac{5}{8}}+(16i-16)\,(-1)^{\frac{1}{8}}\right) \\
& -\left(\frac{1}{8}i+\frac{1}{8}\right)(-1)^{\frac{1}{8}}\log\left(32\,x-(16i+16)\,(-1)^{\frac{5}{8}}-(16i+16)\,(-1)^{\frac{1}{8}}\right)
\end{aligned}$$

input
```
integrate((-x^4+1)/(x^8+1),x, algorithm="fricas")
```

output
```
-1/8*sqrt(2)*(-1)^(1/8)*log(8*sqrt(2)*((-1)^(5/8) - (-1)^(1/8)) + 16*x) +
1/8*sqrt(2)*(-1)^(1/8)*log(-8*sqrt(2)*((-1)^(5/8) - (-1)^(1/8)) + 16*x) +
1/8*I*sqrt(2)*(-1)^(1/8)*log(-8*sqrt(2)*(I*(-1)^(5/8) - I*(-1)^(1/8)) + 16
*x) - 1/8*I*sqrt(2)*(-1)^(1/8)*log(-8*sqrt(2)*(-I*(-1)^(5/8) + I*(-1)^(1/8
)) + 16*x) + (1/8*I + 1/8)*(-1)^(1/8)*log(32*x + (16*I + 16)*(-1)^(5/8) +
(16*I + 16)*(-1)^(1/8)) - (1/8*I - 1/8)*(-1)^(1/8)*log(32*x - (16*I - 16)*
(-1)^(5/8) - (16*I - 16)*(-1)^(1/8)) + (1/8*I - 1/8)*(-1)^(1/8)*log(32*x +
 (16*I - 16)*(-1)^(5/8) + (16*I - 16)*(-1)^(1/8)) - (1/8*I + 1/8)*(-1)^(1/
8)*log(32*x - (16*I + 16)*(-1)^(5/8) - (16*I + 16)*(-1)^(1/8))
```

3.24.6 Sympy [A] (verification not implemented)

Time = 1.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.06

$$\int \frac{1-x^4}{1+x^8}\,dx = -\operatorname{RootSum}\left(1048576t^8+1,\left(t\mapsto t\log\left(4096t^5-4t+x\right)\right)\right)$$

input
```
integrate((-x**4+1)/(x**8+1),x)
```

output
```
-RootSum(1048576*_t**8 + 1, Lambda(_t, _t*log(4096*_t**5 - 4*_t + x)))
```

3.24.7 Maxima [F]

$$\int \frac{1-x^4}{1+x^8}\,dx = \int -\frac{x^4-1}{x^8+1}\,dx$$

input
```
integrate((-x^4+1)/(x^8+1),x, algorithm="maxima")
```

output
```
-integrate((x^4 - 1)/(x^8 + 1), x)
```

3.24.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.71

$$\int \frac{1-x^4}{1+x^8}\,dx = \frac{1}{8}\sqrt{2\sqrt{2}+4}\arctan\left(\frac{2x+\sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}}\right) + \frac{1}{8}\sqrt{2\sqrt{2}+4}\arctan\left(\frac{2x-\sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}}\right) - \frac{1}{8}\sqrt{-2\sqrt{2}+4}\arctan\left(\frac{2x+\sqrt{\sqrt{2}+2}}{\sqrt{-\sqrt{2}+2}}\right) - \frac{1}{8}\sqrt{-2\sqrt{2}+4}\arctan\left(\frac{2x-\sqrt{\sqrt{2}+2}}{\sqrt{-\sqrt{2}+2}}\right) + \frac{1}{16}\sqrt{2\sqrt{2}+4}\log\left(x^2+x\sqrt{\sqrt{2}+2}+1\right) - \frac{1}{16}\sqrt{2\sqrt{2}+4}\log\left(x^2-x\sqrt{\sqrt{2}+2}+1\right) - \frac{1}{16}\sqrt{-2\sqrt{2}+4}\log\left(x^2+x\sqrt{-\sqrt{2}+2}+1\right) + \frac{1}{16}\sqrt{-2\sqrt{2}+4}\log\left(x^2-x\sqrt{-\sqrt{2}+2}+1\right)$$

input
```
integrate((-x^4+1)/(x^8+1),x, algorithm="giac")
```

output
```
1/8*sqrt(2*sqrt(2) + 4)*arctan((2*x + sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2
)) + 1/8*sqrt(2*sqrt(2) + 4)*arctan((2*x - sqrt(-sqrt(2) + 2))/sqrt(sqrt(2
) + 2)) - 1/8*sqrt(-2*sqrt(2) + 4)*arctan((2*x + sqrt(sqrt(2) + 2))/sqrt(-
sqrt(2) + 2)) - 1/8*sqrt(-2*sqrt(2) + 4)*arctan((2*x - sqrt(sqrt(2) + 2))/
sqrt(-sqrt(2) + 2)) + 1/16*sqrt(2*sqrt(2) + 4)*log(x^2 + x*sqrt(sqrt(2) +
2) + 1) - 1/16*sqrt(2*sqrt(2) + 4)*log(x^2 - x*sqrt(sqrt(2) + 2) + 1) - 1/
16*sqrt(-2*sqrt(2) + 4)*log(x^2 + x*sqrt(-sqrt(2) + 2) + 1) + 1/16*sqrt(-2
*sqrt(2) + 4)*log(x^2 - x*sqrt(-sqrt(2) + 2) + 1)
```

3.24.9 Mupad [B] (verification not implemented)

Time = 8.55 (sec) , antiderivative size = 312, normalized size of antiderivative = 0.90

$$\int \frac{1-x^4}{1+x^8}\,dx = -\ln\left(\left(\left(\frac{\sqrt{-2\sqrt{2}-4}}{16}-\frac{\sqrt{4-2\sqrt{2}}}{16}\right)^3\left(65536\,x-16384\sqrt{-2\sqrt{2}-4}+16384\sqrt{4-2\sqrt{2}}\right)-256\right)\left(\frac{\sqrt{-2\sqrt{2}-4}}{16}-\frac{\sqrt{4-2\sqrt{2}}}{16}\right)\right)-\operatorname{atan}\left(-\frac{x\,1\mathrm{i}}{\sqrt{\sqrt{2}-2}}+\frac{x\,1\mathrm{i}}{\sqrt{\sqrt{2}+2}}+\frac{\sqrt{2}\,x\,1\mathrm{i}}{2\sqrt{\sqrt{2}-2}}+\frac{\sqrt{2}\,x\,1\mathrm{i}}{2\sqrt{\sqrt{2}+2}}\right)\left(\frac{\sqrt{2}\sqrt{\sqrt{2}-2}\,1\mathrm{i}}{8}+\frac{\sqrt{2}\sqrt{\sqrt{2}+2}\,1\mathrm{i}}{8}\right)$$
$$+\frac{\operatorname{atan}\left(x\left(\sqrt{2}+2\right)^{3/2}\left(\frac{1}{2}+1\mathrm{i}\right)+\sqrt{2}\,x\left(\sqrt{2}+2\right)^{3/2}\left(-\frac{1}{4}-\frac{3}{4}\mathrm{i}\right)\right)\left(-2+\sqrt{2}\,(1-\mathrm{i})\right)\sqrt{\sqrt{2}+2}\,1\mathrm{i}}{8}$$
$$+\frac{\operatorname{atan}\left(x\left(\sqrt{2}+2\right)^{3/2}\left(1-\frac{1}{2}\mathrm{i}\right)+\sqrt{2}\,x\left(\sqrt{2}+2\right)^{3/2}\left(-\frac{3}{4}+\frac{1}{4}\mathrm{i}\right)\right)\left(\sqrt{2}\,(1+1\mathrm{i})-2\mathrm{i}\right)\sqrt{\sqrt{2}+2}\,1\mathrm{i}}{8}$$
$$+\sqrt{2}\,\ln\left(x+\left(\sqrt{2}+2\right)^{3/2}\left(-1+\frac{1}{2}\mathrm{i}\right)+\sqrt{2}\left(\sqrt{2}+2\right)^{3/2}\left(\frac{3}{4}-\frac{1}{4}\mathrm{i}\right)\right)\left(\frac{\sqrt{\sqrt{2}-2}}{16}+\frac{\sqrt{\sqrt{2}+2}}{16}\right)1\mathrm{i}$$

input
```
int(-(x^4 - 1)/(x^8 + 1),x)
```

output
```
(atan(x*(2^(1/2) + 2)^(3/2)*(1/2 + 1i) - 2^(1/2)*x*(2^(1/2) + 2)^(3/2)*(1/
4 + 3i/4))*(2^(1/2)*(1 - 1i) - 2)*(2^(1/2) + 2)^(1/2)*1i)/8 - atan((x*1i)/
(2^(1/2) + 2)^(1/2) - (x*1i)/(2^(1/2) - 2)^(1/2) + (2^(1/2)*x*1i)/(2*(2^(1
/2) - 2)^(1/2)) + (2^(1/2)*x*1i)/(2*(2^(1/2) + 2)^(1/2)))*((2^(1/2)*(2^(1/
2) - 2)^(1/2)*1i)/8 + (2^(1/2)*(2^(1/2) + 2)^(1/2)*1i)/8) - log(((- 2*2^(1
/2) - 4)^(1/2)/16 - (4 - 2*2^(1/2))^(1/2)/16)^3*(65536*x - 16384*(- 2*2^(1
/2) - 4)^(1/2) + 16384*(4 - 2*2^(1/2))^(1/2)) - 256)*((- 2*2^(1/2) - 4)^(1
/2)/16 - (4 - 2*2^(1/2))^(1/2)/16) + (atan(x*(2^(1/2) + 2)^(3/2)*(1 - 1i/2
) - 2^(1/2)*x*(2^(1/2) + 2)^(3/2)*(3/4 - 1i/4))*(2^(1/2)*(1 + 1i) - 2i)*(2
^(1/2) + 2)^(1/2)*1i)/8 + 2^(1/2)*log(x - (2^(1/2) + 2)^(3/2)*(1 - 1i/2) +
 2^(1/2)*(2^(1/2) + 2)^(3/2)*(3/4 - 1i/4))*((2^(1/2) - 2)^(1/2)/16 + (2^(1
/2) + 2)^(1/2)/16)*1i
```

3.25 $\int \frac{1-x^4}{1-x^4+x^8}\,dx$

3.25.1 Optimal result

Integrand size = 20, antiderivative size = 355

$$\int \frac{1-x^4}{1-x^4+x^8}\,dx = -\frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}+\frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}+\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)+\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(1+\sqrt{2+\sqrt{3}}x+x^2\right)$$

output
```
1/8*ln(1+x^2-x*(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))-1/8*ln
(1+x^2+x*(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))-1/4*arctan((
-2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))/(3/2*2^(1/2)-1/2*
6^(1/2))+1/4*arctan((2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)
))/(3/2*2^(1/2)-1/2*6^(1/2))-1/8*ln(1+x^2-x*(1/2*6^(1/2)+1/2*2^(1/2)))*(1/
2*2^(1/2)+1/6*6^(1/2))+1/8*ln(1+x^2+x*(1/2*6^(1/2)+1/2*2^(1/2)))*(1/2*2^(1
/2)+1/6*6^(1/2))+1/4*arctan((-2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/
2*2^(1/2)))/(3/2*2^(1/2)+1/2*6^(1/2))-1/4*arctan((2*x+1/2*6^(1/2)+1/2*2^(1
/2))/(1/2*6^(1/2)-1/2*2^(1/2)))/(3/2*2^(1/2)+1/2*6^(1/2))
```

3.25.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.16

$$\int \frac{1-x^4}{1-x^4+x^8}\,dx = -\frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{-\log(x-\#1)+\log(x-\#1)\#1^4}{-\#1^3+2\#1^7}\&\right]$$

input
```
Integrate[(1 - x^4)/(1 - x^4 + x^8),x]
```

output
```
-1/4*RootSum[1 - #1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*#1^4)/(-#1^3
+ 2*#1^7) & ]
```

3.25.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 347, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1751, 25, 1483, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x^4}{x^8-x^4+1}\,dx$$

$$\Big\downarrow \text{1751}$$

$$-\frac{\int -\frac{\sqrt{3}-2x^2}{x^4-\sqrt{3}x^2+1}dx}{2\sqrt{3}}-\frac{\int -\frac{2x^2+\sqrt{3}}{x^4+\sqrt{3}x^2+1}dx}{2\sqrt{3}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\sqrt{3}-2x^2}{x^4-\sqrt{3}x^2+1}dx}{2\sqrt{3}}+\frac{\int \frac{2x^2+\sqrt{3}}{x^4+\sqrt{3}x^2+1}dx}{2\sqrt{3}}$$

$$\Big\downarrow \text{1483}$$

$$\frac{\frac{\int \frac{\left(2-\sqrt{3}\right)x+\sqrt{3\left(2-\sqrt{3}\right)}}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\int \frac{\sqrt{3\left(2-\sqrt{3}\right)}-\left(2-\sqrt{3}\right)x}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}+\frac{\frac{\int \frac{\sqrt{3\left(2+\sqrt{3}\right)}-\left(2+\sqrt{3}\right)x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{\int \frac{\left(2+\sqrt{3}\right)x+\sqrt{3\left(2+\sqrt{3}\right)}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}$$

$$\Big\downarrow \text{1142}$$

$$\frac{\frac{\frac{1}{2}\sqrt{2+\sqrt{3}}\int \frac{1}{x^2-\sqrt{2-\sqrt{3}}x+1}dx+\frac{1}{2}\left(2-\sqrt{3}\right)\int -\frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\frac{1}{2}\sqrt{2+\sqrt{3}}\int \frac{1}{x^2+\sqrt{2-\sqrt{3}}x+1}dx-\frac{1}{2}\left(2-\sqrt{3}\right)\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}+\frac{\frac{-\frac{1}{2}\sqrt{2-\sqrt{3}}\int \frac{1}{x^2-\sqrt{2+\sqrt{3}}x+1}dx-\frac{1}{2}\left(2+\sqrt{3}\right)\int -\frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\left(2+\sqrt{3}\right)\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx-\frac{1}{2}\sqrt{2-\sqrt{3}}\int \frac{1}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}$$

$\Big\downarrow$ 25

$$\frac{\frac{\frac{1}{2}\sqrt{2+\sqrt{3}}\int \frac{1}{x^2-\sqrt{2-\sqrt{3}}x+1}dx-\frac{1}{2}\left(2-\sqrt{3}\right)\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\frac{1}{2}\sqrt{2+\sqrt{3}}\int \frac{1}{x^2+\sqrt{2-\sqrt{3}}x+1}dx-\frac{1}{2}\left(2-\sqrt{3}\right)\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}+\frac{\frac{\frac{1}{2}\left(2+\sqrt{3}\right)\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx-\frac{1}{2}\sqrt{2-\sqrt{3}}\int \frac{1}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\left(2+\sqrt{3}\right)\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx-\frac{1}{2}\sqrt{2-\sqrt{3}}\int \frac{1}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}$$

$\Big\downarrow$ 1083

$$\frac{\frac{-\frac{1}{2}\left(2-\sqrt{3}\right)\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx-\sqrt{2+\sqrt{3}}\int \frac{1}{-\left(2x-\sqrt{2-\sqrt{3}}\right)^2-\sqrt{3}-2}d\left(2x-\sqrt{2-\sqrt{3}}\right)}{2\sqrt{2-\sqrt{3}}}+\frac{-\frac{1}{2}\left(2-\sqrt{3}\right)\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx-\sqrt{2+\sqrt{3}}\int \frac{\cdots}{-\left(2x+\sqrt{2\cdots}\right.}}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}$$

$$\frac{\frac{\frac{1}{2}\left(2+\sqrt{3}\right)\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx+\sqrt{2-\sqrt{3}}\int \frac{1}{-\left(2x-\sqrt{2+\sqrt{3}}\right)^2+\sqrt{3}-2}d\left(2x-\sqrt{2+\sqrt{3}}\right)}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\left(2+\sqrt{3}\right)\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx+\sqrt{2-\sqrt{3}}\int \frac{1}{-\left(2x+\sqrt{2+\sqrt{3}}\cdots\right.}}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}$$

$\Big\downarrow$ 217

$$\frac{\frac{\arctan\left(\frac{2x-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{2}\left(2-\sqrt{3}\right)\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{2}\left(2-\sqrt{3}\right)\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}+\frac{\frac{\frac{1}{2}\left(2+\sqrt{3}\right)\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx-\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\left(2+\sqrt{3}\right)\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx-\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}$$

$\Big\downarrow$ 1103

$$\frac{\frac{\arctan\left(\frac{2x-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)+\frac{1}{2}\left(2-\sqrt{3}\right)\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{2\sqrt{2-\sqrt{3}}}+\frac{\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{2}\left(2-\sqrt{3}\right)\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}+\frac{\frac{-\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)-\frac{1}{2}\left(2+\sqrt{3}\right)\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\left(2+\sqrt{3}\right)\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)-\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}$$

3.25. $\int \frac{1-x^4}{1-x^4+x^8}\,dx$

input
```
Int[(1 - x^4)/(1 - x^4 + x^8),x]
```

output
```
((ArcTan[(-Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]] + ((2 - Sqrt[3])*Lo
g[1 - Sqrt[2 - Sqrt[3]]*x + x^2])/2)/(2*Sqrt[2 - Sqrt[3]]) + (ArcTan[(Sqrt
[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]] - ((2 - Sqrt[3])*Log[1 + Sqrt[2 -
Sqrt[3]]*x + x^2])/2)/(2*Sqrt[2 - Sqrt[3]]))/(2*Sqrt[3]) + ((-ArcTan[(-Sqr
t[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]] - ((2 + Sqrt[3])*Log[1 - Sqrt[2 +
 Sqrt[3]]*x + x^2])/2)/(2*Sqrt[2 + Sqrt[3]]) + (-ArcTan[(Sqrt[2 + Sqrt[3]]
 + 2*x)/Sqrt[2 - Sqrt[3]]] + ((2 + Sqrt[3])*Log[1 + Sqrt[2 + Sqrt[3]]*x +
x^2])/2)/(2*Sqrt[2 + Sqrt[3]]))/(2*Sqrt[3])
```

3.25.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 1751
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x
^(n/2))/Simp[d/e + q*x^(n/2) - x^n, x], x], x] + Simp[e/(2*c*q)   Int[(q +
2*x^(n/2))/Simp[d/e - q*x^(n/2) - x^n, x], x], x]] /; FreeQ[{a, b, c, d, e}
, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGt
Q[n/2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

3.25.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.12

method	result	size
default	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{8}-\_Z^{4}+1\right)}\frac{\left(-\_R^{4}+1\right)\ln\left(x-\_R\right)}{2\_R^{7}-\_R^{3}}\right)}{4}$	44
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{8}-\_Z^{4}+1\right)}\frac{\left(-\_R^{4}+1\right)\ln\left(x-\_R\right)}{2\_R^{7}-\_R^{3}}\right)}{4}$	44

input
```
int((-x^4+1)/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum((-_R^4+1)/(2*_R^7-_R^3)*ln(x-_R),_R=RootOf(_Z^8-_Z^4+1))
```

3.25.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 417, normalized size of antiderivative = 1.17

$$
\begin{aligned}
\int \frac{1-x^4}{1-x^4+x^8}\,dx &= \frac{1}{24}\sqrt{6}\sqrt{\sqrt{2}\sqrt{-i\sqrt{3}+1}}\log\left(\sqrt{6}\sqrt{\sqrt{2}\sqrt{-i\sqrt{3}+1}}\left(i\sqrt{3}+3\right)+12\,x\right) \\
&+\frac{1}{24}\sqrt{6}\sqrt{-\sqrt{2}\sqrt{-i\sqrt{3}+1}}\log\left(\sqrt{6}\sqrt{-\sqrt{2}\sqrt{-i\sqrt{3}+1}}\left(i\sqrt{3}+3\right)\right. \\
&\left.+12\,x\right) \\
&-\frac{1}{24}\sqrt{6}\sqrt{\sqrt{2}\sqrt{i\sqrt{3}+1}}\log\left(\sqrt{6}\sqrt{\sqrt{2}\sqrt{i\sqrt{3}+1}}\left(i\sqrt{3}-3\right)+12\,x\right) \\
&-\frac{1}{24}\sqrt{6}\sqrt{-\sqrt{2}\sqrt{i\sqrt{3}+1}}\log\left(\sqrt{6}\sqrt{-\sqrt{2}\sqrt{i\sqrt{3}+1}}\left(i\sqrt{3}-3\right)\right. \\
&\left.+12\,x\right) \\
&+\frac{1}{24}\sqrt{6}\sqrt{\sqrt{2}\sqrt{i\sqrt{3}+1}}\log\left(\sqrt{6}\sqrt{\sqrt{2}\sqrt{i\sqrt{3}+1}}\left(-i\sqrt{3}+3\right)+12\,x\right) \\
&+\frac{1}{24}\sqrt{6}\sqrt{-\sqrt{2}\sqrt{i\sqrt{3}+1}}\log\left(\sqrt{6}\sqrt{-\sqrt{2}\sqrt{i\sqrt{3}+1}}\left(-i\sqrt{3}+3\right)\right. \\
&\left.+12\,x\right) \\
&-\frac{1}{24}\sqrt{6}\sqrt{\sqrt{2}\sqrt{-i\sqrt{3}+1}}\log\left(\sqrt{6}\sqrt{\sqrt{2}\sqrt{-i\sqrt{3}+1}}\left(-i\sqrt{3}-3\right)\right. \\
&\left.+12\,x\right) \\
&-\frac{1}{24}\sqrt{6}\sqrt{-\sqrt{2}\sqrt{-i\sqrt{3}+1}}\log\left(\sqrt{6}\sqrt{-\sqrt{2}\sqrt{-i\sqrt{3}+1}}\left(-i\sqrt{3}-3\right)\right. \\
&\left.+12\,x\right)
\end{aligned}
$$

input
```
integrate((-x^4+1)/(x^8-x^4+1),x, algorithm="fricas")
```

output
```
1/24*sqrt(6)*sqrt(sqrt(2)*sqrt(-I*sqrt(3) + 1))*log(sqrt(6)*sqrt(sqrt(2)*s
qrt(-I*sqrt(3) + 1))*(I*sqrt(3) + 3) + 12*x) + 1/24*sqrt(6)*sqrt(-sqrt(2)*
sqrt(-I*sqrt(3) + 1))*log(sqrt(6)*sqrt(-sqrt(2)*sqrt(-I*sqrt(3) + 1))*(I*s
qrt(3) + 3) + 12*x) - 1/24*sqrt(6)*sqrt(sqrt(2)*sqrt(I*sqrt(3) + 1))*log(s
qrt(6)*sqrt(sqrt(2)*sqrt(I*sqrt(3) + 1))*(I*sqrt(3) - 3) + 12*x) - 1/24*sq
rt(6)*sqrt(-sqrt(2)*sqrt(I*sqrt(3) + 1))*log(sqrt(6)*sqrt(-sqrt(2)*sqrt(I*
sqrt(3) + 1))*(I*sqrt(3) - 3) + 12*x) + 1/24*sqrt(6)*sqrt(sqrt(2)*sqrt(I*s
qrt(3) + 1))*log(sqrt(6)*sqrt(sqrt(2)*sqrt(I*sqrt(3) + 1))*(-I*sqrt(3) + 3
) + 12*x) + 1/24*sqrt(6)*sqrt(-sqrt(2)*sqrt(I*sqrt(3) + 1))*log(sqrt(6)*sq
rt(-sqrt(2)*sqrt(I*sqrt(3) + 1))*(-I*sqrt(3) + 3) + 12*x) - 1/24*sqrt(6)*s
qrt(sqrt(2)*sqrt(-I*sqrt(3) + 1))*log(sqrt(6)*sqrt(sqrt(2)*sqrt(-I*sqrt(3)
 + 1))*(-I*sqrt(3) - 3) + 12*x) - 1/24*sqrt(6)*sqrt(-sqrt(2)*sqrt(-I*sqrt(
3) + 1))*log(sqrt(6)*sqrt(-sqrt(2)*sqrt(-I*sqrt(3) + 1))*(-I*sqrt(3) - 3)
+ 12*x)
```

3.25.6 Sympy [A] (verification not implemented)

Time = 1.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.07

$$\int \frac{1-x^4}{1-x^4+x^8}\,dx = -\operatorname{RootSum}\left(5308416t^8 - 2304t^4 + 1, \left(t \mapsto t\log\left(9216t^5 - 8t + x\right)\right)\right)$$

input
```
integrate((-x**4+1)/(x**8-x**4+1),x)
```

output
```
-RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(9216*_t**5 - 8*
_t + x)))
```

3.25.7 Maxima [F]

$$\int \frac{1-x^4}{1-x^4+x^8}\,dx = \int -\frac{x^4-1}{x^8-x^4+1}\,dx$$

input
```
integrate((-x^4+1)/(x^8-x^4+1),x, algorithm="maxima")
```

output
```
-integrate((x^4 - 1)/(x^8 - x^4 + 1), x)
```

3.25.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.71

$$\begin{aligned}\int \frac{1-x^4}{1-x^4+x^8}\,dx = {} & \frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right) \\ & + \frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right) \\ & + \frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right) \\ & + \frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right) \\ & + \frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right) \\ & - \frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right) \\ & + \frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right) \\ & - \frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)\end{aligned}$$

input
```
integrate((-x^4+1)/(x^8-x^4+1),x, algorithm="giac")
```

output
```
1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqr
t(2))) + 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt
(6) + sqrt(2))) + 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt(
2))/(sqrt(6) - sqrt(2))) + 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6
) - sqrt(2))/(sqrt(6) - sqrt(2))) + 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 + 1
/2*x*(sqrt(6) + sqrt(2)) + 1) - 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*x
*(sqrt(6) + sqrt(2)) + 1) + 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1/2*x*(sq
rt(6) - sqrt(2)) + 1) - 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(6
) - sqrt(2)) + 1)
```

3.25.9 Mupad [B] (verification not implemented)

Time = 8.44 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.59

$$\int \frac{1-x^4}{1-x^4+x^8}\,dx = -\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x}{\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}+\frac{\sqrt{3}\,x\,1\mathrm{i}}{\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}\right)\,\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}\,1\mathrm{i}}{12} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\,1\mathrm{i}}{\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}-\frac{\sqrt{3}\,x}{\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}\right)\,\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}{12} + \frac{2^{3/4}\,\sqrt{3}\,\mathrm{atan}\left(\frac{2^{1/4}\,x}{2\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}-\frac{2^{1/4}\,\sqrt{3}\,x\,1\mathrm{i}}{2\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}\,1\mathrm{i}}{12} + \frac{2^{3/4}\,\sqrt{3}\,\mathrm{atan}\left(\frac{2^{1/4}\,x\,1\mathrm{i}}{2\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}+\frac{2^{1/4}\,\sqrt{3}\,x}{2\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}{12}$$

input
```
int(-(x^4 - 1)/(x^8 - x^4 + 1),x)
```

output
```
(2^(3/4)*3^(1/2)*atan((2^(1/4)*x)/(2*(3^(1/2)*1i + 1)^(1/4)) - (2^(1/4)*3^
(1/2)*x*1i)/(2*(3^(1/2)*1i + 1)^(1/4)))*(3^(1/2)*1i + 1)^(1/4)*1i)/12 - (3
^(1/2)*atan((x*1i)/(8 - 3^(1/2)*8i)^(1/4) - (3^(1/2)*x)/(8 - 3^(1/2)*8i)^(
1/4))*(8 - 3^(1/2)*8i)^(1/4))/12 - (3^(1/2)*atan(x/(8 - 3^(1/2)*8i)^(1/4)
+ (3^(1/2)*x*1i)/(8 - 3^(1/2)*8i)^(1/4))*(8 - 3^(1/2)*8i)^(1/4)*1i)/12 + (
2^(3/4)*3^(1/2)*atan((2^(1/4)*x*1i)/(2*(3^(1/2)*1i + 1)^(1/4)) + (2^(1/4)*
3^(1/2)*x)/(2*(3^(1/2)*1i + 1)^(1/4)))*(3^(1/2)*1i + 1)^(1/4))/12
```

3.26 $\int \frac{1-x^4}{1-2x^4+x^8}\,dx$

3.26.1 Optimal result

Integrand size = 20, antiderivative size = 13

$$\int \frac{1-x^4}{1-2x^4+x^8}\,dx=\frac{\arctan(x)}{2}+\frac{\operatorname{arctanh}(x)}{2}$$

output
```
1/2*arctan(x)+1/2*arctanh(x)
```

3.26.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.92

$$\int \frac{1-x^4}{1-2x^4+x^8}\,dx=\frac{\arctan(x)}{2}-\frac{1}{4}\log(1-x)+\frac{1}{4}\log(1+x)$$

input
```
Integrate[(1 - x^4)/(1 - 2*x^4 + x^8),x]
```

output
```
ArcTan[x]/2 - Log[1 - x]/4 + Log[1 + x]/4
```

3.26.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1380, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x^4}{x^8-2x^4+1}dx$$

$$\Big\downarrow \text{1380}$$

$$\int \frac{1}{1-x^4}dx$$

$$\Big\downarrow \text{756}$$

$$\frac{1}{2}\int \frac{1}{1-x^2}dx+\frac{1}{2}\int \frac{1}{x^2+1}dx$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\int \frac{1}{1-x^2}dx+\frac{\arctan(x)}{2}$$

$$\Big\downarrow \text{219}$$

$$\frac{\arctan(x)}{2}+\frac{\text{arctanh}(x)}{2}$$

input
```
Int[(1 - x^4)/(1 - 2*x^4 + x^8),x]
```

output
```
ArcTan[x]/2 + ArcTanh[x]/2
```

3.26.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

3.26.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
default	$\frac{\arctan(x)}{2}+\frac{\operatorname{arctanh}(x)}{2}$	10
risch	$-\frac{\ln(x-1)}{4}+\frac{\arctan(x)}{2}+\frac{\ln(x+1)}{4}$	18
parallelrisch	$-\frac{\ln(x-1)}{4}+\frac{i\ln(x+i)}{4}-\frac{i\ln(x-i)}{4}+\frac{\ln(x+1)}{4}$	30

input
```
int((-x^4+1)/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*arctan(x)+1/2*arctanh(x)
```

3.26.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{1-x^4}{1-2x^4+x^8}\,dx=\frac{1}{2}\arctan(x)+\frac{1}{4}\log(x+1)-\frac{1}{4}\log(x-1)$$

input
```
integrate((-x^4+1)/(x^8-2*x^4+1),x, algorithm="fricas")
```

output
```
1/2*arctan(x) + 1/4*log(x + 1) - 1/4*log(x - 1)
```

3.26.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(8) = 16$.

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{1-x^4}{1-2x^4+x^8}\,dx = -\frac{\log(x-1)}{4} + \frac{\log(x+1)}{4} + \frac{\operatorname{atan}(x)}{2}$$

input
```
integrate((-x**4+1)/(x**8-2*x**4+1),x)
```

output
```
-log(x - 1)/4 + log(x + 1)/4 + atan(x)/2
```

3.26.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{1-x^4}{1-2x^4+x^8}\,dx = \frac{1}{2}\arctan(x) + \frac{1}{4}\log(x+1) - \frac{1}{4}\log(x-1)$$

input
```
integrate((-x^4+1)/(x^8-2*x^4+1),x, algorithm="maxima")
```

output
```
1/2*arctan(x) + 1/4*log(x + 1) - 1/4*log(x - 1)
```

3.26.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(9) = 18$.

Time = 0.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.46

$$\int \frac{1-x^4}{1-2x^4+x^8}\,dx = \frac{1}{2}\arctan(x) + \frac{1}{4}\log(|x+1|) - \frac{1}{4}\log(|x-1|)$$

input
```
integrate((-x^4+1)/(x^8-2*x^4+1),x, algorithm="giac")
```

output
```
1/2*arctan(x) + 1/4*log(abs(x + 1)) - 1/4*log(abs(x - 1))
```

3.26.9 Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{1-x^4}{1-2x^4+x^8}\,dx=\frac{\operatorname{atan}(x)}{2}+\frac{\operatorname{atanh}(x)}{2}$$

input
```
int(-(x^4 - 1)/(x^8 - 2*x^4 + 1),x)
```

output
```
atan(x)/2 + atanh(x)/2
```

3.27 $\int \frac{1-x^4}{1-3x^4+x^8}\,dx$

3.27.1 Optimal result

Integrand size = 20, antiderivative size = 129

$$\int \frac{1-x^4}{1-3x^4+x^8}\,dx = \frac{\arctan\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)}{\sqrt{10\left(-1+\sqrt{5}\right)}}+\frac{\arctan\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}}+\frac{\operatorname{arctanh}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)}{\sqrt{10\left(-1+\sqrt{5}\right)}}+\frac{\operatorname{arctanh}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}}$$

output
```
arctan(x*2^(1/2)/(5^(1/2)-1)^(1/2))/(-10+10*5^(1/2))^(1/2)+arctanh(x*2^(1/
2)/(5^(1/2)-1)^(1/2))/(-10+10*5^(1/2))^(1/2)+arctan(x*2^(1/2)/(5^(1/2)+1)^
(1/2))/(10+10*5^(1/2))^(1/2)+arctanh(x*2^(1/2)/(5^(1/2)+1)^(1/2))/(10+10*5
^(1/2))^(1/2)
```

3.27.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00

$$\int \frac{1-x^4}{1-3x^4+x^8}\,dx = \frac{\arctan\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)}{\sqrt{10\left(-1+\sqrt{5}\right)}}+\frac{\arctan\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}}+\frac{\operatorname{arctanh}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)}{\sqrt{10\left(-1+\sqrt{5}\right)}}+\frac{\operatorname{arctanh}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}}$$

input
```
Integrate[(1 - x^4)/(1 - 3*x^4 + x^8),x]
```

output
```
ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x]/Sqrt[10*(-1 + Sqrt[5])] + ArcTan[Sqrt[2/(
1 + Sqrt[5])]*x]/Sqrt[10*(1 + Sqrt[5])] + ArcTanh[Sqrt[2/(-1 + Sqrt[5])]*x
]/Sqrt[10*(-1 + Sqrt[5])] + ArcTanh[Sqrt[2/(1 + Sqrt[5])]*x]/Sqrt[10*(1 +
Sqrt[5])]
```

3.27.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.20, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1749, 1406, 216, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x^4}{x^8-3x^4+1} \, dx$$

$$\Big\downarrow \text{1749}$$

$$-\frac{1}{2} \int \frac{1}{x^4-x^2-1} dx - \frac{1}{2} \int \frac{1}{x^4+x^2-1} dx$$

$$\Big\downarrow \text{1406}$$

$$\frac{1}{2} \left(\frac{\int \frac{1}{x^2+\frac{1}{2}\left(-1+\sqrt{5}\right)} dx}{\sqrt{5}} - \frac{\int \frac{1}{x^2+\frac{1}{2}\left(-1-\sqrt{5}\right)} dx}{\sqrt{5}} \right) + \frac{1}{2} \left(\frac{\int \frac{1}{x^2+\frac{1}{2}\left(1+\sqrt{5}\right)} dx}{\sqrt{5}} - \frac{\int \frac{1}{x^2+\frac{1}{2}\left(1-\sqrt{5}\right)} dx}{\sqrt{5}} \right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2} \left(\sqrt{\frac{2}{5\left(\sqrt{5}-1\right)}} \arctan\left(\sqrt{\frac{2}{\sqrt{5}-1}} x \right) - \frac{\int \frac{1}{x^2+\frac{1}{2}\left(-1-\sqrt{5}\right)} dx}{\sqrt{5}} \right) + \frac{1}{2} \left(\sqrt{\frac{2}{5\left(1+\sqrt{5}\right)}} \arctan\left(\sqrt{\frac{2}{1+\sqrt{5}}} x \right) - \frac{\int \frac{1}{x^2+\frac{1}{2}\left(1-\sqrt{5}\right)} dx}{\sqrt{5}} \right)$$

$$\Big\downarrow \text{220}$$

$$\frac{1}{2}\left(\sqrt{\frac{2}{5\left(1+\sqrt{5}\right)}}\arctan\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)+\sqrt{\frac{2}{5\left(\sqrt{5}-1\right)}}\operatorname{arctanh}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)\right)+\frac{1}{2}\left(\sqrt{\frac{2}{5\left(\sqrt{5}-1\right)}}\arctan\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)+\sqrt{\frac{2}{5\left(1+\sqrt{5}\right)}}\operatorname{arctanh}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)\right)$$

input
```
Int[(1 - x^4)/(1 - 3*x^4 + x^8),x]
```

output
```
(Sqrt[2/(5*(1 + Sqrt[5]))]*ArcTan[Sqrt[2/(1 + Sqrt[5])]*x] + Sqrt[2/(5*(-1
 + Sqrt[5]))]*ArcTanh[Sqrt[2/(-1 + Sqrt[5])]*x])/2 + (Sqrt[2/(5*(-1 + Sqrt
[5]))]*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x] + Sqrt[2/(5*(1 + Sqrt[5]))]*ArcTan
h[Sqrt[2/(1 + Sqrt[5])]*x])/2
```

3.27.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

rule 1749
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e
+ q*x^(n/2) + x^n, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x^(n/2) +
 x^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[2*(d/e) - b/c,
0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

3.27.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.50

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(25\_Z^4-5\_Z^2-1\right)}\_R\ln\left(-5\_R^3+3\_R+x\right)\right)}{4}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(25\_Z^4+5\_Z^2-1\right)}\_R\ln\left(5\_R^3+3\_R+x\right)\right)}{4}$
default	$\frac{\sqrt{5}\,\mathrm{arctanh}\left(\frac{2x}{\sqrt{2\sqrt{5}+2}}\right)}{5\sqrt{2\sqrt{5}+2}}+\frac{\sqrt{5}\,\arctan\left(\frac{2x}{\sqrt{2\sqrt{5}-2}}\right)}{5\sqrt{2\sqrt{5}-2}}+\frac{\sqrt{5}\,\arctan\left(\frac{2x}{\sqrt{2\sqrt{5}+2}}\right)}{5\sqrt{2\sqrt{5}+2}}+\frac{\sqrt{5}\,\mathrm{arctanh}\left(\frac{2x}{\sqrt{2\sqrt{5}-2}}\right)}{5\sqrt{2\sqrt{5}-2}}$

input
```
int((-x^4+1)/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(_R*ln(-5*_R^3+3*_R+x),_R=RootOf(25*_Z^4-5*_Z^2-1))+1/4*sum(_R*ln(5
*_R^3+3*_R+x),_R=RootOf(25*_Z^4+5*_Z^2-1))
```

3.27.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 285 vs. 2(93) = 186.

Time = 0.29 (sec) , antiderivative size = 285, normalized size of antiderivative = 2.21

$$\begin{aligned}\int \frac{1-x^4}{1-3x^4+x^8}\,dx &= \frac{1}{40}\sqrt{10}\sqrt{\sqrt{5}-1}\log\left(\sqrt{10}\left(\sqrt{5}+5\right)\sqrt{\sqrt{5}-1}+20\,x\right)\\&\quad-\frac{1}{40}\sqrt{10}\sqrt{\sqrt{5}-1}\log\left(-\sqrt{10}\left(\sqrt{5}+5\right)\sqrt{\sqrt{5}-1}+20\,x\right)\\&\quad-\frac{1}{40}\sqrt{10}\sqrt{\sqrt{5}+1}\log\left(\sqrt{10}\sqrt{\sqrt{5}+1}\left(\sqrt{5}-5\right)+20\,x\right)\\&\quad+\frac{1}{40}\sqrt{10}\sqrt{\sqrt{5}+1}\log\left(-\sqrt{10}\sqrt{\sqrt{5}+1}\left(\sqrt{5}-5\right)+20\,x\right)\\&\quad+\frac{1}{40}\sqrt{10}\sqrt{-\sqrt{5}+1}\log\left(\sqrt{10}\left(\sqrt{5}+5\right)\sqrt{-\sqrt{5}+1}+20\,x\right)\\&\quad-\frac{1}{40}\sqrt{10}\sqrt{-\sqrt{5}+1}\log\left(-\sqrt{10}\left(\sqrt{5}+5\right)\sqrt{-\sqrt{5}+1}+20\,x\right)\\&\quad-\frac{1}{40}\sqrt{10}\sqrt{-\sqrt{5}-1}\log\left(\sqrt{10}\left(\sqrt{5}-5\right)\sqrt{-\sqrt{5}-1}+20\,x\right)\\&\quad+\frac{1}{40}\sqrt{10}\sqrt{-\sqrt{5}-1}\log\left(-\sqrt{10}\left(\sqrt{5}-5\right)\sqrt{-\sqrt{5}-1}+20\,x\right)\end{aligned}$$

input
```
integrate((-x^4+1)/(x^8-3*x^4+1),x, algorithm="fricas")
```

output
```
1/40*sqrt(10)*sqrt(sqrt(5) - 1)*log(sqrt(10)*(sqrt(5) + 5)*sqrt(sqrt(5) -
1) + 20*x) - 1/40*sqrt(10)*sqrt(sqrt(5) - 1)*log(-sqrt(10)*(sqrt(5) + 5)*s
qrt(sqrt(5) - 1) + 20*x) - 1/40*sqrt(10)*sqrt(sqrt(5) + 1)*log(sqrt(10)*sq
rt(sqrt(5) + 1)*(sqrt(5) - 5) + 20*x) + 1/40*sqrt(10)*sqrt(sqrt(5) + 1)*lo
g(-sqrt(10)*sqrt(sqrt(5) + 1)*(sqrt(5) - 5) + 20*x) + 1/40*sqrt(10)*sqrt(-
sqrt(5) + 1)*log(sqrt(10)*(sqrt(5) + 5)*sqrt(-sqrt(5) + 1) + 20*x) - 1/40*
sqrt(10)*sqrt(-sqrt(5) + 1)*log(-sqrt(10)*(sqrt(5) + 5)*sqrt(-sqrt(5) + 1)
 + 20*x) - 1/40*sqrt(10)*sqrt(-sqrt(5) - 1)*log(sqrt(10)*(sqrt(5) - 5)*sqr
t(-sqrt(5) - 1) + 20*x) + 1/40*sqrt(10)*sqrt(-sqrt(5) - 1)*log(-sqrt(10)*(
sqrt(5) - 5)*sqrt(-sqrt(5) - 1) + 20*x)
```

3.27.6 Sympy [A] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.40

$$\int \frac{1-x^4}{1-3x^4+x^8}\,dx = -\operatorname{RootSum}\left(6400t^4-80t^2-1,\left(t\mapsto t\log\left(25600t^5-16t+x\right)\right)\right) - \operatorname{RootSum}\left(6400t^4+80t^2-1,\left(t\mapsto t\log\left(25600t^5-16t+x\right)\right)\right)$$

input
```
integrate((-x**4+1)/(x**8-3*x**4+1),x)
```

output
```
-RootSum(6400*_t**4 - 80*_t**2 - 1, Lambda(_t, _t*log(25600*_t**5 - 16*_t
+ x))) - RootSum(6400*_t**4 + 80*_t**2 - 1, Lambda(_t, _t*log(25600*_t**5
- 16*_t + x)))
```

3.27.7 Maxima [F]

$$\int \frac{1-x^4}{1-3x^4+x^8}\,dx = \int -\frac{x^4-1}{x^8-3\,x^4+1}\,dx$$

input
```
integrate((-x^4+1)/(x^8-3*x^4+1),x, algorithm="maxima")
```

output
```
-integrate((x^4 - 1)/(x^8 - 3*x^4 + 1), x)
```

3.27.8 Giac [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.14

$$\int \frac{1-x^4}{1-3x^4+x^8}\,dx = \frac{1}{20}\sqrt{10\sqrt{5}-10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right) + \frac{1}{20}\sqrt{10\sqrt{5}+10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right) + \frac{1}{40}\sqrt{10\sqrt{5}-10}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right) - \frac{1}{40}\sqrt{10\sqrt{5}-10}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right) + \frac{1}{40}\sqrt{10\sqrt{5}+10}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right) - \frac{1}{40}\sqrt{10\sqrt{5}+10}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)$$

input
```
integrate((-x^4+1)/(x^8-3*x^4+1),x, algorithm="giac")
```

output
```
1/20*sqrt(10*sqrt(5) - 10)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) + 1/20*sqrt(1
0*sqrt(5) + 10)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) + 1/40*sqrt(10*sqrt(5) -
 10)*log(abs(x + sqrt(1/2*sqrt(5) + 1/2))) - 1/40*sqrt(10*sqrt(5) - 10)*lo
g(abs(x - sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(10*sqrt(5) + 10)*log(abs(x
 + sqrt(1/2*sqrt(5) - 1/2))) - 1/40*sqrt(10*sqrt(5) + 10)*log(abs(x - sqrt
(1/2*sqrt(5) - 1/2)))
```

3.27.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 269, normalized size of antiderivative = 2.09

$$\int \frac{1-x^4}{1-3x^4+x^8}\,dx = -\frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{\sqrt{5}-1}\,3\mathrm{i}}{2\left(3\sqrt{5}-7\right)}-\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{\sqrt{5}-1}\,7\mathrm{i}}{10\left(3\sqrt{5}-7\right)}\right)\sqrt{\sqrt{5}-1}\,1\mathrm{i}}{20} - \frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{\sqrt{5}+1}\,3\mathrm{i}}{2\left(3\sqrt{5}+7\right)}+\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{\sqrt{5}+1}\,7\mathrm{i}}{10\left(3\sqrt{5}+7\right)}\right)\sqrt{\sqrt{5}+1}\,1\mathrm{i}}{20} + \frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{1-\sqrt{5}}\,3\mathrm{i}}{2\left(3\sqrt{5}-7\right)}-\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{1-\sqrt{5}}\,7\mathrm{i}}{10\left(3\sqrt{5}-7\right)}\right)\sqrt{1-\sqrt{5}}\,1\mathrm{i}}{20} + \frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{-\sqrt{5}-1}\,3\mathrm{i}}{2\left(3\sqrt{5}+7\right)}+\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{-\sqrt{5}-1}\,7\mathrm{i}}{10\left(3\sqrt{5}+7\right)}\right)\sqrt{-\sqrt{5}-1}\,1\mathrm{i}}{20}$$

input
```
int(-(x^4 - 1)/(x^8 - 3*x^4 + 1),x)
```

output
```
(10^(1/2)*atan((10^(1/2)*x*(1 - 5^(1/2))^(1/2)*3i)/(2*(3*5^(1/2) - 7)) - (
5^(1/2)*10^(1/2)*x*(1 - 5^(1/2))^(1/2)*7i)/(10*(3*5^(1/2) - 7)))*(1 - 5^(1
/2))^(1/2)*1i)/20 - (10^(1/2)*atan((10^(1/2)*x*(5^(1/2) + 1)^(1/2)*3i)/(2*
(3*5^(1/2) + 7)) + (5^(1/2)*10^(1/2)*x*(5^(1/2) + 1)^(1/2)*7i)/(10*(3*5^(1
/2) + 7)))*(5^(1/2) + 1)^(1/2)*1i)/20 - (10^(1/2)*atan((10^(1/2)*x*(5^(1/2
) - 1)^(1/2)*3i)/(2*(3*5^(1/2) - 7)) - (5^(1/2)*10^(1/2)*x*(5^(1/2) - 1)^(
1/2)*7i)/(10*(3*5^(1/2) - 7)))*(5^(1/2) - 1)^(1/2)*1i)/20 + (10^(1/2)*atan
((10^(1/2)*x*(- 5^(1/2) - 1)^(1/2)*3i)/(2*(3*5^(1/2) + 7)) + (5^(1/2)*10^(
1/2)*x*(- 5^(1/2) - 1)^(1/2)*7i)/(10*(3*5^(1/2) + 7)))*(- 5^(1/2) - 1)^(1/
2)*1i)/20
```

3.28 $\int \frac{1-x^4}{1-4x^4+x^8}\,dx$

3.28.1 Optimal result

Integrand size = 20, antiderivative size = 165

$$\int \frac{1-x^4}{1-4x^4+x^8}\,dx = \frac{\arctan\left(\frac{\sqrt[4]{2}x}{\sqrt{-1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{3\left(-1+\sqrt{3}\right)}} + \frac{\arctan\left(\frac{\sqrt[4]{2}x}{\sqrt{1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{3\left(1+\sqrt{3}\right)}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{2}x}{\sqrt{-1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{3\left(-1+\sqrt{3}\right)}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{2}x}{\sqrt{1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{3\left(1+\sqrt{3}\right)}}$$

output
```
1/4*arctan(2^(1/4)*x/(3^(1/2)-1)^(1/2))*2^(3/4)/(-3+3*3^(1/2))^(1/2)+1/4*a
rctanh(2^(1/4)*x/(3^(1/2)-1)^(1/2))*2^(3/4)/(-3+3*3^(1/2))^(1/2)+1/4*arcta
n(2^(1/4)*x/(1+3^(1/2))^(1/2))*2^(3/4)/(3+3*3^(1/2))^(1/2)+1/4*arctanh(2^(
1/4)*x/(1+3^(1/2))^(1/2))*2^(3/4)/(3+3*3^(1/2))^(1/2)
```

3.28.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.33

$$\int \frac{1-x^4}{1-4x^4+x^8}\,dx = -\frac{1}{8}\text{RootSum}\left[1-4\#1^4+\#1^8\&,\frac{-\log(x-\#1)+\log(x-\#1)\#1^4}{-2\#1^3+\#1^7}\&\right]$$

input
```
Integrate[(1 - x^4)/(1 - 4*x^4 + x^8),x]
```

output
```
-1/8*RootSum[1 - 4*#1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*#1^4)/(-2*#
1^3 + #1^7) & ]
```

3.28.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1749, 1406, 216, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x^4}{x^8-4x^4+1}dx$$

$$\Big\downarrow \text{1749}$$

$$-\frac{1}{2}\int \frac{1}{x^4-\sqrt{2}x^2-1}dx-\frac{1}{2}\int \frac{1}{x^4+\sqrt{2}x^2-1}dx$$

$$\Big\downarrow \text{1406}$$

$$\frac{1}{2}\left(\frac{\int \frac{1}{x^2-\frac{1-\sqrt{3}}{\sqrt{2}}}dx}{\sqrt{6}}-\frac{\int \frac{1}{x^2-\frac{1+\sqrt{3}}{\sqrt{2}}}dx}{\sqrt{6}}\right)+\frac{1}{2}\left(\frac{\int \frac{1}{x^2+\frac{1+\sqrt{3}}{\sqrt{2}}}dx}{\sqrt{6}}-\frac{\int \frac{1}{x^2+\frac{1-\sqrt{3}}{\sqrt{2}}}dx}{\sqrt{6}}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt[4]{2}x}{\sqrt{1+\sqrt{3}}}\right)}{\sqrt[4]{2}\sqrt{3\left(1+\sqrt{3}\right)}}-\frac{\int \frac{1}{x^2+\frac{1-\sqrt{3}}{\sqrt{2}}}dx}{\sqrt{6}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt[4]{2}x}{\sqrt{\sqrt{3}-1}}\right)}{\sqrt[4]{2}\sqrt{3\left(\sqrt{3}-1\right)}}-\frac{\int \frac{1}{x^2-\frac{1+\sqrt{3}}{\sqrt{2}}}dx}{\sqrt{6}}\right)$$

$$\Big\downarrow \text{220}$$

$$\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt[4]{2}x}{\sqrt{1+\sqrt{3}}}\right)}{\sqrt[4]{2}\sqrt{3\left(1+\sqrt{3}\right)}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}x}{\sqrt{\sqrt{3}-1}}\right)}{\sqrt[4]{2}\sqrt{3\left(\sqrt{3}-1\right)}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt[4]{2}x}{\sqrt{\sqrt{3}-1}}\right)}{\sqrt[4]{2}\sqrt{3\left(\sqrt{3}-1\right)}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}x}{\sqrt{1+\sqrt{3}}}\right)}{\sqrt[4]{2}\sqrt{3\left(1+\sqrt{3}\right)}}\right)$$

input
```
Int[(1 - x^4)/(1 - 4*x^4 + x^8),x]
```

output
```
(ArcTan[(2^(1/4)*x)/Sqrt[1 + Sqrt[3]]]/(2^(1/4)*Sqrt[3*(1 + Sqrt[3])]) + A
rcTanh[(2^(1/4)*x)/Sqrt[-1 + Sqrt[3]]]/(2^(1/4)*Sqrt[3*(-1 + Sqrt[3])]))/2
 + (ArcTan[(2^(1/4)*x)/Sqrt[-1 + Sqrt[3]]]/(2^(1/4)*Sqrt[3*(-1 + Sqrt[3])]
) + ArcTanh[(2^(1/4)*x)/Sqrt[1 + Sqrt[3]]]/(2^(1/4)*Sqrt[3*(1 + Sqrt[3])]))
)/2
```

3.28.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

rule 1749
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e
+ q*x^(n/2) + x^n, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x^(n/2) +
 x^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[2*(d/e) - b/c,
0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

3.28.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.25

method	result	size
default	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^8-4\_Z^4+1\right)}\frac{\left(-\_R^4+1\right)\ln\left(x-\_R\right)}{\_R^7-2\_R^3}\right)}{8}$	42
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^8-4\_Z^4+1\right)}\frac{\left(-\_R^4+1\right)\ln\left(x-\_R\right)}{\_R^7-2\_R^3}\right)}{8}$	42

input
```
int((-x^4+1)/(x^8-4*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/8*sum((-_R^4+1)/(_R^7-2*_R^3)*ln(x-_R),_R=RootOf(_Z^8-4*_Z^4+1))
```

3.28.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 317 vs. $2(109) = 218$.

Time = 0.27 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.92

$$\begin{aligned}\int \frac{1-x^4}{1-4x^4+x^8}\,dx = & -\frac{1}{24}\sqrt{6}\sqrt{-\sqrt{\sqrt{3}+2}}\log\left(\sqrt{6}\left(\sqrt{3}-3\right)\sqrt{-\sqrt{\sqrt{3}+2}}+6x\right)\\ & +\frac{1}{24}\sqrt{6}\sqrt{-\sqrt{\sqrt{3}+2}}\log\left(-\sqrt{6}\left(\sqrt{3}-3\right)\sqrt{-\sqrt{\sqrt{3}+2}}+6x\right)\\ & +\frac{1}{24}\sqrt{6}\sqrt{-\sqrt{-\sqrt{3}+2}}\log\left(\sqrt{6}\left(\sqrt{3}+3\right)\sqrt{-\sqrt{-\sqrt{3}+2}}+6x\right)\\ & -\frac{1}{24}\sqrt{6}\sqrt{-\sqrt{-\sqrt{3}+2}}\log\left(-\sqrt{6}\left(\sqrt{3}+3\right)\sqrt{-\sqrt{-\sqrt{3}+2}}+6x\right)\\ & -\frac{1}{24}\sqrt{6}\left(\sqrt{3}+2\right)^{\frac{1}{4}}\log\left(\sqrt{6}\left(\sqrt{3}+2\right)^{\frac{1}{4}}\left(\sqrt{3}-3\right)+6x\right)\\ & +\frac{1}{24}\sqrt{6}\left(\sqrt{3}+2\right)^{\frac{1}{4}}\log\left(-\sqrt{6}\left(\sqrt{3}+2\right)^{\frac{1}{4}}\left(\sqrt{3}-3\right)+6x\right)\\ & +\frac{1}{24}\sqrt{6}\left(-\sqrt{3}+2\right)^{\frac{1}{4}}\log\left(\sqrt{6}\left(\sqrt{3}+3\right)\left(-\sqrt{3}+2\right)^{\frac{1}{4}}+6x\right)\\ & -\frac{1}{24}\sqrt{6}\left(-\sqrt{3}+2\right)^{\frac{1}{4}}\log\left(-\sqrt{6}\left(\sqrt{3}+3\right)\left(-\sqrt{3}+2\right)^{\frac{1}{4}}+6x\right)\end{aligned}$$

input
```
integrate((-x^4+1)/(x^8-4*x^4+1),x, algorithm="fricas")
```

output
```
-1/24*sqrt(6)*sqrt(-sqrt(sqrt(3) + 2))*log(sqrt(6)*(sqrt(3) - 3)*sqrt(-sqr
t(sqrt(3) + 2)) + 6*x) + 1/24*sqrt(6)*sqrt(-sqrt(sqrt(3) + 2))*log(-sqrt(6
)*(sqrt(3) - 3)*sqrt(-sqrt(sqrt(3) + 2)) + 6*x) + 1/24*sqrt(6)*sqrt(-sqrt(
-sqrt(3) + 2))*log(sqrt(6)*(sqrt(3) + 3)*sqrt(-sqrt(-sqrt(3) + 2)) + 6*x)
- 1/24*sqrt(6)*sqrt(-sqrt(-sqrt(3) + 2))*log(-sqrt(6)*(sqrt(3) + 3)*sqrt(-
sqrt(-sqrt(3) + 2)) + 6*x) - 1/24*sqrt(6)*(sqrt(3) + 2)^(1/4)*log(sqrt(6)*
(sqrt(3) + 2)^(1/4)*(sqrt(3) - 3) + 6*x) + 1/24*sqrt(6)*(sqrt(3) + 2)^(1/4
)*log(-sqrt(6)*(sqrt(3) + 2)^(1/4)*(sqrt(3) - 3) + 6*x) + 1/24*sqrt(6)*(-s
qrt(3) + 2)^(1/4)*log(sqrt(6)*(sqrt(3) + 3)*(-sqrt(3) + 2)^(1/4) + 6*x) -
1/24*sqrt(6)*(-sqrt(3) + 2)^(1/4)*log(-sqrt(6)*(sqrt(3) + 3)*(-sqrt(3) + 2
)^(1/4) + 6*x)
```

3.28.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.16

$$\int \frac{1-x^4}{1-4x^4+x^8}\,dx = -\operatorname{RootSum}\left(84934656t^8-36864t^4+1,\left(t\mapsto t\log\left(36864t^5-20t+x\right)\right)\right)$$

input
```
integrate((-x**4+1)/(x**8-4*x**4+1),x)
```

output
```
-RootSum(84934656*_t**8 - 36864*_t**4 + 1, Lambda(_t, _t*log(36864*_t**5 -
 20*_t + x)))
```

3.28.7 Maxima [F]

$$\int \frac{1-x^4}{1-4x^4+x^8}\,dx = \int -\frac{x^4-1}{x^8-4\,x^4+1}\,dx$$

input
```
integrate((-x^4+1)/(x^8-4*x^4+1),x, algorithm="maxima")
```

output
```
-integrate((x^4 - 1)/(x^8 - 4*x^4 + 1), x)
```

3.28.8 Giac [F]

$$\int \frac{1-x^4}{1-4x^4+x^8}\,dx=\int -\frac{x^4-1}{x^8-4\,x^4+1}\,dx$$

input
```
integrate((-x^4+1)/(x^8-4*x^4+1),x, algorithm="giac")
```

output
```
integrate(-(x^4 - 1)/(x^8 - 4*x^4 + 1), x)
```

3.28.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 399, normalized size of antiderivative = 2.42

$$\int \frac{1-x^4}{1-4x^4+x^8}\,dx = \frac{\sqrt{6}\,\mathrm{atan}\left(\frac{64\,\sqrt{6}\,x\,\left(\sqrt{3}+2\right)^{1/4}}{80\,\sqrt{\sqrt{3}+2}+48\,\sqrt{3}\,\sqrt{\sqrt{3}+2}}+\frac{112\,\sqrt{3}\,\sqrt{6}\,x\,\left(\sqrt{3}+2\right)^{1/4}}{3\,\left(80\,\sqrt{\sqrt{3}+2}+48\,\sqrt{3}\,\sqrt{\sqrt{3}+2}\right)}\right)\,\left(\sqrt{3}+2\right)^{1/4}}{12}$$
$$+\frac{\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x\,\left(2-\sqrt{3}\right)^{1/4}\,64\mathrm{i}}{48\,\sqrt{3}\,\sqrt{2-\sqrt{3}}-80\,\sqrt{2-\sqrt{3}}}-\frac{\sqrt{3}\,\sqrt{6}\,x\,\left(2-\sqrt{3}\right)^{1/4}\,112\mathrm{i}}{3\,\left(48\,\sqrt{3}\,\sqrt{2-\sqrt{3}}-80\,\sqrt{2-\sqrt{3}}\right)}\right)\,\left(2-\sqrt{3}\right)^{1/4}\,1\mathrm{i}}{12}$$
$$-\frac{\sqrt{6}\,\mathrm{atan}\left(\frac{64\,\sqrt{6}\,x\,\left(2-\sqrt{3}\right)^{1/4}}{48\,\sqrt{3}\,\sqrt{2-\sqrt{3}}-80\,\sqrt{2-\sqrt{3}}}-\frac{112\,\sqrt{3}\,\sqrt{6}\,x\,\left(2-\sqrt{3}\right)^{1/4}}{3\,\left(48\,\sqrt{3}\,\sqrt{2-\sqrt{3}}-80\,\sqrt{2-\sqrt{3}}\right)}\right)\,\left(2-\sqrt{3}\right)^{1/4}}{12}$$
$$-\frac{\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x\,\left(\sqrt{3}+2\right)^{1/4}\,64\mathrm{i}}{80\,\sqrt{\sqrt{3}+2}+48\,\sqrt{3}\,\sqrt{\sqrt{3}+2}}+\frac{\sqrt{3}\,\sqrt{6}\,x\,\left(\sqrt{3}+2\right)^{1/4}\,112\mathrm{i}}{3\,\left(80\,\sqrt{\sqrt{3}+2}+48\,\sqrt{3}\,\sqrt{\sqrt{3}+2}\right)}\right)\,\left(\sqrt{3}+2\right)^{1/4}\,1\mathrm{i}}{12}$$

input
```
int(-(x^4 - 1)/(x^8 - 4*x^4 + 1),x)
```

output
```
(6^(1/2)*atan((6^(1/2)*x*(2 - 3^(1/2))^(1/4)*64i)/(48*3^(1/2)*(2 - 3^(1/2)
)^(1/2) - 80*(2 - 3^(1/2))^(1/2)) - (3^(1/2)*6^(1/2)*x*(2 - 3^(1/2))^(1/4)
*112i)/(3*(48*3^(1/2)*(2 - 3^(1/2))^(1/2) - 80*(2 - 3^(1/2))^(1/2))))*(2 -
 3^(1/2))^(1/4)*1i)/12 - (6^(1/2)*atan((64*6^(1/2)*x*(2 - 3^(1/2))^(1/4))/
(48*3^(1/2)*(2 - 3^(1/2))^(1/2) - 80*(2 - 3^(1/2))^(1/2)) - (112*3^(1/2)*6
^(1/2)*x*(2 - 3^(1/2))^(1/4))/(3*(48*3^(1/2)*(2 - 3^(1/2))^(1/2) - 80*(2 -
 3^(1/2))^(1/2))))*(2 - 3^(1/2))^(1/4))/12 + (6^(1/2)*atan((64*6^(1/2)*x*(
3^(1/2) + 2)^(1/4))/(80*(3^(1/2) + 2)^(1/2) + 48*3^(1/2)*(3^(1/2) + 2)^(1/
2)) + (112*3^(1/2)*6^(1/2)*x*(3^(1/2) + 2)^(1/4))/(3*(80*(3^(1/2) + 2)^(1/
2) + 48*3^(1/2)*(3^(1/2) + 2)^(1/2))))*(3^(1/2) + 2)^(1/4))/12 - (6^(1/2)*
atan((6^(1/2)*x*(3^(1/2) + 2)^(1/4)*64i)/(80*(3^(1/2) + 2)^(1/2) + 48*3^(1
/2)*(3^(1/2) + 2)^(1/2)) + (3^(1/2)*6^(1/2)*x*(3^(1/2) + 2)^(1/4)*112i)/(3
*(80*(3^(1/2) + 2)^(1/2) + 48*3^(1/2)*(3^(1/2) + 2)^(1/2))))*(3^(1/2) + 2)
^(1/4)*1i)/12
```

3.28. $\int \frac{1-x^4}{1-4x^4+x^8}\,dx$

3.29 $\int \frac{1-x^4}{1-5x^4+x^8}\,dx$

3.29.1 Optimal result

Integrand size = 20, antiderivative size = 169

$$\int \frac{1-x^4}{1-5x^4+x^8}\,dx = \frac{\arctan\left(\sqrt{\frac{2}{-\sqrt{3}+\sqrt{7}}}x\right)}{\sqrt{14\left(-\sqrt{3}+\sqrt{7}\right)}} + \frac{\arctan\left(\sqrt{\frac{2}{\sqrt{3}+\sqrt{7}}}x\right)}{\sqrt{14\left(\sqrt{3}+\sqrt{7}\right)}} + \frac{\operatorname{arctanh}\left(\sqrt{\frac{2}{-\sqrt{3}+\sqrt{7}}}x\right)}{\sqrt{14\left(-\sqrt{3}+\sqrt{7}\right)}} + \frac{\operatorname{arctanh}\left(\sqrt{\frac{2}{\sqrt{3}+\sqrt{7}}}x\right)}{\sqrt{14\left(\sqrt{3}+\sqrt{7}\right)}}$$

output
```
arctan(x*2^(1/2)/(7^(1/2)-3^(1/2))^(1/2))/(-14*3^(1/2)+14*7^(1/2))^(1/2)+a
rctanh(x*2^(1/2)/(7^(1/2)-3^(1/2))^(1/2))/(-14*3^(1/2)+14*7^(1/2))^(1/2)+a
rctan(x*2^(1/2)/(7^(1/2)+3^(1/2))^(1/2))/(14*3^(1/2)+14*7^(1/2))^(1/2)+arc
tanh(x*2^(1/2)/(7^(1/2)+3^(1/2))^(1/2))/(14*3^(1/2)+14*7^(1/2))^(1/2)
```

3.29.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.34

$$\int \frac{1-x^4}{1-5x^4+x^8}\,dx = -\frac{1}{4}\text{RootSum}\left[1-5\#1^4+\#1^8\&,\frac{-\log(x-\#1)+\log(x-\#1)\#1^4}{-5\#1^3+2\#1^7}\&\right]$$

input
```
Integrate[(1 - x^4)/(1 - 5*x^4 + x^8),x]
```

output
```
-1/4*RootSum[1 - 5*#1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*#1^4)/(-5*#
1^3 + 2*#1^7) & ]
```

3.29.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.15, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1749, 1406, 216, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x^4}{x^8-5x^4+1}\,dx$$
$$\Big\downarrow \text{1749}$$
$$-\frac{1}{2}\int \frac{1}{x^4-\sqrt{3}x^2-1}dx-\frac{1}{2}\int \frac{1}{x^4+\sqrt{3}x^2-1}dx$$
$$\Big\downarrow \text{1406}$$
$$\frac{1}{2}\left(\frac{\int \frac{1}{x^2+\frac{1}{2}\left(-\sqrt{3}+\sqrt{7}\right)}dx}{\sqrt{7}}-\frac{\int \frac{1}{x^2+\frac{1}{2}\left(-\sqrt{3}-\sqrt{7}\right)}dx}{\sqrt{7}}\right)+\frac{1}{2}\left(\frac{\int \frac{1}{x^2+\frac{1}{2}\left(\sqrt{3}+\sqrt{7}\right)}dx}{\sqrt{7}}-\frac{\int \frac{1}{x^2+\frac{1}{2}\left(\sqrt{3}-\sqrt{7}\right)}dx}{\sqrt{7}}\right)$$
$$\Big\downarrow \text{216}$$
$$\frac{1}{2}\left(\sqrt{\frac{2}{7\left(\sqrt{7}-\sqrt{3}\right)}}\arctan\left(\sqrt{\frac{2}{\sqrt{7}-\sqrt{3}}}x\right)-\frac{\int \frac{1}{x^2+\frac{1}{2}\left(-\sqrt{3}-\sqrt{7}\right)}dx}{\sqrt{7}}\right)+\frac{1}{2}\left(\sqrt{\frac{2}{7\left(\sqrt{3}+\sqrt{7}\right)}}\arctan\left(\sqrt{\frac{2}{\sqrt{3}+\sqrt{7}}}x\right)-\frac{\int \frac{1}{x^2+\frac{1}{2}\left(\sqrt{3}-\sqrt{7}\right)}dx}{\sqrt{7}}\right)$$
$$\Big\downarrow \text{220}$$
$$\frac{1}{2}\left(\sqrt{\frac{2}{7\left(\sqrt{3}+\sqrt{7}\right)}}\arctan\left(\sqrt{\frac{2}{\sqrt{3}+\sqrt{7}}}x\right)+\sqrt{\frac{2}{7\left(\sqrt{7}-\sqrt{3}\right)}}\text{arctanh}\left(\sqrt{\frac{2}{\sqrt{7}-\sqrt{3}}}x\right)\right)+\frac{1}{2}\left(\sqrt{\frac{2}{7\left(\sqrt{7}-\sqrt{3}\right)}}\arctan\left(\sqrt{\frac{2}{\sqrt{7}-\sqrt{3}}}x\right)+\sqrt{\frac{2}{7\left(\sqrt{3}+\sqrt{7}\right)}}\text{arctanh}\left(\sqrt{\frac{2}{\sqrt{3}+\sqrt{7}}}x\right)\right)$$

input
```
Int[(1 - x^4)/(1 - 5*x^4 + x^8),x]
```

output
```
(Sqrt[2/(7*(Sqrt[3] + Sqrt[7]))]*ArcTan[Sqrt[2/(Sqrt[3] + Sqrt[7])]*x] + S
qrt[2/(7*(-Sqrt[3] + Sqrt[7]))]*ArcTanh[Sqrt[2/(-Sqrt[3] + Sqrt[7])]*x])/2
 + (Sqrt[2/(7*(-Sqrt[3] + Sqrt[7]))]*ArcTan[Sqrt[2/(-Sqrt[3] + Sqrt[7])]*x
] + Sqrt[2/(7*(Sqrt[3] + Sqrt[7]))]*ArcTanh[Sqrt[2/(Sqrt[3] + Sqrt[7])]*x]
)/2
```

3.29.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

rule 1749
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e
+ q*x^(n/2) + x^n, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x^(n/2) +
 x^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[2*(d/e) - b/c,
0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

3.29.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.26

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^8-5\_Z^4+1\right)}\frac{\left(-\_R^4+1\right)\ln\left(x-\_R\right)}{2\_R^7-5\_R^3}\right)}{4}$	44
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^8-5\_Z^4+1\right)}\frac{\left(-\_R^4+1\right)\ln\left(x-\_R\right)}{2\_R^7-5\_R^3}\right)}{4}$	44

input
```
int((-x^4+1)/(x^8-5*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum((-_R^4+1)/(2*_R^7-5*_R^3)*ln(x-_R),_R=RootOf(_Z^8-5*_Z^4+1))
```

3.29.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 477 vs. $2(121) = 242$.

Time = 0.30 (sec) , antiderivative size = 477, normalized size of antiderivative = 2.82

$$\begin{aligned}
\int \frac{1-x^4}{1-5x^4+x^8}\,dx
&= -\frac{1}{56}\sqrt{14}\sqrt{\sqrt{2}\sqrt{\sqrt{7}\sqrt{3}+5}}\log\left(\sqrt{14}\left(\sqrt{7}\sqrt{3}-7\right)\sqrt{\sqrt{2}\sqrt{\sqrt{7}\sqrt{3}+5}}+28\,x\right)\\
&\quad+\frac{1}{56}\sqrt{14}\sqrt{\sqrt{2}\sqrt{\sqrt{7}\sqrt{3}+5}}\log\left(-\sqrt{14}\left(\sqrt{7}\sqrt{3}-7\right)\sqrt{\sqrt{2}\sqrt{\sqrt{7}\sqrt{3}+5}}+28\,x\right)\\
&\quad-\frac{1}{56}\sqrt{14}\sqrt{-\sqrt{2}\sqrt{\sqrt{7}\sqrt{3}+5}}\log\left(\sqrt{14}\left(\sqrt{7}\sqrt{3}-7\right)\sqrt{-\sqrt{2}\sqrt{\sqrt{7}\sqrt{3}+5}}+28\,x\right)\\
&\quad+\frac{1}{56}\sqrt{14}\sqrt{-\sqrt{2}\sqrt{\sqrt{7}\sqrt{3}+5}}\log\left(-\sqrt{14}\left(\sqrt{7}\sqrt{3}-7\right)\sqrt{-\sqrt{2}\sqrt{\sqrt{7}\sqrt{3}+5}}+28\,x\right)\\
&\quad+\frac{1}{56}\sqrt{14}\sqrt{\sqrt{2}\sqrt{-\sqrt{7}\sqrt{3}+5}}\log\left(\sqrt{14}\left(\sqrt{7}\sqrt{3}+7\right)\sqrt{\sqrt{2}\sqrt{-\sqrt{7}\sqrt{3}+5}}+28\,x\right)\\
&\quad-\frac{1}{56}\sqrt{14}\sqrt{\sqrt{2}\sqrt{-\sqrt{7}\sqrt{3}+5}}\log\left(-\sqrt{14}\left(\sqrt{7}\sqrt{3}+7\right)\sqrt{\sqrt{2}\sqrt{-\sqrt{7}\sqrt{3}+5}}+28\,x\right)\\
&\quad+\frac{1}{56}\sqrt{14}\sqrt{-\sqrt{2}\sqrt{-\sqrt{7}\sqrt{3}+5}}\log\left(\sqrt{14}\left(\sqrt{7}\sqrt{3}+7\right)\sqrt{-\sqrt{2}\sqrt{-\sqrt{7}\sqrt{3}+5}}+28\,x\right)\\
&\quad-\frac{1}{56}\sqrt{14}\sqrt{-\sqrt{2}\sqrt{-\sqrt{7}\sqrt{3}+5}}\log\left(-\sqrt{14}\left(\sqrt{7}\sqrt{3}+7\right)\sqrt{-\sqrt{2}\sqrt{-\sqrt{7}\sqrt{3}+5}}+28\,x\right)
\end{aligned}$$

input
```
integrate((-x^4+1)/(x^8-5*x^4+1),x, algorithm="fricas")
```

output
```
-1/56*sqrt(14)*sqrt(sqrt(2)*sqrt(sqrt(7)*sqrt(3) + 5))*log(sqrt(14)*(sqrt(
7)*sqrt(3) - 7)*sqrt(sqrt(2)*sqrt(sqrt(7)*sqrt(3) + 5)) + 28*x) + 1/56*sqr
t(14)*sqrt(sqrt(2)*sqrt(sqrt(7)*sqrt(3) + 5))*log(-sqrt(14)*(sqrt(7)*sqrt(
3) - 7)*sqrt(sqrt(2)*sqrt(sqrt(7)*sqrt(3) + 5)) + 28*x) - 1/56*sqrt(14)*sq
rt(-sqrt(2)*sqrt(sqrt(7)*sqrt(3) + 5))*log(sqrt(14)*(sqrt(7)*sqrt(3) - 7)*
sqrt(-sqrt(2)*sqrt(sqrt(7)*sqrt(3) + 5)) + 28*x) + 1/56*sqrt(14)*sqrt(-sqr
t(2)*sqrt(sqrt(7)*sqrt(3) + 5))*log(-sqrt(14)*(sqrt(7)*sqrt(3) - 7)*sqrt(-
sqrt(2)*sqrt(sqrt(7)*sqrt(3) + 5)) + 28*x) + 1/56*sqrt(14)*sqrt(sqrt(2)*sq
rt(-sqrt(7)*sqrt(3) + 5))*log(sqrt(14)*(sqrt(7)*sqrt(3) + 7)*sqrt(sqrt(2)*
sqrt(-sqrt(7)*sqrt(3) + 5)) + 28*x) - 1/56*sqrt(14)*sqrt(sqrt(2)*sqrt(-sqr
t(7)*sqrt(3) + 5))*log(-sqrt(14)*(sqrt(7)*sqrt(3) + 7)*sqrt(sqrt(2)*sqrt(-
sqrt(7)*sqrt(3) + 5)) + 28*x) + 1/56*sqrt(14)*sqrt(-sqrt(2)*sqrt(-sqrt(7)*
sqrt(3) + 5))*log(sqrt(14)*(sqrt(7)*sqrt(3) + 7)*sqrt(-sqrt(2)*sqrt(-sqrt(
7)*sqrt(3) + 5)) + 28*x) - 1/56*sqrt(14)*sqrt(-sqrt(2)*sqrt(-sqrt(7)*sqrt(
3) + 5))*log(-sqrt(14)*(sqrt(7)*sqrt(3) + 7)*sqrt(-sqrt(2)*sqrt(-sqrt(7)*s
qrt(3) + 5)) + 28*x)
```

3.29.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.15

$$\int \frac{1-x^4}{1-5x^4+x^8}\,dx$$
$$= -\operatorname{RootSum}\left(157351936t^8 - 62720t^4 + 1, \left(t \mapsto t\log\left(50176t^5 - 24t + x\right)\right)\right)$$

input
```
integrate((-x**4+1)/(x**8-5*x**4+1),x)
```

output
```
-RootSum(157351936*_t**8 - 62720*_t**4 + 1, Lambda(_t, _t*log(50176*_t**5
- 24*_t + x)))
```

3.29.7 Maxima [F]

$$\int \frac{1-x^4}{1-5x^4+x^8}\,dx = \int -\frac{x^4-1}{x^8-5\,x^4+1}\,dx$$

input
```
integrate((-x^4+1)/(x^8-5*x^4+1),x, algorithm="maxima")
```

output
```
-integrate((x^4 - 1)/(x^8 - 5*x^4 + 1), x)
```

3.29.8 Giac [F]

$$\int \frac{1-x^4}{1-5x^4+x^8}\,dx = \int -\frac{x^4-1}{x^8-5\,x^4+1}\,dx$$

input
```
integrate((-x^4+1)/(x^8-5*x^4+1),x, algorithm="giac")
```

output
```
integrate(-(x^4 - 1)/(x^8 - 5*x^4 + 1), x)
```

3.29.9 Mupad [B] (verification not implemented)

Time = 8.57 (sec) , antiderivative size = 483, normalized size of antiderivative = 2.86

$$\int \frac{1-x^4}{1-5x^4+x^8}\,dx$$
$$= \frac{2^{3/4}\sqrt{7}\,\mathrm{atan}\left(\frac{405\,2^{3/4}\sqrt{7}\,x\left(5-\sqrt{21}\right)^{1/4}}{2\left(243\sqrt{2}\sqrt{5-\sqrt{21}}-54\sqrt{2}\sqrt{21}\sqrt{5-\sqrt{21}}\right)}-\frac{621\,2^{3/4}\sqrt{7}\sqrt{21}\,x\left(5-\sqrt{21}\right)^{1/4}}{14\left(243\sqrt{2}\sqrt{5-\sqrt{21}}-54\sqrt{2}\sqrt{21}\sqrt{5-\sqrt{21}}\right)}\right)\left(5-\sqrt{21}\right)^{1/4}}{28}$$
$$-\frac{2^{3/4}\sqrt{7}\,\mathrm{atan}\left(\frac{2^{3/4}\sqrt{7}\,x\left(5-\sqrt{21}\right)^{1/4}405\mathrm{i}}{2\left(243\sqrt{2}\sqrt{5-\sqrt{21}}-54\sqrt{2}\sqrt{21}\sqrt{5-\sqrt{21}}\right)}-\frac{2^{3/4}\sqrt{7}\sqrt{21}\,x\left(5-\sqrt{21}\right)^{1/4}621\mathrm{i}}{14\left(243\sqrt{2}\sqrt{5-\sqrt{21}}-54\sqrt{2}\sqrt{21}\sqrt{5-\sqrt{21}}\right)}\right)\left(5-\sqrt{21}\right)^{1/4}1\mathrm{i}}{28}$$
$$+\frac{2^{3/4}\sqrt{7}\,\mathrm{atan}\left(\frac{405\,2^{3/4}\sqrt{7}\,x\left(\sqrt{21}+5\right)^{1/4}}{2\left(243\sqrt{2}\sqrt{\sqrt{21}+5}+54\sqrt{2}\sqrt{21}\sqrt{\sqrt{21}+5}\right)}+\frac{621\,2^{3/4}\sqrt{7}\sqrt{21}\,x\left(\sqrt{21}+5\right)^{1/4}}{14\left(243\sqrt{2}\sqrt{\sqrt{21}+5}+54\sqrt{2}\sqrt{21}\sqrt{\sqrt{21}+5}\right)}\right)\left(\sqrt{21}+5\right)^{1/4}}{28}$$
$$-\frac{2^{3/4}\sqrt{7}\,\mathrm{atan}\left(\frac{2^{3/4}\sqrt{7}\,x\left(\sqrt{21}+5\right)^{1/4}405\mathrm{i}}{2\left(243\sqrt{2}\sqrt{\sqrt{21}+5}+54\sqrt{2}\sqrt{21}\sqrt{\sqrt{21}+5}\right)}+\frac{2^{3/4}\sqrt{7}\sqrt{21}\,x\left(\sqrt{21}+5\right)^{1/4}621\mathrm{i}}{14\left(243\sqrt{2}\sqrt{\sqrt{21}+5}+54\sqrt{2}\sqrt{21}\sqrt{\sqrt{21}+5}\right)}\right)\left(\sqrt{21}+5\right)^{1/4}1\mathrm{i}}{28}$$

input
```
int(-(x^4 - 1)/(x^8 - 5*x^4 + 1),x)
```

output
```
(2^(3/4)*7^(1/2)*atan((405*2^(3/4)*7^(1/2)*x*(5 - 21^(1/2))^(1/4))/(2*(243
*2^(1/2)*(5 - 21^(1/2))^(1/2) - 54*2^(1/2)*21^(1/2)*(5 - 21^(1/2))^(1/2)))
 - (621*2^(3/4)*7^(1/2)*21^(1/2)*x*(5 - 21^(1/2))^(1/4))/(14*(243*2^(1/2)*
(5 - 21^(1/2))^(1/2) - 54*2^(1/2)*21^(1/2)*(5 - 21^(1/2))^(1/2))))*(5 - 21
^(1/2))^(1/4))/28 - (2^(3/4)*7^(1/2)*atan((2^(3/4)*7^(1/2)*x*(5 - 21^(1/2)
)^(1/4)*405i)/(2*(243*2^(1/2)*(5 - 21^(1/2))^(1/2) - 54*2^(1/2)*21^(1/2)*(
5 - 21^(1/2))^(1/2))) - (2^(3/4)*7^(1/2)*21^(1/2)*x*(5 - 21^(1/2))^(1/4)*6
21i)/(14*(243*2^(1/2)*(5 - 21^(1/2))^(1/2) - 54*2^(1/2)*21^(1/2)*(5 - 21^(
1/2))^(1/2))))*(5 - 21^(1/2))^(1/4)*1i)/28 + (2^(3/4)*7^(1/2)*atan((405*2^
(3/4)*7^(1/2)*x*(21^(1/2) + 5)^(1/4))/(2*(243*2^(1/2)*(21^(1/2) + 5)^(1/2)
 + 54*2^(1/2)*21^(1/2)*(21^(1/2) + 5)^(1/2))) + (621*2^(3/4)*7^(1/2)*21^(1
/2)*x*(21^(1/2) + 5)^(1/4))/(14*(243*2^(1/2)*(21^(1/2) + 5)^(1/2) + 54*2^(
1/2)*21^(1/2)*(21^(1/2) + 5)^(1/2))))*(21^(1/2) + 5)^(1/4))/28 - (2^(3/4)*
7^(1/2)*atan((2^(3/4)*7^(1/2)*x*(21^(1/2) + 5)^(1/4)*405i)/(2*(243*2^(1/2)
*(21^(1/2) + 5)^(1/2) + 54*2^(1/2)*21^(1/2)*(21^(1/2) + 5)^(1/2))) + (2^(3
/4)*7^(1/2)*21^(1/2)*x*(21^(1/2) + 5)^(1/4)*621i)/(14*(243*2^(1/2)*(21^(1/
2) + 5)^(1/2) + 54*2^(1/2)*21^(1/2)*(21^(1/2) + 5)^(1/2))))*(21^(1/2) + 5)
^(1/4)*1i)/28
```

3.29. $\int \frac{1-x^4}{1-5x^4+x^8}\,dx$

3.30 $\int \frac{1-x^4}{1-6x^4+x^8} dx$

3.30.1 Optimal result

Integrand size = 20, antiderivative size = 125

$$\int \frac{1-x^4}{1-6x^4+x^8} dx = \frac{\arctan\left(\frac{x}{\sqrt{-1+\sqrt{2}}}\right)}{4\sqrt{2\left(-1+\sqrt{2}\right)}} + \frac{\arctan\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}} + \frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{-1+\sqrt{2}}}\right)}{4\sqrt{2\left(-1+\sqrt{2}\right)}} + \frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}$$

output
```
1/4*arctan(x/(2^(1/2)-1)^(1/2))/(-2+2*2^(1/2))^(1/2)+1/4*arctanh(x/(2^(1/2
)-1)^(1/2))/(-2+2*2^(1/2))^(1/2)+1/4*arctan(x/(1+2^(1/2))^(1/2))/(2+2*2^(1
/2))^(1/2)+1/4*arctanh(x/(1+2^(1/2))^(1/2))/(2+2*2^(1/2))^(1/2)
```

3.30.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.91

$$\int \frac{1-x^4}{1-6x^4+x^8} dx = \frac{\sqrt{1+\sqrt{2}}\arctan\left(\frac{x}{\sqrt{-1+\sqrt{2}}}\right) + \sqrt{-1+\sqrt{2}}\arctan\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right) + \sqrt{1+\sqrt{2}}\operatorname{arctanh}\left(\frac{x}{\sqrt{-1+\sqrt{2}}}\right) + \sqrt{-1+\sqrt{2}}}{4\sqrt{2}}$$

input
```
Integrate[(1 - x^4)/(1 - 6*x^4 + x^8),x]
```

output
```
(Sqrt[1 + Sqrt[2]]*ArcTan[x/Sqrt[-1 + Sqrt[2]]] + Sqrt[-1 + Sqrt[2]]*ArcTa
n[x/Sqrt[1 + Sqrt[2]]] + Sqrt[1 + Sqrt[2]]*ArcTanh[x/Sqrt[-1 + Sqrt[2]]] +
 Sqrt[-1 + Sqrt[2]]*ArcTanh[x/Sqrt[1 + Sqrt[2]]])/(4*Sqrt[2])
```

3.30.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1749, 1406, 216, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x^4}{x^8-6x^4+1}dx$$

$$\Big\downarrow \text{1749}$$

$$-\frac{1}{2}\int \frac{1}{x^4-2x^2-1}dx-\frac{1}{2}\int \frac{1}{x^4+2x^2-1}dx$$

$$\Big\downarrow \text{1406}$$

$$\frac{1}{2}\left(\frac{\int \frac{1}{x^2+\sqrt{2}-1}dx}{2\sqrt{2}}-\frac{\int \frac{1}{x^2-\sqrt{2}-1}dx}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\int \frac{1}{x^2+\sqrt{2}+1}dx}{2\sqrt{2}}-\frac{\int \frac{1}{x^2-\sqrt{2}+1}dx}{2\sqrt{2}}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\left(\frac{\arctan\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)}{2\sqrt{2\left(\sqrt{2}-1\right)}}-\frac{\int \frac{1}{x^2-\sqrt{2}-1}dx}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{2\sqrt{2\left(1+\sqrt{2}\right)}}-\frac{\int \frac{1}{x^2-\sqrt{2}+1}dx}{2\sqrt{2}}\right)$$

$$\Big\downarrow \text{220}$$

$$\frac{1}{2}\left(\frac{\arctan\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{2\sqrt{2\left(1+\sqrt{2}\right)}}+\frac{\text{arctanh}\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)}{2\sqrt{2\left(\sqrt{2}-1\right)}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)}{2\sqrt{2\left(\sqrt{2}-1\right)}}+\frac{\text{arctanh}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{2\sqrt{2\left(1+\sqrt{2}\right)}}\right)$$

input
```
Int[(1 - x^4)/(1 - 6*x^4 + x^8),x]
```

output
```
(ArcTan[x/Sqrt[1 + Sqrt[2]]]/(2*Sqrt[2*(1 + Sqrt[2])]) + ArcTanh[x/Sqrt[-1
 + Sqrt[2]]]/(2*Sqrt[2*(-1 + Sqrt[2])]))/2 + (ArcTan[x/Sqrt[-1 + Sqrt[2]]]
/(2*Sqrt[2*(-1 + Sqrt[2])]) + ArcTanh[x/Sqrt[1 + Sqrt[2]]]/(2*Sqrt[2*(1 +
Sqrt[2])]))/2
```

3.30.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

rule 1749
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e
+ q*x^(n/2) + x^n, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x^(n/2) +
 x^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[2*(d/e) - b/c,
0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

3.30.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.51

method	result
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(4\_Z^4-4\_Z^2-1\right)}\_R\ln\left(-2\_R^3+3\_R+x\right)\right)}{8}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(4\_Z^4+4\_Z^2-1\right)}\_R\ln\left(2\_R^3+3\_R+x\right)\right)}{8}$
default	$\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{8\sqrt{1+\sqrt{2}}}+\frac{\sqrt{2}\,\arctan\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)}{8\sqrt{\sqrt{2}-1}}+\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)}{8\sqrt{\sqrt{2}-1}}+\frac{\sqrt{2}\,\arctan\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{8\sqrt{1+\sqrt{2}}}$

input
```
int((-x^4+1)/(x^8-6*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/8*sum(_R*ln(-2*_R^3+3*_R+x),_R=RootOf(4*_Z^4-4*_Z^2-1))+1/8*sum(_R*ln(2*
_R^3+3*_R+x),_R=RootOf(4*_Z^4+4*_Z^2-1))
```

3.30.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(85) = 170$.

Time = 0.28 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.96

$$\begin{aligned}\int \frac{1-x^4}{1-6x^4+x^8}\,dx =& \frac{1}{16}\sqrt{2}\sqrt{\sqrt{2}-1}\log\left(\left(\sqrt{2}+1\right)\sqrt{\sqrt{2}-1}+x\right)\\ &-\frac{1}{16}\sqrt{2}\sqrt{\sqrt{2}-1}\log\left(-\left(\sqrt{2}+1\right)\sqrt{\sqrt{2}-1}+x\right)\\ &+\frac{1}{16}\sqrt{2}\sqrt{\sqrt{2}+1}\log\left(\sqrt{\sqrt{2}+1}\left(\sqrt{2}-1\right)+x\right)\\ &-\frac{1}{16}\sqrt{2}\sqrt{\sqrt{2}+1}\log\left(-\sqrt{\sqrt{2}+1}\left(\sqrt{2}-1\right)+x\right)\\ &+\frac{1}{16}\sqrt{2}\sqrt{-\sqrt{2}+1}\log\left(\left(\sqrt{2}+1\right)\sqrt{-\sqrt{2}+1}+x\right)\\ &-\frac{1}{16}\sqrt{2}\sqrt{-\sqrt{2}+1}\log\left(-\left(\sqrt{2}+1\right)\sqrt{-\sqrt{2}+1}+x\right)\\ &+\frac{1}{16}\sqrt{2}\sqrt{-\sqrt{2}-1}\log\left(\left(\sqrt{2}-1\right)\sqrt{-\sqrt{2}-1}+x\right)\\ &-\frac{1}{16}\sqrt{2}\sqrt{-\sqrt{2}-1}\log\left(-\left(\sqrt{2}-1\right)\sqrt{-\sqrt{2}-1}+x\right)\end{aligned}$$

input
```
integrate((-x^4+1)/(x^8-6*x^4+1),x, algorithm="fricas")
```

output
```
1/16*sqrt(2)*sqrt(sqrt(2) - 1)*log((sqrt(2) + 1)*sqrt(sqrt(2) - 1) + x) -
1/16*sqrt(2)*sqrt(sqrt(2) - 1)*log(-(sqrt(2) + 1)*sqrt(sqrt(2) - 1) + x) +
 1/16*sqrt(2)*sqrt(sqrt(2) + 1)*log(sqrt(sqrt(2) + 1)*(sqrt(2) - 1) + x) -
 1/16*sqrt(2)*sqrt(sqrt(2) + 1)*log(-sqrt(sqrt(2) + 1)*(sqrt(2) - 1) + x)
+ 1/16*sqrt(2)*sqrt(-sqrt(2) + 1)*log((sqrt(2) + 1)*sqrt(-sqrt(2) + 1) + x
) - 1/16*sqrt(2)*sqrt(-sqrt(2) + 1)*log(-(sqrt(2) + 1)*sqrt(-sqrt(2) + 1)
+ x) + 1/16*sqrt(2)*sqrt(-sqrt(2) - 1)*log((sqrt(2) - 1)*sqrt(-sqrt(2) - 1
) + x) - 1/16*sqrt(2)*sqrt(-sqrt(2) - 1)*log(-(sqrt(2) - 1)*sqrt(-sqrt(2)
- 1) + x)
```

3.30.6 Sympy [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.41

$$\int \frac{1-x^4}{1-6x^4+x^8}\,dx = -\operatorname{RootSum}\left(16384t^4-256t^2-1,\left(t\mapsto t\log\left(65536t^5-28t+x\right)\right)\right) - \operatorname{RootSum}\left(16384t^4+256t^2-1,\left(t\mapsto t\log\left(65536t^5-28t+x\right)\right)\right)$$

input
```
integrate((-x**4+1)/(x**8-6*x**4+1),x)
```

output
```
-RootSum(16384*_t**4 - 256*_t**2 - 1, Lambda(_t, _t*log(65536*_t**5 - 28*_
t + x))) - RootSum(16384*_t**4 + 256*_t**2 - 1, Lambda(_t, _t*log(65536*_t
**5 - 28*_t + x)))
```

3.30.7 Maxima [F]

$$\int \frac{1-x^4}{1-6x^4+x^8}\,dx = \int -\frac{x^4-1}{x^8-6\,x^4+1}\,dx$$

input
```
integrate((-x^4+1)/(x^8-6*x^4+1),x, algorithm="maxima")
```

output
```
-integrate((x^4 - 1)/(x^8 - 6*x^4 + 1), x)
```

3.30.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.08

$$\begin{aligned}\int \frac{1-x^4}{1-6x^4+x^8}\,dx = {} & \frac{1}{8}\sqrt{2\sqrt{2}-2}\arctan\left(\frac{x}{\sqrt{\sqrt{2}+1}}\right) \\ & + \frac{1}{8}\sqrt{2\sqrt{2}+2}\arctan\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right) \\ & + \frac{1}{16}\sqrt{2\sqrt{2}-2}\log\left(\left|x+\sqrt{\sqrt{2}+1}\right|\right) \\ & - \frac{1}{16}\sqrt{2\sqrt{2}-2}\log\left(\left|x-\sqrt{\sqrt{2}+1}\right|\right) \\ & + \frac{1}{16}\sqrt{2\sqrt{2}+2}\log\left(\left|x+\sqrt{\sqrt{2}-1}\right|\right) \\ & - \frac{1}{16}\sqrt{2\sqrt{2}+2}\log\left(\left|x-\sqrt{\sqrt{2}-1}\right|\right)\end{aligned}$$

input
```
integrate((-x^4+1)/(x^8-6*x^4+1),x, algorithm="giac")
```

output
```
1/8*sqrt(2*sqrt(2) - 2)*arctan(x/sqrt(sqrt(2) + 1)) + 1/8*sqrt(2*sqrt(2) +
 2)*arctan(x/sqrt(sqrt(2) - 1)) + 1/16*sqrt(2*sqrt(2) - 2)*log(abs(x + sqr
t(sqrt(2) + 1))) - 1/16*sqrt(2*sqrt(2) - 2)*log(abs(x - sqrt(sqrt(2) + 1))
) + 1/16*sqrt(2*sqrt(2) + 2)*log(abs(x + sqrt(sqrt(2) - 1))) - 1/16*sqrt(2
*sqrt(2) + 2)*log(abs(x - sqrt(sqrt(2) - 1)))
```

3.30.9 Mupad [B] (verification not implemented)

Time = 8.51 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.96

$$\int \frac{1-x^4}{1-6x^4+x^8}\,dx = -\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{x\,\sqrt{1-\sqrt{2}}\,4352\mathrm{i}}{3072\,\sqrt{2}-4352}-\frac{\sqrt{2}\,x\,\sqrt{1-\sqrt{2}}\,3072\mathrm{i}}{3072\,\sqrt{2}-4352}\right)\,\sqrt{1-\sqrt{2}}\,1\mathrm{i}}{8}$$
$$+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{x\,\sqrt{-\sqrt{2}-1}\,4352\mathrm{i}}{3072\,\sqrt{2}+4352}+\frac{\sqrt{2}\,x\,\sqrt{-\sqrt{2}-1}\,3072\mathrm{i}}{3072\,\sqrt{2}+4352}\right)\,\sqrt{-\sqrt{2}-1}\,1\mathrm{i}}{8}$$
$$+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{x\,\sqrt{\sqrt{2}-1}\,4352\mathrm{i}}{3072\,\sqrt{2}-4352}-\frac{\sqrt{2}\,x\,\sqrt{\sqrt{2}-1}\,3072\mathrm{i}}{3072\,\sqrt{2}-4352}\right)\,\sqrt{\sqrt{2}-1}\,1\mathrm{i}}{8}$$
$$-\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{x\,\sqrt{\sqrt{2}+1}\,4352\mathrm{i}}{3072\,\sqrt{2}+4352}+\frac{\sqrt{2}\,x\,\sqrt{\sqrt{2}+1}\,3072\mathrm{i}}{3072\,\sqrt{2}+4352}\right)\,\sqrt{\sqrt{2}+1}\,1\mathrm{i}}{8}$$

input
```
int(-(x^4 - 1)/(x^8 - 6*x^4 + 1),x)
```

output
```
(2^(1/2)*atan((x*(- 2^(1/2) - 1)^(1/2)*4352i)/(3072*2^(1/2) + 4352) + (2^(
1/2)*x*(- 2^(1/2) - 1)^(1/2)*3072i)/(3072*2^(1/2) + 4352))*(- 2^(1/2) - 1)
^(1/2)*1i)/8 - (2^(1/2)*atan((x*(1 - 2^(1/2))^(1/2)*4352i)/(3072*2^(1/2) -
 4352) - (2^(1/2)*x*(1 - 2^(1/2))^(1/2)*3072i)/(3072*2^(1/2) - 4352))*(1 -
 2^(1/2))^(1/2)*1i)/8 + (2^(1/2)*atan((x*(2^(1/2) - 1)^(1/2)*4352i)/(3072*
2^(1/2) - 4352) - (2^(1/2)*x*(2^(1/2) - 1)^(1/2)*3072i)/(3072*2^(1/2) - 43
52))*(2^(1/2) - 1)^(1/2)*1i)/8 - (2^(1/2)*atan((x*(2^(1/2) + 1)^(1/2)*4352
i)/(3072*2^(1/2) + 4352) + (2^(1/2)*x*(2^(1/2) + 1)^(1/2)*3072i)/(3072*2^(
1/2) + 4352))*(2^(1/2) + 1)^(1/2)*1i)/8
```

3.31 $\int \frac{-1+\sqrt{3}+2x^4}{1-x^4+x^8}\,dx$

3.31.1 Optimal result

Integrand size = 25, antiderivative size = 135

$$\int \frac{-1+\sqrt{3}+2x^4}{1-x^4+x^8}\,dx=-\frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2}}+\frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2}}-\frac{\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)}{2\sqrt{2}}+\frac{\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)}{2\sqrt{2}}$$

output
```
-1/2*arctan((-2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/2)))*2^(1
/2)+1/2*arctan((2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/2)))*2^
(1/2)-1/4*ln(1+x^2-x*(1/2*6^(1/2)-1/2*2^(1/2)))*2^(1/2)+1/4*ln(1+x^2+x*(1/
2*6^(1/2)-1/2*2^(1/2)))*2^(1/2)
```

3.31.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.53

$$\int \frac{-1+\sqrt{3}+2x^4}{1-x^4+x^8}\,dx=\frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{-\log(x-\#1)+\sqrt{3}\log(x-\#1)+2\log(x-\#1)\#1^4}{-\#1^3+2\#1^7}\&\right]$$

input
```
Integrate[(-1 + Sqrt[3] + 2*x^4)/(1 - x^4 + x^8),x]
```

output
```
RootSum[1 - #1^4 + #1^8 & , (-Log[x - #1] + Sqrt[3]*Log[x - #1] + 2*Log[x
- #1]*#1^4)/(-#1^3 + 2*#1^7) & ]/4
```

3.31.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.40, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {1753, 1475, 1083, 217, 1478, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^4+\sqrt{3}-1}{x^8-x^4+1}dx$$

$$\Big\downarrow \; 1753$$

$$\frac{\int \frac{\left(3-\sqrt{3}\right)x^2-\sqrt{3}+3}{x^4-\sqrt{3}x^2+1}dx}{2\sqrt{3}}+\frac{\int \frac{-\left(\left(3-\sqrt{3}\right)x^2\right)-\sqrt{3}+3}{x^4+\sqrt{3}x^2+1}dx}{2\sqrt{3}}$$

$$\Big\downarrow \; 1475$$

$$\frac{\frac{1}{2}\left(3-\sqrt{3}\right)\int \frac{1}{x^2-\sqrt{2+\sqrt{3}}x+1}dx+\frac{1}{2}\left(3-\sqrt{3}\right)\int \frac{1}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{3}}+\frac{\int \frac{-\left(\left(3-\sqrt{3}\right)x^2\right)-\sqrt{3}+3}{x^4+\sqrt{3}x^2+1}dx}{2\sqrt{3}}$$

$$\Big\downarrow \; 1083$$

$$\frac{\int \frac{-\left(\left(3-\sqrt{3}\right)x^2\right)-\sqrt{3}+3}{x^4+\sqrt{3}x^2+1}dx}{2\sqrt{3}}+\frac{-\left(\left(3-\sqrt{3}\right)\int \frac{1}{-\left(2x-\sqrt{2+\sqrt{3}}\right)^2+\sqrt{3}-2}d\left(2x-\sqrt{2+\sqrt{3}}\right)\right)-\left(3-\sqrt{3}\right)\int \frac{1}{-\left(2x+\sqrt{2+\sqrt{3}}\right)^2+\sqrt{3}-2}d\left(2x+\sqrt{2+\sqrt{3}}\right)}{2\sqrt{3}}$$

$$\Big\downarrow \; 217$$

$$\frac{\int \frac{-\left(\left(3-\sqrt{3}\right)x^2\right)-\sqrt{3}+3}{x^4+\sqrt{3}x^2+1}dx}{2\sqrt{3}}+\frac{\frac{\left(3-\sqrt{3}\right)\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2-\sqrt{3}}}+\frac{\left(3-\sqrt{3}\right)\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}$$

$$\Big\downarrow \; 1478$$

$$\frac{-\sqrt{\frac{3}{2}}\int -\frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx-\sqrt{\frac{3}{2}}\int -\frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{3}}+\frac{\frac{\left(3-\sqrt{3}\right)\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2-\sqrt{3}}}+\frac{\left(3-\sqrt{3}\right)\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{\frac{3}{2}}\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx+\sqrt{\frac{3}{2}}\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{3}}+\frac{\frac{\left(3-\sqrt{3}\right)\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2-\sqrt{3}}}+\frac{\left(3-\sqrt{3}\right)\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}$$

$$\Big\downarrow \text{1103}$$

$$\frac{\frac{\left(3-\sqrt{3}\right)\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2-\sqrt{3}}}+\frac{\left(3-\sqrt{3}\right)\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}+\frac{\sqrt{\frac{3}{2}}\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)-\sqrt{\frac{3}{2}}\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{2\sqrt{3}}$$

input
```
Int[(-1 + Sqrt[3] + 2*x^4)/(1 - x^4 + x^8),x]
```

output
```
(((3 - Sqrt[3])*ArcTan[(-Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]])/Sqrt
[2 - Sqrt[3]] + ((3 - Sqrt[3])*ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - S
qrt[3]]])/Sqrt[2 - Sqrt[3]])/(2*Sqrt[3]) + (-(Sqrt[3/2]*Log[1 - Sqrt[2 - S
qrt[3]]*x + x^2]) + Sqrt[3/2]*Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2])/(2*Sqrt[
3])
```

3.31.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

rule 1753
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c
*q*r)   Int[(d*r - (d - e*q)*x^(n/2))/(q - r*x^(n/2) + x^n), x], x] + Simp[
1/(2*c*q*r)   Int[(d*r + (d - e*q)*x^(n/2))/(q + r*x^(n/2) + x^n), x], x]]]
 /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[n/2, 0] && NegQ[b^2 - 4*a*c]
```

3.31. $\int \frac{-1+\sqrt{3}+2x^4}{1-x^4+x^8}\,dx$

3.31.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.35

method	result
default	$\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^{8}-\_Z^{4}+1\right)}\frac{\left(-1+2\_R^{4}+\sqrt{3}\right)\ln\left(x-\_R\right)}{2\_R^{7}-\_R^{3}}\right)}{4}$
risch	$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\,x}{\sqrt{3}-1}\right)}{2}+\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\,x^{3}}{\sqrt{3}-1}-\frac{\sqrt{2}\,x}{\sqrt{3}-1}+\frac{\sqrt{3}\,\sqrt{2}\,x}{2}-\frac{x\sqrt{2}}{2}\right)}{2}+\frac{\sqrt{2}\,\ln\left(2x^{2}+\left(\sqrt{3}\,\sqrt{2}-\sqrt{2}\right)x+2\right)}{4}-\frac{\sqrt{2}\,\ln\left(2x^{2}+\left(-\sqrt{3}\,\sqrt{2}\right.\right.}{4}$

input
```
int((-1+2*x^4+3^(1/2))/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(1/(2*_R^7-_R^3)*(-1+2*_R^4+3^(1/2))*ln(x-_R),_R=RootOf(_Z^8-_Z^4+1
))
```

3.31.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.13

$$\int\frac{-1+\sqrt{3}+2x^4}{1-x^4+x^8}\,dx$$
$$=\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{3}\sqrt{2}x^3+\frac{1}{2}\sqrt{2}(x^3-2x)\right)+\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{3}\sqrt{2}x+\frac{1}{2}\sqrt{2}x\right)$$
$$+\frac{1}{4}\sqrt{2}\log\left(\frac{x^8+4x^6+5x^4+4x^2-\sqrt{2}(x^7+4x^5+4x^3+x)-\sqrt{3}(2x^6+4x^4+2x^2-\sqrt{2}(x^7+2x^5}{x^8-x^4+1}\right.$$

input
```
integrate((-1+2*x^4+3^(1/2))/(x^8-x^4+1),x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*arctan(1/2*sqrt(3)*sqrt(2)*x^3 + 1/2*sqrt(2)*(x^3 - 2*x)) + 1/
2*sqrt(2)*arctan(1/2*sqrt(3)*sqrt(2)*x + 1/2*sqrt(2)*x) + 1/4*sqrt(2)*log(
(x^8 + 4*x^6 + 5*x^4 + 4*x^2 - sqrt(2)*(x^7 + 4*x^5 + 4*x^3 + x) - sqrt(3)
*(2*x^6 + 4*x^4 + 2*x^2 - sqrt(2)*(x^7 + 2*x^5 + 2*x^3 + x)) + 1)/(x^8 - x
^4 + 1))
```

3.31.6 Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.21

$$\int \frac{-1+\sqrt{3}+2x^4}{1-x^4+x^8}\,dx = \frac{\sqrt{2}\cdot\left(2\,\mathrm{atan}\left(x\left(\frac{\sqrt{6}}{1+\sqrt{3}}+\frac{2\sqrt{2}}{1+\sqrt{3}}\right)\right)+2\,\mathrm{atan}\left(x^3\left(\frac{\sqrt{6}}{1+\sqrt{3}}+\frac{2\sqrt{2}}{1+\sqrt{3}}\right)-\sqrt{2}x\right)\right)}{4} - \frac{\sqrt{2}\log\left(x^2-\frac{\sqrt{2}x\left(\frac{2}{\sqrt{3}+2}+\frac{2\sqrt{3}}{\sqrt{3}+2}\right)}{4}+1\right)}{4} + \frac{\sqrt{2}\log\left(x^2+\frac{\sqrt{2}x\left(\frac{2}{\sqrt{3}+2}+\frac{2\sqrt{3}}{\sqrt{3}+2}\right)}{4}+1\right)}{4}$$

input
```
integrate((-1+2*x**4+3**(1/2))/(x**8-x**4+1),x)
```

output
```
sqrt(2)*(2*atan(x*(sqrt(6)/(1 + sqrt(3)) + 2*sqrt(2)/(1 + sqrt(3)))) + 2*a
tan(x**3*(sqrt(6)/(1 + sqrt(3)) + 2*sqrt(2)/(1 + sqrt(3))) - sqrt(2)*x))/4
 - sqrt(2)*log(x**2 - sqrt(2)*x*(2/(sqrt(3) + 2) + 2*sqrt(3)/(sqrt(3) + 2)
)/4 + 1)/4 + sqrt(2)*log(x**2 + sqrt(2)*x*(2/(sqrt(3) + 2) + 2*sqrt(3)/(sq
rt(3) + 2))/4 + 1)/4
```

3.31.7 Maxima [F]

$$\int \frac{-1+\sqrt{3}+2x^4}{1-x^4+x^8}\,dx = \int \frac{2\,x^4+\sqrt{3}-1}{x^8-x^4+1}\,dx$$

input
```
integrate((-1+2*x^4+3^(1/2))/(x^8-x^4+1),x, algorithm="maxima")
```

output
```
integrate((2*x^4 + sqrt(3) - 1)/(x^8 - x^4 + 1), x)
```

3.31.8 Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.79

$$\int \frac{-1+\sqrt{3}+2x^4}{1-x^4+x^8}\,dx = \frac{1}{2}\sqrt{2}\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{2}\sqrt{2}\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right) + \frac{1}{4}\sqrt{2}\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right) - \frac{1}{4}\sqrt{2}\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)$$

input
```
integrate((-1+2*x^4+3^(1/2))/(x^8-x^4+1),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*arctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(2))) + 1/2*sq
rt(2)*arctan((4*x - sqrt(6) - sqrt(2))/(sqrt(6) - sqrt(2))) + 1/4*sqrt(2)*
log(x^2 + 1/2*x*(sqrt(6) - sqrt(2)) + 1) - 1/4*sqrt(2)*log(x^2 - 1/2*x*(sq
rt(6) - sqrt(2)) + 1)
```

3.31.9 Mupad [B] (verification not implemented)

Time = 8.80 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.99

$$\int \frac{-1+\sqrt{3}+2x^4}{1-x^4+x^8}\,dx = \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{72\sqrt{2}\,x}{144\sqrt{3}-144\sqrt{3}\,x^2-288\,x^2+288}+\frac{72\sqrt{2}\sqrt{3}\,x}{144\sqrt{3}-144\sqrt{3}\,x^2-288\,x^2+288}\right)}{2} + \frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{72\sqrt{2}\,x}{144\sqrt{3}+144\sqrt{3}\,x^2+288\,x^2+288}+\frac{72\sqrt{2}\sqrt{3}\,x}{144\sqrt{3}+144\sqrt{3}\,x^2+288\,x^2+288}\right)}{2}$$

input
```
int((3^(1/2) + 2*x^4 - 1)/(x^8 - x^4 + 1),x)
```

output
```
(2^(1/2)*atan((72*2^(1/2)*x)/(144*3^(1/2) - 144*3^(1/2)*x^2 - 288*x^2 + 28
8) + (72*2^(1/2)*3^(1/2)*x)/(144*3^(1/2) - 144*3^(1/2)*x^2 - 288*x^2 + 288
)))/2 + (2^(1/2)*atanh((72*2^(1/2)*x)/(144*3^(1/2) + 144*3^(1/2)*x^2 + 288
*x^2 + 288) + (72*2^(1/2)*3^(1/2)*x)/(144*3^(1/2) + 144*3^(1/2)*x^2 + 288*
x^2 + 288)))/2
```

3.32 $\int \frac{1+\left(1+\sqrt{3}\right)x^4}{1-x^4+x^8}\,dx$

3.32.1 Optimal result

Integrand size = 26, antiderivative size = 164

$$\int \frac{1+\left(1+\sqrt{3}\right)x^4}{1-x^4+x^8}\,dx = -\frac{1}{2}\sqrt{2+\sqrt{3}}\arctan\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)+\frac{1}{2}\sqrt{2+\sqrt{3}}\arctan\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)-\frac{1}{4}\sqrt{2+\sqrt{3}}\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)+\frac{1}{4}\sqrt{2+\sqrt{3}}\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)$$

output
```
-1/2*arctan((-2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2
*6^(1/2)+1/2*2^(1/2))+1/2*arctan((2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2
)-1/2*2^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))-1/4*ln(1+x^2-x*(1/2*6^(1/2)-1/2*
2^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))+1/4*ln(1+x^2+x*(1/2*6^(1/2)-1/2*2^(1/2
)))*(1/2*6^(1/2)+1/2*2^(1/2))
```

3.32.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.44

$$\int \frac{1+\left(1+\sqrt{3}\right)x^4}{1-x^4+x^8}\,dx = \frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{\log(x-\#1)+\log(x-\#1)\#1^4+\sqrt{3}\log(x-\#1)\#1^4}{-\#1^3+2\#1^7}\&\right]$$

input
```
Integrate[(1 + (1 + Sqrt[3])*x^4)/(1 - x^4 + x^8),x]
```

output
```
RootSum[1 - #1^4 + #1^8 & , (Log[x - #1] + Log[x - #1]*#1^4 + Sqrt[3]*Log[
x - #1]*#1^4)/(-#1^3 + 2*#1^7) & ]/4
```

3.32.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {1753, 27, 1475, 1083, 217, 1478, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(1+\sqrt{3}\right)x^4+1}{x^8-x^4+1}dx$$

$$\Big\downarrow \text{1753}$$

$$\frac{\int \frac{\sqrt{3}(x^2+1)}{x^4-\sqrt{3}x^2+1}dx}{2\sqrt{3}}+\frac{\int \frac{\sqrt{3}(1-x^2)}{x^4+\sqrt{3}x^2+1}dx}{2\sqrt{3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \frac{x^2+1}{x^4-\sqrt{3}x^2+1}dx+\frac{1}{2}\int \frac{1-x^2}{x^4+\sqrt{3}x^2+1}dx$$

$$\Big\downarrow \text{1475}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2-\sqrt{2+\sqrt{3}}x+1}dx+\frac{1}{2}\int \frac{1}{x^2+\sqrt{2+\sqrt{3}}x+1}dx\right)+\frac{1}{2}\int \frac{1-x^2}{x^4+\sqrt{3}x^2+1}dx$$

$$\Big\downarrow \ 1083$$

$$\frac{1}{2}\int \frac{1-x^2}{x^4+\sqrt{3}x^2+1}dx+$$
$$\frac{1}{2}\left(-\int \frac{1}{-\left(2x-\sqrt{2+\sqrt{3}}\right)^2+\sqrt{3}-2}d\left(2x-\sqrt{2+\sqrt{3}}\right)-\int \frac{1}{-\left(2x+\sqrt{2+\sqrt{3}}\right)^2+\sqrt{3}-2}d\left(2x+\sqrt{2+\sqrt{}\right.\right.$$

$$\Big\downarrow \ 217$$

$$\frac{1}{2}\int \frac{1-x^2}{x^4+\sqrt{3}x^2+1}dx+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2-\sqrt{3}}}+\frac{\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2-\sqrt{3}}}\right)$$

$$\Big\downarrow \ 1478$$

$$\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}-\frac{\int -\frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}\right)+$$
$$\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2-\sqrt{3}}}+\frac{\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2-\sqrt{3}}}\right)$$

$$\Big\downarrow \ 25$$

$$\frac{1}{2}\left(\frac{\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2-\sqrt{3}}}+\frac{\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2-\sqrt{3}}}\right)$$

$$\Big\downarrow \ 1103$$

$$\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2-\sqrt{3}}}+\frac{\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2-\sqrt{3}}}\right)+$$
$$\frac{1}{2}\left(\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)}{2\sqrt{2-\sqrt{3}}}-\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{2\sqrt{2-\sqrt{3}}}\right)$$

input
```
Int[(1 + (1 + Sqrt[3])*x^4)/(1 - x^4 + x^8),x]
```

output
```
(ArcTan[(-Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/Sqrt[2 - Sqrt[3]] +
ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/Sqrt[2 - Sqrt[3]])/2 +
 (-1/2*Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2]/Sqrt[2 - Sqrt[3]] + Log[1 + Sqrt
[2 - Sqrt[3]]*x + x^2]/(2*Sqrt[2 - Sqrt[3]]))/2
```

3.32. $\int \frac{1+\left(1+\sqrt{3}\right)x^4}{1-x^4+x^8}\,dx$

3.32.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

3.32. $\int \frac{1+\left(1+\sqrt{3}\right)x^4}{1-x^4+x^8}\,dx$

rule 1753
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c
*q*r)   Int[(d*r - (d - e*q)*x^(n/2))/(q - r*x^(n/2) + x^n), x], x] + Simp[
1/(2*c*q*r)   Int[(d*r + (d - e*q)*x^(n/2))/(q + r*x^(n/2) + x^n), x], x]]]
 /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[n/2, 0] && NegQ[b^2 - 4*a*c]
```

3.32.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.21 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.38

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}-\_Z^{4}+1\right)}\frac{\left(2\_R^{4}+2\sqrt{3}\,\_R^{4}+\left(1+\sqrt{3}\right)\left(\sqrt{3}-1\right)\right)\ln\left(x-\_R\right)}{2\_R^{7}-\_R^{3}}\right)}{8}$	62

input
```
int((1+x^4*(1+3^(1/2)))/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/8*sum(1/(2*_R^7-_R^3)*(2*_R^4+2*3^(1/2)*_R^4+(1+3^(1/2))*(3^(1/2)-1))*ln
(x-_R),_R=RootOf(_Z^8-_Z^4+1))
```

3.32.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.96

$$\int \frac{1+\left(1+\sqrt{3}\right)x^4}{1-x^4+x^8}\,dx$$
$$= -\frac{1}{2}\sqrt{\sqrt{3}+2}\arctan\left(-\left(x^3-\sqrt{3}x+x\right)\sqrt{\sqrt{3}+2}\right)+\frac{1}{2}\sqrt{\sqrt{3}+2}\arctan\left(x\sqrt{\sqrt{3}+2}\right)$$
$$+\frac{1}{4}\sqrt{\sqrt{3}+2}\log\left(\frac{x^8+4\,x^6+5\,x^4+4\,x^2-2\,\sqrt{3}(x^6+2\,x^4+x^2)+2\,\left(2\,x^7+5\,x^5+5\,x^3-\sqrt{3}(x^7+3\,x^5}{x^8-x^4+1}\right.$$

input
```
integrate((1+x^4*(1+3^(1/2)))/(x^8-x^4+1),x, algorithm="fricas")
```

output
```
-1/2*sqrt(sqrt(3) + 2)*arctan(-(x^3 - sqrt(3)*x + x)*sqrt(sqrt(3) + 2)) +
1/2*sqrt(sqrt(3) + 2)*arctan(x*sqrt(sqrt(3) + 2)) + 1/4*sqrt(sqrt(3) + 2)*
log((x^8 + 4*x^6 + 5*x^4 + 4*x^2 - 2*sqrt(3)*(x^6 + 2*x^4 + x^2) + 2*(2*x^
7 + 5*x^5 + 5*x^3 - sqrt(3)*(x^7 + 3*x^5 + 3*x^3 + x) + 2*x)*sqrt(sqrt(3)
+ 2) + 1)/(x^8 - x^4 + 1))
```

3.32.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{1+\left(1+\sqrt{3}\right)x^4}{1-x^4+x^8}\,dx = \text{Exception raised: PolynomialError}$$

input
```
integrate((1+x**4*(1+3**(1/2)))/(x**8-x**4+1),x)
```

output
```
Exception raised: PolynomialError >> 1/(239467000838037598029035598269032
5810751919767151652506842000402903189411594 24*_t**88 + 138256337395873345762
80342370533073164132612616075147807283055647306312738 4064*sqrt(3)*_t**88
- 5732624312622
```

3.32.7 Maxima [F]

$$\int \frac{1+\left(1+\sqrt{3}\right)x^4}{1-x^4+x^8}\,dx = \int \frac{x^4\left(\sqrt{3}+1\right)+1}{x^8-x^4+1}\,dx$$

input
```
integrate((1+x^4*(1+3^(1/2)))/(x^8-x^4+1),x, algorithm="maxima")
```

output
```
integrate((x^4*(sqrt(3) + 1) + 1)/(x^8 - x^4 + 1), x)
```

3.32.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.75

$$\int \frac{1+\left(1+\sqrt{3}\right)x^4}{1-x^4+x^8}\,dx = \frac{1}{4}\left(\sqrt{6}+\sqrt{2}\right)\arctan\left(\frac{4\,x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right) + \frac{1}{4}\left(\sqrt{6}+\sqrt{2}\right)\arctan\left(\frac{4\,x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right) + \frac{1}{8}\left(\sqrt{6}+\sqrt{2}\right)\log\left(x^2+\frac{1}{2}\,x\left(\sqrt{6}-\sqrt{2}\right)+1\right) - \frac{1}{8}\left(\sqrt{6}+\sqrt{2}\right)\log\left(x^2-\frac{1}{2}\,x\left(\sqrt{6}-\sqrt{2}\right)+1\right)$$

input
```
integrate((1+x^4*(1+3^(1/2)))/(x^8-x^4+1),x, algorithm="giac")
```

output
```
1/4*(sqrt(6) + sqrt(2))*arctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(2
))) + 1/4*(sqrt(6) + sqrt(2))*arctan((4*x - sqrt(6) - sqrt(2))/(sqrt(6) -
sqrt(2))) + 1/8*(sqrt(6) + sqrt(2))*log(x^2 + 1/2*x*(sqrt(6) - sqrt(2)) +
1) - 1/8*(sqrt(6) + sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) - sqrt(2)) + 1)
```

3.32.9 Mupad [B] (verification not implemented)

Time = 8.70 (sec) , antiderivative size = 1, normalized size of antiderivative = 0.01

$$\int \frac{1+\left(1+\sqrt{3}\right)x^4}{1-x^4+x^8}\,dx = 0$$

input
```
int((x^4*(3^(1/2) + 1) + 1)/(x^8 - x^4 + 1),x)
```

output
```
0
```

3.33 $\int \frac{3-2\sqrt{3}+\left(-3+\sqrt{3}\right)x^4}{1-x^4+x^8}\,dx$

3.33.1 Optimal result

Integrand size = 33, antiderivative size = 180

$$\begin{aligned}\int \frac{3-2\sqrt{3}+\left(-3+\sqrt{3}\right)x^4}{1-x^4+x^8}\,dx &= \frac{1}{2}\sqrt{3\left(2-\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)\\ &\quad-\frac{1}{2}\sqrt{3\left(2-\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)\\ &\quad+\frac{1}{4}\sqrt{3\left(2-\sqrt{3}\right)}\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)\\ &\quad-\frac{1}{4}\sqrt{3\left(2-\sqrt{3}\right)}\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)\end{aligned}$$

output
```
1/2*arctan((-2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/2)))*(3/2*
2^(1/2)-1/2*6^(1/2))-1/2*arctan((2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)
-1/2*2^(1/2)))*(3/2*2^(1/2)-1/2*6^(1/2))+1/4*ln(1+x^2-x*(1/2*6^(1/2)-1/2*2
^(1/2)))*(3/2*2^(1/2)-1/2*6^(1/2))-1/4*ln(1+x^2+x*(1/2*6^(1/2)-1/2*2^(1/2)
))*(3/2*2^(1/2)-1/2*6^(1/2))
```

3.33.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.49

$$\int \frac{3-2\sqrt{3}+\left(-3+\sqrt{3}\right)x^4}{1-x^4+x^8}\,dx = \frac{1}{4}\text{RootSum}\Bigg[1-\#1^4 +\#1^8\&, \frac{3\log(x-\#1)-2\sqrt{3}\log(x-\#1)-3\log(x-\#1)\#1^4+\sqrt{3}\log(x-\#1)\#1^4}{-\#1^3+2\#1^7}\&\Bigg]$$

input
```
Integrate[(3 - 2*Sqrt[3] + (-3 + Sqrt[3])*x^4)/(1 - x^4 + x^8),x]
```

output
```
RootSum[1 - #1^4 + #1^8 & , (3*Log[x - #1] - 2*Sqrt[3]*Log[x - #1] - 3*Log
[x - #1]*#1^4 + Sqrt[3]*Log[x - #1]*#1^4)/(-#1^3 + 2*#1^7) & ]/4
```

3.33.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {1753, 27, 1475, 1083, 217, 1478, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(\sqrt{3}-3\right)x^4-2\sqrt{3}+3}{x^8-x^4+1}\,dx$$

$$\Big\downarrow \; 1753$$

$$\frac{\int -\frac{3\left(\left(2-\sqrt{3}\right)x^2-\sqrt{3}+2\right)}{x^4-\sqrt{3}x^2+1}dx}{2\sqrt{3}}+\frac{\int -\frac{3\left(-\left(\left(2-\sqrt{3}\right)x^2\right)-\sqrt{3}+2\right)}{x^4+\sqrt{3}x^2+1}dx}{2\sqrt{3}}$$

$$\Big\downarrow \; 27$$

$$-\frac{1}{2}\sqrt{3}\int \frac{\left(2-\sqrt{3}\right)x^2-\sqrt{3}+2}{x^4-\sqrt{3}x^2+1}dx-\frac{1}{2}\sqrt{3}\int \frac{-\left(\left(2-\sqrt{3}\right)x^2\right)-\sqrt{3}+2}{x^4+\sqrt{3}x^2+1}dx$$

$$\Big\downarrow \; 1475$$

$$-\frac{1}{2}\sqrt{3}\left(\frac{1}{2}\left(2-\sqrt{3}\right)\int \frac{1}{x^2-\sqrt{2+\sqrt{3}}x+1}dx+\frac{1}{2}\left(2-\sqrt{3}\right)\int \frac{1}{x^2+\sqrt{2+\sqrt{3}}x+1}dx\right)-\frac{1}{2}\sqrt{3}\int \frac{-\left(\left(2-\sqrt{3}\right)x^2\right)-\sqrt{3}+2}{x^4+\sqrt{3}x^2+1}dx$$

$$\Big\downarrow \; 1083$$

$$-\frac{1}{2}\sqrt{3}\int \frac{-\left(\left(2-\sqrt{3}\right)x^2\right)-\sqrt{3}+2}{x^4+\sqrt{3}x^2+1}dx-\frac{1}{2}\sqrt{3}\left(-\left(\left(2-\sqrt{3}\right)\int \frac{1}{-\left(2x-\sqrt{2+\sqrt{3}}\right)^2+\sqrt{3}-2}d\left(2x-\sqrt{2+\sqrt{3}}\right)\right)-\left(2-\sqrt{3}\right)\int \frac{1}{-\left(2x+\sqrt{2+\sqrt{3}}\right)^2}\right.$$

$$\Big\downarrow \; 217$$

$$-\frac{1}{2}\sqrt{3}\int \frac{-\left(\left(2-\sqrt{3}\right)x^2\right)-\sqrt{3}+2}{x^4+\sqrt{3}x^2+1}dx-\frac{1}{2}\sqrt{3}\left(\sqrt{2-\sqrt{3}}\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)+\sqrt{2-\sqrt{3}}\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)\right)$$

$$\Big\downarrow \; 1478$$

$$-\frac{1}{2}\sqrt{3}\left(-\frac{1}{2}\sqrt{2-\sqrt{3}}\int -\frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx-\frac{1}{2}\sqrt{2-\sqrt{3}}\int -\frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx\right)-\frac{1}{2}\sqrt{3}\left(\sqrt{2-\sqrt{3}}\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)+\sqrt{2-\sqrt{3}}\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)\right)$$

$$\Big\downarrow \; 25$$

$$-\frac{1}{2}\sqrt{3}\left(\frac{1}{2}\sqrt{2-\sqrt{3}}\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx+\frac{1}{2}\sqrt{2-\sqrt{3}}\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx\right)-\frac{1}{2}\sqrt{3}\left(\sqrt{2-\sqrt{3}}\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)+\sqrt{2-\sqrt{3}}\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)\right)$$

$$\Big\downarrow \; 1103$$

$$-\frac{1}{2}\sqrt{3}\left(\sqrt{2-\sqrt{3}}\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)+\sqrt{2-\sqrt{3}}\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)\right)-\frac{1}{2}\sqrt{3}\left(\frac{1}{2}\sqrt{2-\sqrt{3}}\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)-\frac{1}{2}\sqrt{2-\sqrt{3}}\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)\right)$$

input
```
Int[(3 - 2*Sqrt[3] + (-3 + Sqrt[3])*x^4)/(1 - x^4 + x^8),x]
```

3.33. $\int \frac{3-2\sqrt{3}+\left(-3+\sqrt{3}\right)x^4}{1-x^4+x^8}\,dx$

output
```
-1/2*(Sqrt[3]*(Sqrt[2 - Sqrt[3]]*ArcTan[(-Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2
- Sqrt[3]]] + Sqrt[2 - Sqrt[3]]*ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 -
Sqrt[3]]])) - (Sqrt[3]*(-1/2*(Sqrt[2 - Sqrt[3]]*Log[1 - Sqrt[2 - Sqrt[3]]*
x + x^2]) + (Sqrt[2 - Sqrt[3]]*Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2])/2))/2
```

3.33.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

rule 1753
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c
*q*r)   Int[(d*r - (d - e*q)*x^(n/2))/(q - r*x^(n/2) + x^n), x], x] + Simp[
1/(2*c*q*r)   Int[(d*r + (d - e*q)*x^(n/2))/(q + r*x^(n/2) + x^n), x], x]]]
 /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[n/2, 0] && NegQ[b^2 - 4*a*c]
```

3.33.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.18 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.34

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}-\_Z^{4}+1\right)}\frac{\left(-6\_R^{4}+2\sqrt{3}\,\_R^{4}+\left(-3+\sqrt{3}\right)\left(\sqrt{3}-1\right)\right)\ln\left(x-\_R\right)}{2\_R^{7}-\_R^{3}}\right)}{8}$	62

input
```
int((3+x^4*(-3+3^(1/2))-2*3^(1/2))/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/8*sum(1/(2*_R^7-_R^3)*(-6*_R^4+2*3^(1/2)*_R^4+(-3+3^(1/2))*(3^(1/2)-1))*
ln(x-_R),_R=RootOf(_Z^8-_Z^4+1))
```

3.33.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.01

$$\int \frac{3-2\sqrt{3}+\left(-3+\sqrt{3}\right)x^4}{1-x^4+x^8}\,dx$$
$$= -\frac{1}{2}\sqrt{-3\sqrt{3}+6}\arctan\left(\frac{1}{3}\left(3\,x^3+\sqrt{3}(2\,x^3-x)-3\,x\right)\sqrt{-3\sqrt{3}+6}\right)$$
$$-\frac{1}{2}\sqrt{-3\sqrt{3}+6}\arctan\left(\frac{1}{3}\left(2\sqrt{3}x+3\,x\right)\sqrt{-3\sqrt{3}+6}\right)$$
$$+\frac{1}{4}\sqrt{-3\sqrt{3}+6}\log\left(\frac{3\,x^8+12\,x^6+15\,x^4+12\,x^2-6\sqrt{3}(x^6+2\,x^4+x^2)+2\left(3\,x^5+3\,x^3-\sqrt{3}(x^7+x^5\right.}{x^8-x^4+1}\right.$$

input
```
integrate((3+x^4*(-3+3^(1/2))-2*3^(1/2))/(x^8-x^4+1),x, algorithm="fricas"
)
```

output
```
-1/2*sqrt(-3*sqrt(3) + 6)*arctan(1/3*(3*x^3 + sqrt(3)*(2*x^3 - x) - 3*x)*s
qrt(-3*sqrt(3) + 6)) - 1/2*sqrt(-3*sqrt(3) + 6)*arctan(1/3*(2*sqrt(3)*x +
3*x)*sqrt(-3*sqrt(3) + 6)) + 1/4*sqrt(-3*sqrt(3) + 6)*log((3*x^8 + 12*x^6
+ 15*x^4 + 12*x^2 - 6*sqrt(3)*(x^6 + 2*x^4 + x^2) + 2*(3*x^5 + 3*x^3 - sqr
t(3)*(x^7 + x^5 + x^3 + x))*sqrt(-3*sqrt(3) + 6) + 3)/(x^8 - x^4 + 1))
```

3.33.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{3-2\sqrt{3}+\left(-3+\sqrt{3}\right)x^4}{1-x^4+x^8}\,dx = \text{Exception raised: PolynomialError}$$

input
```
integrate((3+x**4*(-3+3**(1/2))-2*3**(1/2))/(x**8-x**4+1),x)
```

output
```
Exception raised: PolynomialError >> 1/(-369443695440637751966679695 36*_t*
*32 + 213298417013062322820533452 80*sqrt(3)*_t**32 - 167111083173036783803
087978496*sqrt(3)*_t**28 + 289444886563568182740740210688*_t**28 - 9921139
603646460044679
```

3.33.7 Maxima [F]

$$\int \frac{3-2\sqrt{3}+\left(-3+\sqrt{3}\right)x^4}{1-x^4+x^8}\,dx=\int \frac{x^4\left(\sqrt{3}-3\right)-2\sqrt{3}+3}{x^8-x^4+1}\,dx$$

input
```
integrate((3+x^4*(-3+3^(1/2))-2*3^(1/2))/(x^8-x^4+1),x, algorithm="maxima"
)
```

output
```
integrate((x^4*(sqrt(3) - 3) - 2*sqrt(3) + 3)/(x^8 - x^4 + 1), x)
```

3.33.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.73

$$\begin{aligned}\int \frac{3-2\sqrt{3}+\left(-3+\sqrt{3}\right)x^4}{1-x^4+x^8}\,dx=&\ \frac{1}{4}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)\\&+\frac{1}{4}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)\\&+\frac{1}{8}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)\\&-\frac{1}{8}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)\end{aligned}$$

input
```
integrate((3+x^4*(-3+3^(1/2))-2*3^(1/2))/(x^8-x^4+1),x, algorithm="giac")
```

output
```
1/4*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt
(2))) + 1/4*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6) - sqrt(2))/(sqrt(6
) - sqrt(2))) + 1/8*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1/2*x*(sqrt(6) - sqrt(
2)) + 1) - 1/8*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) - sqrt(2)) +
 1)
```

3.33.9 Mupad [B] (verification not implemented)

Time = 8.76 (sec) , antiderivative size = 1, normalized size of antiderivative = 0.01

$$\int \frac{3-2\sqrt{3}+\left(-3+\sqrt{3}\right)x^4}{1-x^4+x^8}\,dx = 0$$

input
```
int((x^4*(3^(1/2) - 3) - 2*3^(1/2) + 3)/(x^8 - x^4 + 1),x)
```

output
```
0
```

3.34 $\int \frac{d+\frac{e}{x}}{c+\frac{a}{x^2}} \, dx$

3.34.1 Optimal result

Integrand size = 17, antiderivative size = 49

$$\int \frac{d+\frac{e}{x}}{c+\frac{a}{x^2}} \, dx = \frac{dx}{c} - \frac{\sqrt{a} d \arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{c^{3/2}} + \frac{e \log(a+cx^2)}{2c}$$

output
```
d*x/c+1/2*e*ln(c*x^2+a)/c-d*arctan(x*c^(1/2)/a^(1/2))*a^(1/2)/c^(3/2)
```

3.34.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{d+\frac{e}{x}}{c+\frac{a}{x^2}} \, dx = \frac{dx}{c} - \frac{\sqrt{a} d \arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{c^{3/2}} + \frac{e \log(a+cx^2)}{2c}$$

input
```
Integrate[(d + e/x)/(c + a/x^2),x]
```

output
```
(d*x)/c - (Sqrt[a]*d*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/c^(3/2) + (e*Log[a + c*x
^2])/(2*c)
```

3.34.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1728, 523, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+\frac{e}{x}}{\frac{a}{x^2}+c}dx$$

$$\Big\downarrow \ 1728$$

$$\int \frac{x(dx+e)}{a+cx^2}dx$$

$$\Big\downarrow \ 523$$

$$\int \left(\frac{d}{c}-\frac{ad-cex}{c\left(a+cx^2\right)}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{\sqrt{a}d\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{c^{3/2}}+\frac{e\log\left(a+cx^2\right)}{2c}+\frac{dx}{c}$$

input
```
Int[(d + e/x)/(c + a/x^2),x]
```

output
```
(d*x)/c - (Sqrt[a]*d*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/c^(3/2) + (e*Log[a + c*x
^2])/(2*c)
```

3.34.3.1 Defintions of rubi rules used

rule 523
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_)))/((a_) + (b_.)*(x_)^2), x_Symbol] :> In
t[ExpandIntegrand[x^m*((c + d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}
, x] && IntegerQ[m]
```

rule 1728
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> Int[x^(n*(2*p + q))*(e + d/x^n)^q*(c + a/x^(2*n))^p, x] /; FreeQ[{a,
 c, d, e, n}, x] && EqQ[n2, 2*n] && IntegersQ[p, q] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.34.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{dx}{c}+\frac{\frac{e\ln\left(c\,x^{2}+a\right)}{2}-\frac{da\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}}}{c}$	42
risch	$\frac{dx}{c}+\frac{\ln\left(-\sqrt{-ac}\,x-a\right)d\sqrt{-ac}}{2c^{2}}+\frac{\ln\left(-\sqrt{-ac}\,x-a\right)e}{2c}-\frac{\ln\left(\sqrt{-ac}\,x-a\right)d\sqrt{-ac}}{2c^{2}}+\frac{\ln\left(\sqrt{-ac}\,x-a\right)e}{2c}$	98

input
```
int((d+e/x)/(c+a/x^2),x,method=_RETURNVERBOSE)
```

output
```
d*x/c+1/c*(1/2*e*ln(c*x^2+a)-d*a/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2)))
```

3.34.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.20

$$\int \frac{d+\frac{e}{x}}{c+\frac{a}{x^2}}\,dx=\left[\frac{d\sqrt{-\frac{a}{c}}\log\left(\frac{cx^2-2\,cx\sqrt{-\frac{a}{c}}-a}{cx^2+a}\right)+2\,dx+e\log\left(cx^2+a\right)}{2\,c},\ -\frac{2\,d\sqrt{\frac{a}{c}}\arctan\left(\frac{cx\sqrt{\frac{a}{c}}}{a}\right)-2\,dx-e\log\left(cx^2+a\right)}{2\,c}\right]$$

input
```
integrate((d+e/x)/(c+a/x^2),x, algorithm="fricas")
```

output
```
[1/2*(d*sqrt(-a/c)*log((c*x^2 - 2*c*x*sqrt(-a/c) - a)/(c*x^2 + a)) + 2*d*x
 + e*log(c*x^2 + a))/c, -1/2*(2*d*sqrt(a/c)*arctan(c*x*sqrt(a/c)/a) - 2*d*
x - e*log(c*x^2 + a))/c]
```

3.34.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(42) = 84$.

Time = 0.14 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.29

$$\int \frac{d+\frac{e}{x}}{c+\frac{a}{x^2}}\,dx = \left(\frac{e}{2c} - \frac{d\sqrt{-ac^3}}{2c^3}\right)\log\left(x + \frac{-2c\left(\frac{e}{2c} - \frac{d\sqrt{-ac^3}}{2c^3}\right) + e}{d}\right) + \left(\frac{e}{2c} + \frac{d\sqrt{-ac^3}}{2c^3}\right)\log\left(x + \frac{-2c\left(\frac{e}{2c} + \frac{d\sqrt{-ac^3}}{2c^3}\right) + e}{d}\right) + \frac{dx}{c}$$

input
```
integrate((d+e/x)/(c+a/x**2),x)
```

output
```
(e/(2*c) - d*sqrt(-a*c**3)/(2*c**3))*log(x + (-2*c*(e/(2*c) - d*sqrt(-a*c*
*3)/(2*c**3)) + e)/d) + (e/(2*c) + d*sqrt(-a*c**3)/(2*c**3))*log(x + (-2*c
*(e/(2*c) + d*sqrt(-a*c**3)/(2*c**3)) + e)/d) + d*x/c
```

3.34.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

$$\int \frac{d+\frac{e}{x}}{c+\frac{a}{x^2}}\,dx = -\frac{ad\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}c} + \frac{dx}{c} + \frac{e\log\left(cx^2+a\right)}{2\,c}$$

input
```
integrate((d+e/x)/(c+a/x^2),x, algorithm="maxima")
```

output
```
-a*d*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*c) + d*x/c + 1/2*e*log(c*x^2 + a)/c
```

3.34.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

$$\int \frac{d+\frac{e}{x}}{c+\frac{a}{x^2}}\,dx = -\frac{ad\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}c} + \frac{dx}{c} + \frac{e\log\left(cx^2+a\right)}{2\,c}$$

input
```
integrate((d+e/x)/(c+a/x^2),x, algorithm="giac")
```

output
```
-a*d*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*c) + d*x/c + 1/2*e*log(c*x^2 + a)/c
```

3.34.9 Mupad [B] (verification not implemented)

Time = 8.50 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

$$\int \frac{d+\frac{e}{x}}{c+\frac{a}{x^2}}\,dx = \frac{e\,\ln\left(c\,x^2+a\right)}{2\,c}+\frac{d\,x}{c}-\frac{\sqrt{a}\,d\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{c^{3/2}}$$

input
```
int((d + e/x)/(c + a/x^2),x)
```

output
```
(e*log(a + c*x^2))/(2*c) + (d*x)/c - (a^(1/2)*d*atan((c^(1/2)*x)/a^(1/2)))
/c^(3/2)
```

3.35 $\int \frac{d+\frac{e}{x}}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx$

3.35.1 Optimal result

Integrand size = 22, antiderivative size = 86

$$\int \frac{d+\frac{e}{x}}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx=\frac{dx}{c}-\frac{(b^2d-2acd-bce)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{(bd-ce)\log(a+bx+cx^2)}{2c^2}$$

output
```
d*x/c-1/2*(b*d-c*e)*ln(c*x^2+b*x+a)/c^2-(-2*a*c*d+b^2*d-b*c*e)*arctanh((2*
c*x+b)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(1/2)
```

3.35.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00

$$\int \frac{d+\frac{e}{x}}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx=\frac{2cdx+\frac{2(b^2d-2acd-bce)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+(-bd+ce)\log(a+x(b+cx))}{2c^2}$$

input
```
Integrate[(d + e/x)/(c + a/x^2 + b/x),x]
```

output
```
(2*c*d*x + (2*(b^2*d - 2*a*c*d - b*c*e)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a
*c]])/Sqrt[-b^2 + 4*a*c] + (-(b*d) + c*e)*Log[a + x*(b + c*x)])/(2*c^2)
```

3.35.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1727, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+\frac{e}{x}}{\frac{a}{x^2}+\frac{b}{x}+c} \, dx$$

$$\Big\downarrow \text{ 1727}$$

$$\int \frac{x(dx+e)}{a+bx+cx^2}dx$$

$$\Big\downarrow \text{ 1200}$$

$$\int \left(\frac{d}{c}-\frac{ad+x(bd-ce)}{c\left(a+bx+cx^2\right)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-2acd+b^2d-bce\right)}{c^2\sqrt{b^2-4ac}}-\frac{(bd-ce)\log\left(a+bx+cx^2\right)}{2c^2}+\frac{dx}{c}$$

input
```
Int[(d + e/x)/(c + a/x^2 + b/x),x]
```

output
```
(d*x)/c - ((b^2*d - 2*a*c*d - b*c*e)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]
])/(c^2*Sqrt[b^2 - 4*a*c]) - ((b*d - c*e)*Log[a + b*x + c*x^2])/(2*c^2)
```

3.35.3.1 Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1727
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (e_.)*(x_)^(
n_))^(q_.), x_Symbol] :> Int[x^(n*(2*p + q))*(e + d/x^n)^q*(c + b/x^n + a/x
^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && IntegersQ[
p, q] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.35.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.05

method	result	size
default	$\frac{dx}{c}+\frac{\frac{(-bd+ec)\ln\left(c\,x^2+bx+a\right)}{2c}+\frac{2\left(-da-\frac{(-bd+ec)b}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{c}$	90
risch	Expression too large to display	1357

input
```
int((d+e/x)/(c+a/x^2+b/x),x,method=_RETURNVERBOSE)
```

output
```
d*x/c+1/c*(1/2*(-b*d+c*e)/c*ln(c*x^2+b*x+a)+2*(-d*a-1/2*(-b*d+c*e)*b/c)/(4
*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

3.35.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 291, normalized size of antiderivative = 3.38

$$\int \frac{d+\frac{e}{x}}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx$$
$$= \left[\frac{2\left(b^2c-4\,ac^2\right)dx+\left(bce-\left(b^2-2\,ac\right)d\right)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)-((b^3-4\,abc)}{2\left(b^2c^2-4\,ac^3\right)}\right.$$

input
```
integrate((d+e/x)/(c+a/x^2+b/x),x, algorithm="fricas")
```

output
```
[1/2*(2*(b^2*c - 4*a*c^2)*d*x + (b*c*e - (b^2 - 2*a*c)*d)*sqrt(b^2 - 4*a*c
)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/
(c*x^2 + b*x + a)) - ((b^3 - 4*a*b*c)*d - (b^2*c - 4*a*c^2)*e)*log(c*x^2 +
 b*x + a))/(b^2*c^2 - 4*a*c^3), 1/2*(2*(b^2*c - 4*a*c^2)*d*x + 2*(b*c*e -
(b^2 - 2*a*c)*d)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)
/(b^2 - 4*a*c)) - ((b^3 - 4*a*b*c)*d - (b^2*c - 4*a*c^2)*e)*log(c*x^2 + b*
x + a))/(b^2*c^2 - 4*a*c^3)]
```

3.35.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 423 vs. $2(82) = 164$.

Time = 0.71 (sec) , antiderivative size = 423, normalized size of antiderivative = 4.92

$$\int \frac{d+\frac{e}{x}}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx = \left(-\frac{\sqrt{-4ac+b^2}\cdot(2acd-b^2d+bce)}{2c^2\cdot(4ac-b^2)} - \frac{bd-ce}{2c^2}\right)\log\left(x+\frac{-abd-4ac^2\left(-\frac{\sqrt{-4ac+b^2}\cdot(2acd-b^2d+bce)}{2c^2\cdot(4ac-b^2)}-\frac{bd-ce}{2c^2}\right)+2ace+b^2c\left(-\frac{\sqrt{-4ac+b^2}\cdot(2acd-b^2d+bce)}{2c^2\cdot(4ac-b^2)}\right.}{2acd-b^2d+bce}\right.$$
$$+\left(\frac{\sqrt{-4ac+b^2}\cdot(2acd-b^2d+bce)}{2c^2\cdot(4ac-b^2)} - \frac{bd-ce}{2c^2}\right)\log\left(x+\frac{-abd-4ac^2\left(\frac{\sqrt{-4ac+b^2}\cdot(2acd-b^2d+bce)}{2c^2\cdot(4ac-b^2)}-\frac{bd-ce}{2c^2}\right)+2ace+b^2c\left(\frac{\sqrt{-4ac+b^2}\cdot(2acd-b^2d+bce)}{2c^2\cdot(4ac-b^2)}\right. -}{2acd-b^2d+bce}\right.$$
$$+\frac{dx}{c}$$

input
```
integrate((d+e/x)/(c+a/x**2+b/x),x)
```

output
```
(-sqrt(-4*a*c + b**2)*(2*a*c*d - b**2*d + b*c*e)/(2*c**2*(4*a*c - b**2)) -
 (b*d - c*e)/(2*c**2))*log(x + (-a*b*d - 4*a*c**2*(-sqrt(-4*a*c + b**2)*(2
*a*c*d - b**2*d + b*c*e)/(2*c**2*(4*a*c - b**2)) - (b*d - c*e)/(2*c**2)) +
 2*a*c*e + b**2*c*(-sqrt(-4*a*c + b**2)*(2*a*c*d - b**2*d + b*c*e)/(2*c**2
*(4*a*c - b**2)) - (b*d - c*e)/(2*c**2)))/(2*a*c*d - b**2*d + b*c*e)) + (s
qrt(-4*a*c + b**2)*(2*a*c*d - b**2*d + b*c*e)/(2*c**2*(4*a*c - b**2)) - (b
*d - c*e)/(2*c**2))*log(x + (-a*b*d - 4*a*c**2*(sqrt(-4*a*c + b**2)*(2*a*c
*d - b**2*d + b*c*e)/(2*c**2*(4*a*c - b**2)) - (b*d - c*e)/(2*c**2)) + 2*a
*c*e + b**2*c*(sqrt(-4*a*c + b**2)*(2*a*c*d - b**2*d + b*c*e)/(2*c**2*(4*a
*c - b**2)) - (b*d - c*e)/(2*c**2)))/(2*a*c*d - b**2*d + b*c*e)) + d*x/c
```

3.35.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{d+\frac{e}{x}}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d+e/x)/(c+a/x^2+b/x),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.35.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.97

$$\int \frac{d+\frac{e}{x}}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx = \frac{dx}{c} - \frac{(bd-ce)\log\left(cx^2+bx+a\right)}{2\,c^2} + \frac{\left(b^2d-2\,acd-bce\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}c^2}$$

input
```
integrate((d+e/x)/(c+a/x^2+b/x),x, algorithm="giac")
```

output
```
d*x/c - 1/2*(b*d - c*e)*log(c*x^2 + b*x + a)/c^2 + (b^2*d - 2*a*c*d - b*c*
e)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)
```

3.35.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.48

$$\int \frac{d+\frac{e}{x}}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx = \frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(d\,b^3-e\,b^2\,c-4\,a\,d\,b\,c+4\,a\,e\,c^2\right)}{2\,\left(4\,a\,c^3-b^2\,c^2\right)} + \frac{d\,x}{c} - \frac{\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)\,\left(-d\,b^2+c\,e\,b+2\,a\,c\,d\right)}{c^2\,\sqrt{4\,a\,c-b^2}}$$

input
```
int((d + e/x)/(c + a/x^2 + b/x),x)
```

output
```
(log(a + b*x + c*x^2)*(b^3*d + 4*a*c^2*e - b^2*c*e - 4*a*b*c*d))/(2*(4*a*c
^3 - b^2*c^2)) + (d*x)/c - (atan(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*a*c -
b^2)^(1/2))*(2*a*c*d - b^2*d + b*c*e))/(c^2*(4*a*c - b^2)^(1/2))
```

3.35. $\int \frac{d+\frac{e}{x}}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx$

3.36 $\int \frac{d+\frac{e}{x^2}}{c+\frac{a}{x^4}}\,dx$

3.36.1 Optimal result

Integrand size = 17, antiderivative size = 253

$$\begin{aligned}\int \frac{d+\frac{e}{x^2}}{c+\frac{a}{x^4}}\,dx=&\frac{dx}{c}+\frac{\left(\sqrt{a}d-\sqrt{c}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}c^{5/4}}\\&-\frac{\left(\sqrt{a}d-\sqrt{c}e\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}c^{5/4}}\\&+\frac{\left(\sqrt{a}d+\sqrt{c}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}c^{5/4}}\\&-\frac{\left(\sqrt{a}d+\sqrt{c}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}c^{5/4}}\end{aligned}$$

output
```
d*x/c-1/4*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(d*a^(1/2)-e*c^(1/2))/a^(1/
4)/c^(5/4)*2^(1/2)-1/4*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(d*a^(1/2)-e*c^
(1/2))/a^(1/4)/c^(5/4)*2^(1/2)+1/8*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x
^2*c^(1/2))*(d*a^(1/2)+e*c^(1/2))/a^(1/4)/c^(5/4)*2^(1/2)-1/8*ln(a^(1/4)*c
^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(d*a^(1/2)+e*c^(1/2))/a^(1/4)/c^(5/4
)*2^(1/2)
```

3.36.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.16

$$\int \frac{d+\frac{e}{x^2}}{c+\frac{a}{x^4}}\,dx = \frac{dx}{c} + \frac{\left(-a^{5/4}\sqrt{c}d + a^{3/4}ce\right)\arctan\left(\frac{-\sqrt{2}\sqrt[4]{a}+2\sqrt[4]{c}x}{\sqrt{2}\sqrt[4]{a}}\right)}{2\sqrt{2}ac^{7/4}} + \frac{\left(-a^{5/4}\sqrt{c}d + a^{3/4}ce\right)\arctan\left(\frac{\sqrt{2}\sqrt[4]{a}+2\sqrt[4]{c}x}{\sqrt{2}\sqrt[4]{a}}\right)}{2\sqrt{2}ac^{7/4}} + \frac{\left(a^{5/4}\sqrt{c}d + a^{3/4}ce\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}ac^{7/4}} - \frac{\left(a^{5/4}\sqrt{c}d + a^{3/4}ce\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}ac^{7/4}}$$

input
```
Integrate[(d + e/x^2)/(c + a/x^4),x]
```

output
```
(d*x)/c + ((-(a^(5/4)*Sqrt[c]*d) + a^(3/4)*c*e)*ArcTan[(-(Sqrt[2]*a^(1/4))
 + 2*c^(1/4)*x)/(Sqrt[2]*a^(1/4))])/(2*Sqrt[2]*a*c^(7/4)) + ((-(a^(5/4)*Sq
rt[c]*d) + a^(3/4)*c*e)*ArcTan[(Sqrt[2]*a^(1/4) + 2*c^(1/4)*x)/(Sqrt[2]*a^
(1/4))])/(2*Sqrt[2]*a*c^(7/4)) + ((a^(5/4)*Sqrt[c]*d + a^(3/4)*c*e)*Log[Sq
rt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a*c^(7/4)) -
((a^(5/4)*Sqrt[c]*d + a^(3/4)*c*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x
 + Sqrt[c]*x^2])/(4*Sqrt[2]*a*c^(7/4))
```

3.36.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.96, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.647, Rules used = {1728, 1603, 1482, 27, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+\frac{e}{x^2}}{\frac{a}{x^4}+c}\,dx$$

$$\Big\downarrow \text{1728}$$

$$\int \frac{x^2\left(dx^2+e\right)}{a+cx^4}dx$$

$$\Big\downarrow \text{1603}$$

$$\frac{dx}{c}-\frac{\int \frac{ad-cex^2}{cx^4+a}dx}{c}$$

$$\Big\downarrow \text{1482}$$

$$\frac{dx}{c}-\frac{\frac{1}{2}\left(\frac{\sqrt{a}d}{\sqrt{c}}+e\right)\int \frac{\sqrt{c}(\sqrt{a}-\sqrt{c}x^2)}{cx^4+a}dx+\frac{1}{2}\left(\frac{\sqrt{a}d}{\sqrt{c}}-e\right)\int \frac{\sqrt{c}(\sqrt{c}x^2+\sqrt{a})}{cx^4+a}dx}{c}$$

$$\Big\downarrow \text{27}$$

$$\frac{dx}{c}-\frac{\frac{1}{2}\sqrt{c}\left(\frac{\sqrt{a}d}{\sqrt{c}}+e\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx+\frac{1}{2}\sqrt{c}\left(\frac{\sqrt{a}d}{\sqrt{c}}-e\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{cx^4+a}dx}{c}$$

$$\Big\downarrow \text{1476}$$

$$\frac{dx}{c}-\frac{\frac{1}{2}\sqrt{c}\left(\frac{\sqrt{a}d}{\sqrt{c}}-e\right)\left(\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}\right)+\frac{1}{2}\sqrt{c}\left(\frac{\sqrt{a}d}{\sqrt{c}}+e\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{c}$$

$$\Big\downarrow \text{1082}$$

$$\frac{dx}{c}-\frac{\frac{1}{2}\sqrt{c}\left(\frac{\sqrt{a}d}{\sqrt{c}}+e\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx+\frac{1}{2}\sqrt{c}\left(\frac{\sqrt{a}d}{\sqrt{c}}-e\right)\left(\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right.}{c}$$

$$\Big\downarrow \text{217}$$

$$\frac{dx}{c}-\frac{\frac{1}{2}\sqrt{c}\left(\frac{\sqrt{a}d}{\sqrt{c}}+e\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx+\frac{1}{2}\sqrt{c}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)\left(\frac{\sqrt{a}d}{\sqrt{c}}-e\right)}{c}$$

$$\Big\downarrow \text{1479}$$

3.36. $\int \frac{d+\frac{e}{x^2}}{c+\frac{a}{x^4}}\,dx$

$$\frac{\frac{1}{2}\sqrt{c}\left(\frac{\sqrt{a}d}{\sqrt{c}}+e\right)\left(-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)+\frac{1}{2}\sqrt{c}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1\right.}{\sqrt{2}}\right.}{c}\frac{dx}{c}-$$

$\downarrow$ 25

$$\frac{\frac{1}{2}\sqrt{c}\left(\frac{\sqrt{a}d}{\sqrt{c}}+e\right)\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)+\frac{1}{2}\sqrt{c}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{}}{}\right.}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{}}\right.}{c}\frac{dx}{c}-$$

$\downarrow$ 27

$$\frac{\frac{1}{2}\sqrt{c}\left(\frac{\sqrt{a}d}{\sqrt{c}}+e\right)\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt[4]{a}\sqrt{c}}\right)+\frac{1}{2}\sqrt{c}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)\left(\frac{\sqrt{a}}{\sqrt{c}}\right.}{c}\frac{dx}{c}-$$

$\downarrow$ 1103

$$\frac{\frac{1}{2}\sqrt{c}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)\left(\frac{\sqrt{a}d}{\sqrt{c}}-e\right)+\frac{1}{2}\sqrt{c}\left(\frac{\sqrt{a}d}{\sqrt{c}}+e\right)\left(\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\right.}{}\right.}{c}\frac{dx}{c}-$$

input

```
Int[(d + e/x^2)/(c + a/x^4),x]
```

3.36. $\int \frac{d+\frac{e}{x^2}}{c+\frac{a}{x^4}}\,dx$

output
```
(d*x)/c - ((Sqrt[c]*((Sqrt[a]*d)/Sqrt[c] - e)*(-(ArcTan[1 - (Sqrt[2]*c^(1/
4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*c^(1/4))) + ArcTan[1 + (Sqrt[2]*c^(1/4)*x)
/a^(1/4)]/(Sqrt[2]*a^(1/4)*c^(1/4))))/2 + (Sqrt[c]*((Sqrt[a]*d)/Sqrt[c] +
e)*(-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(Sqrt[2]*a
^(1/4)*c^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(
2*Sqrt[2]*a^(1/4)*c^(1/4))))/2)/c
```

3.36.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

3.36. $\int \frac{d+\frac{e}{x^2}}{c+\frac{a}{x^4}}\,dx$

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 1603
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + c*x^4)^(p + 1)/(c*(m + 4*p + 3))),
x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
 1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ
[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[
m])
```

rule 1728
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> Int[x^(n*(2*p + q))*(e + d/x^n)^q*(c + a/x^(2*n))^p, x] /; FreeQ[{a,
 c, d, e, n}, x] && EqQ[n2, 2*n] && IntegersQ[p, q] && NegQ[n]
```

3.36.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.18

method	result
risch	$\frac{dx}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{4}+a\right)}\frac{\left(\_R^{2}ce-da\right)\ln\left(x-\_R\right)}{\_R^{3}}}{4c^{2}}$
default	$\frac{dx}{c}+\frac{-\frac{d\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8}+\frac{e\sqrt{2}\left(\ln\left(\frac{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\cdots\right.}{8\left(\frac{a}{c}\right)^{\frac{1}{4}}\cdots}}{c}$

input
```
int((d+e/x^2)/(c+a/x^4),x,method=_RETURNVERBOSE)
```

output
```
d*x/c+1/4/c^2*sum((_R^2*c*e-a*d)/_R^3*ln(x-_R),_R=RootOf(_Z^4*c+a))
```

3.36.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 754 vs. 2(172) = 344.

Time = 0.30 (sec) , antiderivative size = 754, normalized size of antiderivative = 2.98

$$\int \frac{d+\frac{e}{x^2}}{c+\frac{a}{x^4}}\,dx$$
$$= \frac{c\sqrt{\frac{c^2\sqrt{-\frac{a^2d^4-2\,acd^2e^2+c^2e^4}{ac^5}}+2\,de}{c^2}}\log\left(-(a^2d^4-c^2e^4)x+\left(ac^4e\sqrt{-\frac{a^2d^4-2\,acd^2e^2+c^2e^4}{ac^5}}+a^2cd^3-ac^2de^2\right)\sqrt{\frac{c^2\sqrt{-}}{}}\right.}{}$$

input
```
integrate((d+e/x^2)/(c+a/x^4),x, algorithm="fricas")
```

output
```
1/4*(c*sqrt((c^2*sqrt(-(a^2*d^4 - 2*a*c*d^2*e^2 + c^2*e^4)/(a*c^5)) + 2*d*
e)/c^2)*log(-(a^2*d^4 - c^2*e^4)*x + (a*c^4*e*sqrt(-(a^2*d^4 - 2*a*c*d^2*e
^2 + c^2*e^4)/(a*c^5)) + a^2*c*d^3 - a*c^2*d*e^2)*sqrt((c^2*sqrt(-(a^2*d^4
 - 2*a*c*d^2*e^2 + c^2*e^4)/(a*c^5)) + 2*d*e)/c^2)) - c*sqrt((c^2*sqrt(-(a
^2*d^4 - 2*a*c*d^2*e^2 + c^2*e^4)/(a*c^5)) + 2*d*e)/c^2)*log(-(a^2*d^4 - c
^2*e^4)*x - (a*c^4*e*sqrt(-(a^2*d^4 - 2*a*c*d^2*e^2 + c^2*e^4)/(a*c^5)) +
a^2*c*d^3 - a*c^2*d*e^2)*sqrt((c^2*sqrt(-(a^2*d^4 - 2*a*c*d^2*e^2 + c^2*e^
4)/(a*c^5)) + 2*d*e)/c^2)) - c*sqrt(-(c^2*sqrt(-(a^2*d^4 - 2*a*c*d^2*e^2 +
 c^2*e^4)/(a*c^5)) - 2*d*e)/c^2)*log(-(a^2*d^4 - c^2*e^4)*x + (a*c^4*e*sqr
t(-(a^2*d^4 - 2*a*c*d^2*e^2 + c^2*e^4)/(a*c^5)) - a^2*c*d^3 + a*c^2*d*e^2)
*sqrt(-(c^2*sqrt(-(a^2*d^4 - 2*a*c*d^2*e^2 + c^2*e^4)/(a*c^5)) - 2*d*e)/c^
2)) + c*sqrt(-(c^2*sqrt(-(a^2*d^4 - 2*a*c*d^2*e^2 + c^2*e^4)/(a*c^5)) - 2*
d*e)/c^2)*log(-(a^2*d^4 - c^2*e^4)*x - (a*c^4*e*sqrt(-(a^2*d^4 - 2*a*c*d^2
*e^2 + c^2*e^4)/(a*c^5)) - a^2*c*d^3 + a*c^2*d*e^2)*sqrt(-(c^2*sqrt(-(a^2*
d^4 - 2*a*c*d^2*e^2 + c^2*e^4)/(a*c^5)) - 2*d*e)/c^2)) + 4*d*x)/c
```

3.36.6 Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.43

$$\int \frac{d+\frac{e}{x^2}}{c+\frac{a}{x^4}}\,dx$$
$$=\text{RootSum}\left(256t^4ac^5-64t^2ac^3de+a^2d^4+2acd^2e^2+c^2e^4,\left(t\mapsto t\log\left(x+\frac{-64t^3ac^4e-4ta^2cd^3+12tac\right.\right.\right.}{a^2d^4-c^2e^4}$$
$$+\frac{dx}{c}$$

input
```
integrate((d+e/x**2)/(c+a/x**4),x)
```

output
```
RootSum(256*_t**4*a*c**5 - 64*_t**2*a*c**3*d*e + a**2*d**4 + 2*a*c*d**2*e*
*2 + c**2*e**4, Lambda(_t, _t*log(x + (-64*_t**3*a*c**4*e - 4*_t*a**2*c*d*
*3 + 12*_t*a*c**2*d*e**2)/(a**2*d**4 - c**2*e**4)))) + d*x/c
```

3.36.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.95

$$\int \frac{d+\frac{e}{x^2}}{c+\frac{a}{x^4}}\,dx=\frac{dx}{c}$$
$$-\frac{\frac{2\sqrt{2}(a\sqrt{c}d-\sqrt{a}ce)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}(a\sqrt{c}d-\sqrt{a}ce)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{\sqrt{2}(a\sqrt{c}d+\sqrt{a}ce)\log\left(\sqrt{c}x^2\right.}{a^{\frac{3}{4}}c^{\frac{3}{4}}}}{8c}$$

input
```
integrate((d+e/x^2)/(c+a/x^4),x, algorithm="maxima")
```

output
```
d*x/c - 1/8*(2*sqrt(2)*(a*sqrt(c)*d - sqrt(a)*c*e)*arctan(1/2*sqrt(2)*(2*s
qrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(s
qrt(a)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*(a*sqrt(c)*d - sqrt(a)*c*e)*arctan(1/
2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(
sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + sqrt(2)*(a*sqrt(c)*d + sqrt(a)*c*
e)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)
) - sqrt(2)*(a*sqrt(c)*d + sqrt(a)*c*e)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*
c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)))/c
```

3.36.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.96

$$\int \frac{d+\frac{e}{x^2}}{c+\frac{a}{x^4}}\,dx = \frac{dx}{c} - \frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}\,acd - (ac^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac^3} - \frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}\,acd - (ac^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac^3} - \frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}\,acd + (ac^3)^{\frac{3}{4}}\,e\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\,ac^3} + \frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}\,acd + (ac^3)^{\frac{3}{4}}\,e\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\,ac^3}$$

input
```
integrate((d+e/x^2)/(c+a/x^4),x, algorithm="giac")
```

output
```
d*x/c - 1/4*sqrt(2)*((a*c^3)^(1/4)*a*c*d - (a*c^3)^(3/4)*e)*arctan(1/2*sqr
t(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^3) - 1/4*sqrt(2)*((a*c^
3)^(1/4)*a*c*d - (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^
(1/4))/(a/c)^(1/4))/(a*c^3) - 1/8*sqrt(2)*((a*c^3)^(1/4)*a*c*d + (a*c^3)^(
3/4)*e)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^3) + 1/8*sqrt(2)
*((a*c^3)^(1/4)*a*c*d + (a*c^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) +
 sqrt(a/c))/(a*c^3)
```

3.36.9 Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 555, normalized size of antiderivative = 2.19

$$\int \frac{d+\frac{e}{x^2}}{c+\frac{a}{x^4}}\,dx = \frac{d\,x}{c} - 2\,\mathrm{atanh}\left(\frac{8\,a^2\,c\,d^2\,x\,\sqrt{\frac{d^2\,\sqrt{-a\,c^5}}{16\,c^5}+\frac{d\,e}{8\,c^2}-\frac{e^2\,\sqrt{-a\,c^5}}{16\,a\,c^4}}}{2\,a^2\,d^2\,e-2\,a\,c\,e^3+\frac{2\,a^2\,d^3\,\sqrt{-a\,c^5}}{c^3}-\frac{2\,a\,d\,e^2\,\sqrt{-a\,c^5}}{c^2}} - \frac{8\,a\,c^2\,e^2\,x\,\sqrt{\frac{d^2\,\sqrt{-a\,c^5}}{16\,c^5}+\frac{d\,e}{8\,c^2}-\frac{e^2\,\sqrt{-a\,c^5}}{16\,a\,c^4}}}{2\,a^2\,d^2\,e-2\,a\,c\,e^3+\frac{2\,a^2\,d^3\,\sqrt{-a\,c^5}}{c^3}-\frac{2\,a\,d\,e^2\,\sqrt{-a\,c^5}}{c^2}}\right)\sqrt{\frac{a\,d^2\,\sqrt{-a\,c^5}-c\,e^2\,\sqrt{-a\,c^5}+2\,a\,c^3\,d\,e}{16\,a\,c^5}} - 2\,\mathrm{atanh}\left(\frac{8\,a^2\,c\,d^2\,x\,\sqrt{\frac{d\,e}{8\,c^2}-\frac{d^2\,\sqrt{-a\,c^5}}{16\,c^5}+\frac{e^2\,\sqrt{-a\,c^5}}{16\,a\,c^4}}}{2\,a^2\,d^2\,e-2\,a\,c\,e^3-\frac{2\,a^2\,d^3\,\sqrt{-a\,c^5}}{c^3}+\frac{2\,a\,d\,e^2\,\sqrt{-a\,c^5}}{c^2}} - \frac{8\,a\,c^2\,e^2\,x\,\sqrt{\frac{d\,e}{8\,c^2}-\frac{d^2\,\sqrt{-a\,c^5}}{16\,c^5}+\frac{e^2\,\sqrt{-a\,c^5}}{16\,a\,c^4}}}{2\,a^2\,d^2\,e-2\,a\,c\,e^3-\frac{2\,a^2\,d^3\,\sqrt{-a\,c^5}}{c^3}+\frac{2\,a\,d\,e^2\,\sqrt{-a\,c^5}}{c^2}}\right)\sqrt{\frac{c\,e^2\,\sqrt{-a\,c^5}-a\,d^2\,\sqrt{-a\,c^5}+2\,a\,c^3\,d\,e}{16\,a\,c^5}}$$

input
```
int((d + e/x^2)/(c + a/x^4),x)
```

output
```
(d*x)/c - 2*atanh((8*a^2*c*d^2*x*((d^2*(-a*c^5)^(1/2))/(16*c^5) + (d*e)/(8
*c^2) - (e^2*(-a*c^5)^(1/2))/(16*a*c^4))^(1/2))/(2*a^2*d^2*e - 2*a*c*e^3 +
 (2*a^2*d^3*(-a*c^5)^(1/2))/c^3 - (2*a*d*e^2*(-a*c^5)^(1/2))/c^2) - (8*a*c
^2*e^2*x*((d^2*(-a*c^5)^(1/2))/(16*c^5) + (d*e)/(8*c^2) - (e^2*(-a*c^5)^(1
/2))/(16*a*c^4))^(1/2))/(2*a^2*d^2*e - 2*a*c*e^3 + (2*a^2*d^3*(-a*c^5)^(1/
2))/c^3 - (2*a*d*e^2*(-a*c^5)^(1/2))/c^2))*((a*d^2*(-a*c^5)^(1/2) - c*e^2*
(-a*c^5)^(1/2) + 2*a*c^3*d*e)/(16*a*c^5))^(1/2) - 2*atanh((8*a^2*c*d^2*x*(
(d*e)/(8*c^2) - (d^2*(-a*c^5)^(1/2))/(16*c^5) + (e^2*(-a*c^5)^(1/2))/(16*a
*c^4))^(1/2))/(2*a^2*d^2*e - 2*a*c*e^3 - (2*a^2*d^3*(-a*c^5)^(1/2))/c^3 +
(2*a*d*e^2*(-a*c^5)^(1/2))/c^2) - (8*a*c^2*e^2*x*((d*e)/(8*c^2) - (d^2*(-a
*c^5)^(1/2))/(16*c^5) + (e^2*(-a*c^5)^(1/2))/(16*a*c^4))^(1/2))/(2*a^2*d^2
*e - 2*a*c*e^3 - (2*a^2*d^3*(-a*c^5)^(1/2))/c^3 + (2*a*d*e^2*(-a*c^5)^(1/2
))/c^2))*((c*e^2*(-a*c^5)^(1/2) - a*d^2*(-a*c^5)^(1/2) + 2*a*c^3*d*e)/(16*
a*c^5))^(1/2)
```

3.37 $\int \frac{d+\frac{e}{x^2}}{c+\frac{a}{x^4}+\frac{b}{x^2}}\,dx$

3.37.1 Optimal result

Integrand size = 22, antiderivative size = 208

$$\int \frac{d+\frac{e}{x^2}}{c+\frac{a}{x^4}+\frac{b}{x^2}}\,dx = \frac{dx}{c} - \frac{\left(bd-ce-\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(bd-ce+\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
d*x/c-1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b*d-c*e+
(2*a*c*d-b^2*d+b*c*e)/(-4*a*c+b^2)^(1/2))/c^(3/2)*2^(1/2)/(b-(-4*a*c+b^2)^
(1/2))^(1/2)-1/2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b
*d-c*e+(-2*a*c*d+b^2*d-b*c*e)/(-4*a*c+b^2)^(1/2))/c^(3/2)*2^(1/2)/(b+(-4*a
*c+b^2)^(1/2))^(1/2)
```

3.37.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.21

$$\int \frac{d+\frac{e}{x^2}}{c+\frac{a}{x^4}+\frac{b}{x^2}}\,dx = \frac{dx}{c}-\frac{\left(-b^2d+2acd+b\sqrt{b^2-4ac}d+bce-c\sqrt{b^2-4ac}e\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(b^2d-2acd+b\sqrt{b^2-4ac}d-bce-c\sqrt{b^2-4ac}e\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}$$

input
```
Integrate[(d + e/x^2)/(c + a/x^4 + b/x^2),x]
```

output
```
(d*x)/c - ((-(b^2*d) + 2*a*c*d + b*Sqrt[b^2 - 4*a*c]*d + b*c*e - c*Sqrt[b^
2 - 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sq
rt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((b^2*d - 2
*a*c*d + b*Sqrt[b^2 - 4*a*c]*d - b*c*e - c*Sqrt[b^2 - 4*a*c]*e)*ArcTan[(Sq
rt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 -
 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

3.37.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1727, 1602, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+\frac{e}{x^2}}{\frac{a}{x^4}+\frac{b}{x^2}+c}dx$$

$$\Big\downarrow \text{ 1727}$$

$$\int \frac{x^2\left(dx^2+e\right)}{a+bx^2+cx^4}dx$$

$$\Big\downarrow \text{ 1602}$$

$$\frac{dx}{c}-\frac{\int \frac{(bd-ce)x^2+ad}{cx^4+bx^2+a}dx}{c}$$

$$\Big\downarrow \text{1480}$$

$$\frac{dx}{c}-\frac{\frac{1}{2}\left(-\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)\int\frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}\left(\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)\int\frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{c}$$

$$\Big\downarrow \text{218}$$

$$\frac{dx}{c}-\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(-\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{c}$$

input
```
Int[(d + e/x^2)/(c + a/x^4 + b/x^2),x]
```

output
```
(d*x)/c - (((b*d - c*e - (b^2*d - 2*a*c*d - b*c*e)/Sqrt[b^2 - 4*a*c])*ArcT
an[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt
[b - Sqrt[b^2 - 4*a*c]]) + ((b*d - c*e + (b^2*d - 2*a*c*d - b*c*e)/Sqrt[b^
2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt
[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/c
```

3.37.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

rule 1727
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (e_.)*(x_)^(
n_))^(q_.), x_Symbol] :> Int[x^(n*(2*p + q))*(e + d/x^n)^q*(c + b/x^n + a/x
^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && IntegersQ[
p, q] && NegQ[n]
```

3.37.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.31

method	result
risch	$\frac{dx}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left(\left(-bd+ec\right)\_R^2-da\right)\ln\left(x-\_R\right)}{2c\_R^3+\_Rb}}{2c}$
default	$\frac{dx}{c}+\frac{\left(-bd\sqrt{-4ac+b^2}+ec\sqrt{-4ac+b^2}+2acd-b^2d+ebc\right)\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\,c\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\left(-bd\sqrt{-4ac+b^2}+ec\sqrt{-4ac+b^2}-2ac\right.}{2\sqrt{-4ac+b^2}}$

input
```
int((d+e/x^2)/(a/x^4+b/x^2+c),x,method=_RETURNVERBOSE)
```

output
```
d*x/c+1/2/c*sum(((-b*d+c*e)*_R^2-d*a)/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_
Z^4*c+_Z^2*b+a))
```

3.37.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2540 vs. 2(172) = 344.

Time = 0.46 (sec) , antiderivative size = 2540, normalized size of antiderivative = 12.21

$$\int \frac{d+\frac{e}{x^2}}{c+\frac{a}{x^4}+\frac{b}{x^2}}\,dx = \text{Too large to display}$$

input
```
integrate((d+e/x^2)/(c+a/x^4+b/x^2),x, algorithm="fricas")
```

output
```
1/2*(sqrt(1/2)*c*sqrt(-(b*c^2*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(b^2*c - 2*a*c
^2)*d*e + (b^2*c^3 - 4*a*c^4)*sqrt(-(4*b*c^3*d*e^3 - c^4*e^4 - (b^4 - 2*a*
b^2*c + a^2*c^2)*d^4 + 4*(b^3*c - a*b*c^2)*d^3*e - 2*(3*b^2*c^2 - a*c^3)*d
^2*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(2*(3*b^2*c*d^2*e^2
- 3*b*c^2*d*e^3 + c^3*e^4 + (a*b^2 - a^2*c)*d^4 - (b^3 + a*b*c)*d^3*e)*x +
 sqrt(1/2)*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d^3 - 2*(b^3*c - 4*a*b*c^2)*d^2*
e + (b^2*c^2 - 4*a*c^3)*d*e^2 - ((b^3*c^3 - 4*a*b*c^4)*d - 2*(b^2*c^4 - 4*
a*c^5)*e)*sqrt(-(4*b*c^3*d*e^3 - c^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4
 + 4*(b^3*c - a*b*c^2)*d^3*e - 2*(3*b^2*c^2 - a*c^3)*d^2*e^2)/(b^2*c^6 - 4
*a*c^7)))*sqrt(-(b*c^2*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(b^2*c - 2*a*c^2)*d*e
 + (b^2*c^3 - 4*a*c^4)*sqrt(-(4*b*c^3*d*e^3 - c^4*e^4 - (b^4 - 2*a*b^2*c +
 a^2*c^2)*d^4 + 4*(b^3*c - a*b*c^2)*d^3*e - 2*(3*b^2*c^2 - a*c^3)*d^2*e^2)
/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) - sqrt(1/2)*c*sqrt(-(b*c^2*e^
2 + (b^3 - 3*a*b*c)*d^2 - 2*(b^2*c - 2*a*c^2)*d*e + (b^2*c^3 - 4*a*c^4)*sq
rt(-(4*b*c^3*d*e^3 - c^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(b^3*c
- a*b*c^2)*d^3*e - 2*(3*b^2*c^2 - a*c^3)*d^2*e^2)/(b^2*c^6 - 4*a*c^7)))/(b
^2*c^3 - 4*a*c^4))*log(2*(3*b^2*c*d^2*e^2 - 3*b*c^2*d*e^3 + c^3*e^4 + (a*b
^2 - a^2*c)*d^4 - (b^3 + a*b*c)*d^3*e)*x - sqrt(1/2)*((b^4 - 5*a*b^2*c + 4
*a^2*c^2)*d^3 - 2*(b^3*c - 4*a*b*c^2)*d^2*e + (b^2*c^2 - 4*a*c^3)*d*e^2 -
((b^3*c^3 - 4*a*b*c^4)*d - 2*(b^2*c^4 - 4*a*c^5)*e)*sqrt(-(4*b*c^3*d*e^...
```

3.37.6 Sympy [F(-1)]

Timed out.

$$\int \frac{d+\frac{e}{x^2}}{c+\frac{a}{x^4}+\frac{b}{x^2}}\,dx=\text{Timed out}$$

input
```
integrate((d+e/x**2)/(c+a/x**4+b/x**2),x)
```

output
```
Timed out
```

3.37.7 Maxima [F]

$$\int \frac{d+\frac{e}{x^2}}{c+\frac{a}{x^4}+\frac{b}{x^2}}\,dx=\int \frac{d+\frac{e}{x^2}}{c+\frac{b}{x^2}+\frac{a}{x^4}}\,dx$$

input
```
integrate((d+e/x^2)/(c+a/x^4+b/x^2),x, algorithm="maxima")
```

output
```
d*x/c + integrate(-((b*d - c*e)*x^2 + a*d)/(c*x^4 + b*x^2 + a), x)/c
```

3.37.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3179 vs. $2(172) = 344$.

Time = 1.02 (sec) , antiderivative size = 3179, normalized size of antiderivative = 15.28

$$\int \frac{d+\frac{e}{x^2}}{c+\frac{a}{x^4}+\frac{b}{x^2}}\,dx=\text{Too large to display}$$

input
```
integrate((d+e/x^2)/(c+a/x^4+b/x^2),x, algorithm="giac")
```

output
```
d*x/c + 1/8*((2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*
c^2*d - (2*b^4*c^3 - 16*a*b^2*c^4 + 32*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*b^2*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3 + 8*(b^2 - 4*a*c)*a*c^4)*c^2*e - 2
*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 8*sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a*b^3*c^3 - 2*a*b^4*c^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^3*c^4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + sqrt(2)*...
```

3.37.9 Mupad [B] (verification not implemented)

Time = 9.23 (sec) , antiderivative size = 6366, normalized size of antiderivative = 30.61

$$\int \frac{d+\frac{e}{x^2}}{c+\frac{a}{x^4}+\frac{b}{x^2}}\,dx = \text{Too large to display}$$

input
```
int((d + e/x^2)/(c + a/x^4 + b/x^2),x)
```

output
```
(d*x)/c - atan(((((16*a^2*c^3*d - 4*a*b^2*c^2*d)/c - (2*x*(4*b^3*c^3 - 16*
a*b*c^4)*(-(b^5*d^2 - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^2*e^2 - c^2
*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*b^4*c*d*e - 7*a*b^3*c
*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^3*e^2 - 16*a^2*c^3*d*e +
 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b
^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(b^5*d^2 - b^2*d^2*(-(4*a*c - b^2)^3)^
(1/2) + b^3*c^2*e^2 - c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2
- 2*b^4*c*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c
^3*e^2 - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^
(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*x*(b^4*d^2 - 2
*a*c^3*e^2 + 2*a^2*c^2*d^2 + b^2*c^2*e^2 - 2*b^3*c*d*e - 4*a*b^2*c*d^2 + 6
*a*b*c^2*d*e))/c)*(-(b^5*d^2 - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^2*
e^2 - c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*b^4*c*d*e -
7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^3*e^2 - 16*a^2*
c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^
2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i - (((16*a^2*c^3*d - 4*a*b^2*c^2*
d)/c + (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(b^5*d^2 - b^2*d^2*(-(4*a*c - b^2)^
3)^(1/2) + b^3*c^2*e^2 - c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d
^2 - 2*b^4*c*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*
b*c^3*e^2 - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^...
```

3.38 $\int \frac{d+\frac{e}{x^3}}{c+\frac{a}{x^6}}\,dx$

3.38.1 Optimal result

Integrand size = 17, antiderivative size = 311

$$\int \frac{d+\frac{e}{x^3}}{c+\frac{a}{x^6}}\,dx = \frac{dx}{c} - \frac{\sqrt[6]{a}d\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{3c^{7/6}} + \frac{\left(\sqrt{a}d-\sqrt{3}\sqrt{c}e\right)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{6\sqrt[3]{a}c^{7/6}} - \frac{\left(\sqrt{a}d+\sqrt{3}\sqrt{c}e\right)\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{6\sqrt[3]{a}c^{7/6}} - \frac{e\log\left(\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{6\sqrt[3]{a}c^{2/3}} + \frac{\left(\sqrt{3}\sqrt{a}d+\sqrt{c}e\right)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{12\sqrt[3]{a}c^{7/6}} - \frac{\left(\sqrt{3}\sqrt{a}d-\sqrt{c}e\right)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{12\sqrt[3]{a}c^{7/6}}$$

output
```
d*x/c-1/3*a^(1/6)*d*arctan(c^(1/6)*x/a^(1/6))/c^(7/6)-1/6*e*ln(a^(1/3)+c^(
1/3)*x^2)/a^(1/3)/c^(2/3)-1/12*ln(a^(1/3)+c^(1/3)*x^2+a^(1/6)*c^(1/6)*x*3^
(1/2))*(d*3^(1/2)*a^(1/2)-e*c^(1/2))/a^(1/3)/c^(7/6)+1/12*ln(a^(1/3)+c^(1/
3)*x^2-a^(1/6)*c^(1/6)*x*3^(1/2))*(d*3^(1/2)*a^(1/2)+e*c^(1/2))/a^(1/3)/c^
(7/6)-1/6*arctan(2*c^(1/6)*x/a^(1/6)-3^(1/2))*(d*a^(1/2)-e*3^(1/2)*c^(1/2)
)/a^(1/3)/c^(7/6)-1/6*arctan(2*c^(1/6)*x/a^(1/6)+3^(1/2))*(d*a^(1/2)+e*3^(
1/2)*c^(1/2))/a^(1/3)/c^(7/6)
```

3.38.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.11

$$\int \frac{d+\frac{e}{x^3}}{c+\frac{a}{x^6}}\,dx = \frac{dx}{c}-\frac{\sqrt[6]{a}d\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{3c^{7/6}} + \frac{\left(-a^{7/6}\sqrt{c}d+\sqrt{3}a^{2/3}ce\right)\arctan\left(\frac{-\sqrt{3}\sqrt[6]{a}+2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{6ac^{5/3}} + \frac{\left(-a^{7/6}\sqrt{c}d-\sqrt{3}a^{2/3}ce\right)\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}+2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{6ac^{5/3}} - \frac{e\log\left(\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{6\sqrt[3]{a}c^{2/3}} - \frac{\left(-\sqrt{3}a^{7/6}\sqrt{c}d-a^{2/3}ce\right)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{12ac^{5/3}} - \frac{\left(\sqrt{3}a^{7/6}\sqrt{c}d-a^{2/3}ce\right)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{12ac^{5/3}}$$

input
```
Integrate[(d + e/x^3)/(c + a/x^6),x]
```

output
```
(d*x)/c - (a^(1/6)*d*ArcTan[(c^(1/6)*x)/a^(1/6)])/(3*c^(7/6)) + ((-(a^(7/6
)*Sqrt[c]*d) + Sqrt[3]*a^(2/3)*c*e)*ArcTan[(-(Sqrt[3]*a^(1/6)) + 2*c^(1/6)
*x)/a^(1/6)])/(6*a*c^(5/3)) + ((-(a^(7/6)*Sqrt[c]*d) - Sqrt[3]*a^(2/3)*c*e
)*ArcTan[(Sqrt[3]*a^(1/6) + 2*c^(1/6)*x)/a^(1/6)])/(6*a*c^(5/3)) - (e*Log[
a^(1/3) + c^(1/3)*x^2])/(6*a^(1/3)*c^(2/3)) - ((-(Sqrt[3]*a^(7/6)*Sqrt[c]*
d) - a^(2/3)*c*e)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/
(12*a*c^(5/3)) - ((Sqrt[3]*a^(7/6)*Sqrt[c]*d - a^(2/3)*c*e)*Log[a^(1/3) +
Sqrt[3]*a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(12*a*c^(5/3))
```

3.38.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.12, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.765$, Rules used = {1728, 1827, 1746, 27, 452, 218, 240, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+\frac{e}{x^3}}{\frac{a}{x^6}+c}\,dx$$

$$\Big\downarrow \ 1728$$

$$\int \frac{x^3\left(dx^3+e\right)}{a+cx^6}dx$$

$$\Big\downarrow \ 1827$$

$$\frac{dx}{c}-\frac{\int \frac{ad-cex^3}{cx^6+a}dx}{c}$$

$$\Big\downarrow \ 1746$$

$$\frac{dx}{c}-\frac{\frac{\int \frac{\sqrt[3]{a}\sqrt[3]{c}\left(a^{2/3}d+c^{2/3}ex\right)}{\sqrt[3]{c}x^2+\sqrt[3]{a}}dx}{3a^{2/3}\sqrt[3]{c}}+\frac{\int \frac{\sqrt[3]{c}\left(2a^{2/3}d-\sqrt[6]{c}\left(\sqrt{3}\sqrt{a}d+\sqrt{c}e\right)x\right)}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a^{2/3}\sqrt[3]{c}}+\frac{\int \frac{\sqrt[3]{c}\left(2a^{2/3}d+\sqrt[6]{c}\left(\sqrt{3}\sqrt{a}d-\sqrt{c}e\right)x\right)}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a^{2/3}\sqrt[3]{c}}}{c}$$

$$\Big\downarrow \ 27$$

$$\frac{dx}{c}-\frac{\frac{\int \frac{a^{2/3}d+c^{2/3}ex}{\sqrt[3]{c}x^2+\sqrt[3]{a}}dx}{3\sqrt[3]{a}}+\frac{\int \frac{2a^{2/3}d-\sqrt[6]{c}\left(\sqrt{3}\sqrt{a}d+\sqrt{c}e\right)x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a^{2/3}}+\frac{\int \frac{2a^{2/3}d+\sqrt[6]{c}\left(\sqrt{3}\sqrt{a}d-\sqrt{c}e\right)x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a^{2/3}}}{c}$$

$$\Big\downarrow \ 452$$

$$\frac{dx}{c}-\frac{\frac{a^{2/3}d\int \frac{1}{\sqrt[3]{c}x^2+\sqrt[3]{a}}dx+c^{2/3}e\int \frac{x}{\sqrt[3]{c}x^2+\sqrt[3]{a}}dx}{3\sqrt[3]{a}}+\frac{\int \frac{2a^{2/3}d-\sqrt[6]{c}\left(\sqrt{3}\sqrt{a}d+\sqrt{c}e\right)x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a^{2/3}}+\frac{\int \frac{2a^{2/3}d+\sqrt[6]{c}\left(\sqrt{3}\sqrt{a}d-\sqrt{c}e\right)x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a^{2/3}}}{c}$$

$$\Big\downarrow \ 218$$

$$\frac{dx}{c}-\frac{\frac{\int \frac{2a^{2/3}d-\sqrt[6]{c}\left(\sqrt{3}\sqrt{a}d+\sqrt{c}e\right)x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a^{2/3}}+\frac{\int \frac{2a^{2/3}d+\sqrt[6]{c}\left(\sqrt{3}\sqrt{a}d-\sqrt{c}e\right)x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a^{2/3}}+\frac{c^{2/3}e\int \frac{x}{\sqrt[3]{c}x^2+\sqrt[3]{a}}dx+\frac{\sqrt{a}d\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{\sqrt[6]{c}}}{3\sqrt[3]{a}}}{c}$$

$$\Big\downarrow \ 240$$

$$\frac{dx}{c}-\frac{\frac{\int \frac{2a^{2/3}d-\sqrt[6]{c}\left(\sqrt{3}\sqrt{a}d+\sqrt{c}e\right)x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a^{2/3}}+\frac{\int \frac{2a^{2/3}d+\sqrt[6]{c}\left(\sqrt{3}\sqrt{a}d-\sqrt{c}e\right)x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a^{2/3}}+\frac{\frac{\sqrt{a}d\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{\sqrt[6]{c}}+\frac{1}{2}\sqrt[3]{c}e\log\left(\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{3\sqrt[3]{a}}}{c}$$

3.38. $\int \frac{d+\frac{e}{x^3}}{c+\frac{a}{x^6}}dx$

$\downarrow$ 1142

$$\frac{dx}{c}-\frac{\frac{\frac{1}{2}\sqrt[6]{a}\left(\sqrt{a}d-\sqrt{3}\sqrt{ce}\right)\int \frac{1}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx-\frac{\sqrt[3]{a}\left(\sqrt{3}\sqrt{a}d+\sqrt{ce}\right)\int -\frac{\sqrt[6]{c}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{c}x\right)}{\sqrt[3]{a}\left(\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1\right)}dx}{2\sqrt[6]{c}}}{6a^{2/3}}+\frac{\frac{1}{2}\sqrt[6]{a}\left(\sqrt{a}d+\sqrt{3}\sqrt{ce}\right)\int \frac{1}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+\cdots}\cdots}{\cdots}}{c}$$

$\downarrow$ 25

$$\frac{dx}{c}-\frac{\frac{\frac{1}{2}\sqrt[6]{a}\left(\sqrt{a}d-\sqrt{3}\sqrt{ce}\right)\int \frac{1}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx+\frac{\sqrt[3]{a}\left(\sqrt{3}\sqrt{a}d+\sqrt{ce}\right)\int \frac{\sqrt[6]{c}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{c}x\right)}{\sqrt[3]{a}\left(\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1\right)}dx}{2\sqrt[6]{c}}}{6a^{2/3}}+\frac{\frac{1}{2}\sqrt[6]{a}\left(\sqrt{a}d+\sqrt{3}\sqrt{ce}\right)\int \frac{1}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{\cdots}}\cdots}\cdots}{\cdots}}{c}$$

$\downarrow$ 27

$$\frac{dx}{c}-\frac{\frac{\frac{1}{2}\sqrt[6]{a}\left(\sqrt{a}d-\sqrt{3}\sqrt{ce}\right)\int \frac{1}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx+\frac{1}{2}\left(\sqrt{3}\sqrt{a}d+\sqrt{ce}\right)\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{c}x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a^{2/3}}+\frac{\frac{1}{2}\sqrt[6]{a}\left(\sqrt{a}d+\sqrt{3}\sqrt{ce}\right)\int \frac{1}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx\cdots}{6a^{2\cdots}}}{c}$$

$\downarrow$ 1082

$$\frac{dx}{c}-\frac{\frac{\frac{1}{2}\left(\sqrt{3}\sqrt{a}d+\sqrt{ce}\right)\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{c}x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx+\frac{\sqrt[3]{a}\left(\sqrt{a}d-\sqrt{3}\sqrt{ce}\right)\int \frac{1}{-\left(1-\frac{2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)^2-\frac{1}{3}}d\left(1-\frac{2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)}{\sqrt{3}\sqrt[6]{c}}}{6a^{2/3}}+\frac{\frac{1}{2}\left(\sqrt{3}\sqrt{a}d-\sqrt{ce}\right)\int \frac{2\sqrt[6]{c}x+\sqrt{3}\sqrt[6]{a}}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+\cdots}\cdots}{\cdots}}{c}$$

$\downarrow$ 217

3.38. $\int \frac{d+\frac{e}{x^3}}{c+\frac{a}{x^6}}\,dx$

$$\frac{dx}{c}-\frac{\frac{\frac{1}{2}\left(\sqrt{3}\sqrt{a}d+\sqrt{c}e\right)\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{c}x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx-\frac{\sqrt[3]{a}\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)\right)\left(\sqrt{a}d-\sqrt{3}\sqrt{c}e\right)}{\sqrt[6]{c}}}{6a^{2/3}}+\frac{\frac{1}{2}\left(\sqrt{3}\sqrt{a}d-\sqrt{c}e\right)\int \frac{2\sqrt[6]{c}x+\sqrt{3}\sqrt[6]{a}}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx+\sqrt[3]{a}\cdots}{6a^{2/3}}}{c}$$

$$\Big\downarrow \text{1103}$$

$$\frac{dx}{c}-\frac{\frac{-\frac{\sqrt[3]{a}\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)\right)\left(\sqrt{a}d-\sqrt{3}\sqrt{c}e\right)}{\sqrt[6]{c}}-\frac{\sqrt[3]{a}\left(\sqrt{3}\sqrt{a}d+\sqrt{c}e\right)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{2\sqrt[6]{c}}}{6a^{2/3}}+\frac{\frac{\sqrt[3]{a}\arctan\left(\sqrt{3}\left(\frac{2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}+1\right)\right)\left(\sqrt{a}d+\sqrt{3}\cdots\right)}{\sqrt[6]{c}}\cdots}{\cdots}}{c}$$

input
```
Int[(d + e/x^3)/(c + a/x^6),x]
```

output
```
(d*x)/c - (((Sqrt[a]*d*ArcTan[(c^(1/6)*x)/a^(1/6)])/c^(1/6) + (c^(1/3)*e*L
og[a^(1/3) + c^(1/3)*x^2])/2)/(3*a^(1/3)) + (-((a^(1/3)*(Sqrt[a]*d - Sqrt[
3]*Sqrt[c]*e)*ArcTan[Sqrt[3]*(1 - (2*c^(1/6)*x)/(Sqrt[3]*a^(1/6)))])/c^(1/
6)) - (a^(1/3)*(Sqrt[3]*Sqrt[a]*d + Sqrt[c]*e)*Log[a^(1/3) - Sqrt[3]*a^(1/
6)*c^(1/6)*x + c^(1/3)*x^2])/(2*c^(1/6)))/(6*a^(2/3)) + ((a^(1/3)*(Sqrt[a]
*d + Sqrt[3]*Sqrt[c]*e)*ArcTan[Sqrt[3]*(1 + (2*c^(1/6)*x)/(Sqrt[3]*a^(1/6)
))])/c^(1/6) + (a^(1/3)*(Sqrt[3]*Sqrt[a]*d - Sqrt[c]*e)*Log[a^(1/3) + Sqrt
[3]*a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(2*c^(1/6)))/(6*a^(2/3)))/c
```

3.38.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1728
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> Int[x^(n*(2*p + q))*(e + d/x^n)^q*(c + a/x^(2*n))^p, x] /; FreeQ[{a,
 c, d, e, n}, x] && EqQ[n2, 2*n] && IntegersQ[p, q] && NegQ[n]
```

rule 1746
```
Int[((d_) + (e_.)*(x_)^3)/((a_) + (c_.)*(x_)^6), x_Symbol] :> With[{q = Rt[
c/a, 6]}, Simp[1/(3*a*q^2)   Int[(q^2*d - e*x)/(1 + q^2*x^2), x], x] + (Sim
p[1/(6*a*q^2)   Int[(2*q^2*d - (Sqrt[3]*q^3*d - e)*x)/(1 - Sqrt[3]*q*x + q^
2*x^2), x], x] + Simp[1/(6*a*q^2)   Int[(2*q^2*d + (Sqrt[3]*q^3*d + e)*x)/(
1 + Sqrt[3]*q*x + q^2*x^2), x], x])] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2
 + a*e^2, 0] && PosQ[c/a]
```

3.38. $\int \frac{d+\frac{e}{x^3}}{c+\frac{a}{x^6}} \, dx$

rule 1827
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (c_.)*(x_)^(n2_))^(
p_), x_Symbol] :> Simp[e*f^(n - 1)*(f*x)^(m - n + 1)*((a + c*x^(2*n))^(p +
1)/(c*(m + n*(2*p + 1) + 1))), x] - Simp[f^n/(c*(m + n*(2*p + 1) + 1))   In
t[(f*x)^(m - n)*(a + c*x^(2*n))^p*(a*e*(m - n + 1) - c*d*(m + n*(2*p + 1) +
 1)*x^n), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && EqQ[n2, 2*n] && IGtQ[n,
 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && IntegerQ[p]
```

3.38.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.16 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.14

method	result
risch	$\frac{dx}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}c+a\right)}\frac{\left(\_R^{3}ce-da\right)\ln\left(x-\_R\right)}{\_R^{5}}}{6c^{2}}$
default	$\frac{dx}{c}+\frac{c\ln\left(x^{2}-\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}x+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)\left(\frac{a}{c}\right)^{\frac{2}{3}}e}{12a}+\frac{\ln\left(x^{2}-\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}x+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}d}{12}+\frac{c\left(\frac{a}{c}\right)^{\frac{2}{3}}\arctan\left(\frac{2x}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}-\sqrt{3}\right)\sqrt{3}e}{6a}-\frac{\left(\frac{a}{c}\right)^{\frac{1}{6}}\arctan\left(\ldots\right.}{6}$

input
```
int((d+e/x^3)/(c+a/x^6),x,method=_RETURNVERBOSE)
```

output
```
d*x/c+1/6/c^2*sum((_R^3*c*e-a*d)/_R^5*ln(x-_R),_R=RootOf(_Z^6*c+a))
```

3.38.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1608 vs. 2(213) = 426.

Time = 0.39 (sec) , antiderivative size = 1608, normalized size of antiderivative = 5.17

$$\int \frac{d+\frac{e}{x^3}}{c+\frac{a}{x^6}}\,dx = \text{Too large to display}$$

input
```
integrate((d+e/x^3)/(c+a/x^6),x, algorithm="fricas")
```

output
```
1/12*(2*c*((a*c^3*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7))
 + 3*a*d^2*e - c*e^3)/(a*c^3))^(1/3)*log(-(a^2*d^5 - 2*a*c*d^3*e^2 - 3*c^2
*d*e^4)*x + (a*c^5*e*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^
7)) + a^2*c*d^4 - 3*a*c^2*d^2*e^2)*((a*c^3*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2
+ 9*c^2*d^2*e^4)/(a*c^7)) + 3*a*d^2*e - c*e^3)/(a*c^3))^(1/3)) - (sqrt(-3)
*c + c)*((a*c^3*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)) +
 3*a*d^2*e - c*e^3)/(a*c^3))^(1/3)*log(-(a^2*d^5 - 2*a*c*d^3*e^2 - 3*c^2*d
*e^4)*x - 1/2*(a^2*c*d^4 - 3*a*c^2*d^2*e^2 + sqrt(-3)*(a^2*c*d^4 - 3*a*c^2
*d^2*e^2) + (sqrt(-3)*a*c^5*e + a*c^5*e)*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2 +
9*c^2*d^2*e^4)/(a*c^7)))*((a*c^3*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^
2*e^4)/(a*c^7)) + 3*a*d^2*e - c*e^3)/(a*c^3))^(1/3)) + (sqrt(-3)*c - c)*((
a*c^3*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)) + 3*a*d^2*e
 - c*e^3)/(a*c^3))^(1/3)*log(-(a^2*d^5 - 2*a*c*d^3*e^2 - 3*c^2*d*e^4)*x -
1/2*(a^2*c*d^4 - 3*a*c^2*d^2*e^2 - sqrt(-3)*(a^2*c*d^4 - 3*a*c^2*d^2*e^2)
- (sqrt(-3)*a*c^5*e - a*c^5*e)*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*
e^4)/(a*c^7)))*((a*c^3*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*
c^7)) + 3*a*d^2*e - c*e^3)/(a*c^3))^(1/3)) + 2*c*(-(a*c^3*sqrt(-(a^2*d^6 -
 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)) - 3*a*d^2*e + c*e^3)/(a*c^3))^(1/
3)*log(-(a^2*d^5 - 2*a*c*d^3*e^2 - 3*c^2*d*e^4)*x - (a*c^5*e*sqrt(-(a^2*d^
6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)) - a^2*c*d^4 + 3*a*c^2*d^2*e...
```

3.38.6 Sympy [A] (verification not implemented)

Time = 5.67 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.54

$$\int \frac{d+\frac{e}{x^3}}{c+\frac{a}{x^6}}\,dx$$
$$=\text{RootSum}\left(46656t^6a^2c^7+t^3\left(-1296a^2c^4d^2e+432ac^5e^3\right)+a^3d^6+3a^2cd^4e^2+3ac^2d^2e^4+c^3e^6,\left(t\mapsto t\log\right.\right.$$
$$+\frac{dx}{c}$$

input
```
integrate((d+e/x**3)/(c+a/x**6),x)
```

output
```
RootSum(46656*_t**6*a**2*c**7 + _t**3*(-1296*a**2*c**4*d**2*e + 432*a*c**5
*e**3) + a**3*d**6 + 3*a**2*c*d**4*e**2 + 3*a*c**2*d**2*e**4 + c**3*e**6,
Lambda(_t, _t*log(x + (-1296*_t**4*a*c**5*e - 6*_t*a**2*c*d**4 + 36*_t*a*c
**2*d**2*e**2 - 6*_t*c**3*e**4)/(a**2*d**5 - 2*a*c*d**3*e**2 - 3*c**2*d*e*
*4)))) + d*x/c
```

3.38.7 Maxima [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 295, normalized size of antiderivative = 0.95

$$\int \frac{d+\frac{e}{x^3}}{c+\frac{a}{x^6}}\,dx = \frac{dx}{c} - \frac{\frac{2\,c^{\frac{1}{3}}e\log\left(c^{\frac{1}{3}}x^2+a^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}} + \frac{4\,a^{\frac{1}{3}}d\arctan\left(\frac{c^{\frac{1}{3}}x}{\sqrt{a^{\frac{1}{3}}c^{\frac{1}{3}}}}\right)}{\sqrt{a^{\frac{1}{3}}c^{\frac{1}{3}}}} + \frac{\left(\sqrt{3}a^{\frac{7}{6}}\sqrt{c}d-a^{\frac{2}{3}}ce\right)\log\left(c^{\frac{1}{3}}x^2+\sqrt{3}a^{\frac{1}{6}}c^{\frac{1}{6}}x+a^{\frac{1}{3}}\right)}{ac^{\frac{2}{3}}} - \frac{\left(\sqrt{3}a^{\frac{7}{6}}\sqrt{c}d+a^{\frac{2}{3}}ce\right)\log\left(c^{\frac{1}{3}}x^2-\right.}{ac^{\frac{2}{3}}}}{12\,c}$$

input
```
integrate((d+e/x^3)/(c+a/x^6),x, algorithm="maxima")
```

output
```
d*x/c - 1/12*(2*c^(1/3)*e*log(c^(1/3)*x^2 + a^(1/3))/a^(1/3) + 4*a^(1/3)*d
*arctan(c^(1/3)*x/sqrt(a^(1/3)*c^(1/3)))/sqrt(a^(1/3)*c^(1/3)) + (sqrt(3)*
a^(7/6)*sqrt(c)*d - a^(2/3)*c*e)*log(c^(1/3)*x^2 + sqrt(3)*a^(1/6)*c^(1/6)
*x + a^(1/3))/(a*c^(2/3)) - (sqrt(3)*a^(7/6)*sqrt(c)*d + a^(2/3)*c*e)*log(
c^(1/3)*x^2 - sqrt(3)*a^(1/6)*c^(1/6)*x + a^(1/3))/(a*c^(2/3)) + 2*(sqrt(3
)*a^(5/6)*c^(7/6)*e + a^(4/3)*c^(2/3)*d)*arctan((2*c^(1/3)*x + sqrt(3)*a^(
1/6)*c^(1/6))/sqrt(a^(1/3)*c^(1/3)))/(a*c^(2/3)*sqrt(a^(1/3)*c^(1/3))) - 2
*(sqrt(3)*a^(5/6)*c^(7/6)*e - a^(4/3)*c^(2/3)*d)*arctan((2*c^(1/3)*x - sqr
t(3)*a^(1/6)*c^(1/6))/sqrt(a^(1/3)*c^(1/3)))/(a*c^(2/3)*sqrt(a^(1/3)*c^(1/
3))))/c
```

3.38.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 290, normalized size of antiderivative = 0.93

$$\begin{aligned}\int \frac{d+\frac{e}{x^3}}{c+\frac{a}{x^6}}\,dx = & -\frac{e|c|\log\left(x^2+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{6\,(ac^5)^{\frac{1}{3}}} + \frac{dx}{c} - \frac{(ac^5)^{\frac{1}{6}}\,d\arctan\left(\frac{x}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{3\,c^2} \\ & - \frac{\left((ac^5)^{\frac{1}{6}}\,ac^2d+\sqrt{3}(ac^5)^{\frac{2}{3}}\,e\right)\arctan\left(\frac{2\,x+\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6\,ac^4} \\ & - \frac{\left((ac^5)^{\frac{1}{6}}\,ac^2d-\sqrt{3}(ac^5)^{\frac{2}{3}}\,e\right)\arctan\left(\frac{2\,x-\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6\,ac^4} \\ & - \frac{\left(\sqrt{3}(ac^5)^{\frac{1}{6}}\,ac^2d-(ac^5)^{\frac{2}{3}}\,e\right)\log\left(x^2+\sqrt{3}x\left(\frac{a}{c}\right)^{\frac{1}{6}}+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12\,ac^4} \\ & + \frac{\left(\sqrt{3}(ac^5)^{\frac{1}{6}}\,ac^2d+(ac^5)^{\frac{2}{3}}\,e\right)\log\left(x^2-\sqrt{3}x\left(\frac{a}{c}\right)^{\frac{1}{6}}+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12\,ac^4}\end{aligned}$$

input
```
integrate((d+e/x^3)/(c+a/x^6),x, algorithm="giac")
```

output
```
-1/6*e*abs(c)*log(x^2 + (a/c)^(1/3))/(a*c^5)^(1/3) + d*x/c - 1/3*(a*c^5)^(
1/6)*d*arctan(x/(a/c)^(1/6))/c^2 - 1/6*((a*c^5)^(1/6)*a*c^2*d + sqrt(3)*(a
*c^5)^(2/3)*e)*arctan((2*x + sqrt(3)*(a/c)^(1/6))/(a/c)^(1/6))/(a*c^4) - 1
/6*((a*c^5)^(1/6)*a*c^2*d - sqrt(3)*(a*c^5)^(2/3)*e)*arctan((2*x - sqrt(3)
*(a/c)^(1/6))/(a/c)^(1/6))/(a*c^4) - 1/12*(sqrt(3)*(a*c^5)^(1/6)*a*c^2*d -
 (a*c^5)^(2/3)*e)*log(x^2 + sqrt(3)*x*(a/c)^(1/6) + (a/c)^(1/3))/(a*c^4) +
 1/12*(sqrt(3)*(a*c^5)^(1/6)*a*c^2*d + (a*c^5)^(2/3)*e)*log(x^2 - sqrt(3)*
x*(a/c)^(1/6) + (a/c)^(1/3))/(a*c^4)
```

3.38.9 Mupad [B] (verification not implemented)

Time = 9.37 (sec) , antiderivative size = 1308, normalized size of antiderivative = 4.21

$$\int \frac{d+\frac{e}{x^3}}{c+\frac{a}{x^6}}\,dx$$

$$= \ln\left(e\,x\,\sqrt{-a^3\,c^7}-a^2\,c^4\,\left(-\frac{a\,c^5\,e^3+a\,d^3\,\sqrt{-a^3\,c^7}-3\,a^2\,c^4\,d^2\,e-3\,c\,d\,e^2\,\sqrt{-a^3\,c^7}}{a^2\,c^7}\right)^{1/3}+a^2\,c^3\,d\,x\right)\,\left(-\frac{a\,c^5\,e^3+a\,d^3\,\sqrt{-a^3\,c^7}-3\,a^2\,c^4\,d^2\,e-3\,c\,d\,e^2\,\sqrt{-a^3\,c^7}}{216\,a^2\,c^7}\right)^{1/3}$$

$$+\ln\left(e\,x\,\sqrt{-a^3\,c^7}+a^2\,c^4\,\left(-\frac{a\,c^5\,e^3-a\,d^3\,\sqrt{-a^3\,c^7}-3\,a^2\,c^4\,d^2\,e+3\,c\,d\,e^2\,\sqrt{-a^3\,c^7}}{a^2\,c^7}\right)^{1/3}-a^2\,c^3\,d\,x\right)\,\left(-\right.$$

input
```
int((d + e/x^3)/(c + a/x^6),x)
```

output
```
log(e*x*(-a^3*c^7)^(1/2) - a^2*c^4*(-(a*c^5*e^3 + a*d^3*(-a^3*c^7)^(1/2) -
 3*a^2*c^4*d^2*e - 3*c*d*e^2*(-a^3*c^7)^(1/2))/(a^2*c^7))^(1/3) + a^2*c^3*
d*x)*(-(a*c^5*e^3 + a*d^3*(-a^3*c^7)^(1/2) - 3*a^2*c^4*d^2*e - 3*c*d*e^2*(
-a^3*c^7)^(1/2))/(216*a^2*c^7))^(1/3) + log(e*x*(-a^3*c^7)^(1/2) + a^2*c^4
*(-(a*c^5*e^3 - a*d^3*(-a^3*c^7)^(1/2) - 3*a^2*c^4*d^2*e + 3*c*d*e^2*(-a^3
*c^7)^(1/2))/(a^2*c^7))^(1/3) - a^2*c^3*d*x)*(-(a*c^5*e^3 - a*d^3*(-a^3*c^
7)^(1/2) - 3*a^2*c^4*d^2*e + 3*c*d*e^2*(-a^3*c^7)^(1/2))/(216*a^2*c^7))^(1
/3) + log(2*e*x*(-a^3*c^7)^(1/2) + a^2*c^4*(-(a*c^5*e^3 + a*d^3*(-a^3*c^7)
^(1/2) - 3*a^2*c^4*d^2*e - 3*c*d*e^2*(-a^3*c^7)^(1/2))/(a^2*c^7))^(1/3) -
3^(1/2)*a^2*c^4*(-(a*c^5*e^3 + a*d^3*(-a^3*c^7)^(1/2) - 3*a^2*c^4*d^2*e -
3*c*d*e^2*(-a^3*c^7)^(1/2))/(a^2*c^7))^(1/3)*1i + 2*a^2*c^3*d*x)*((3^(1/2)
*1i)/2 - 1/2)*(-(a*c^5*e^3 + a*d^3*(-a^3*c^7)^(1/2) - 3*a^2*c^4*d^2*e - 3*
c*d*e^2*(-a^3*c^7)^(1/2))/(216*a^2*c^7))^(1/3) - log(2*e*x*(-a^3*c^7)^(1/2
) + a^2*c^4*(-(a*c^5*e^3 + a*d^3*(-a^3*c^7)^(1/2) - 3*a^2*c^4*d^2*e - 3*c*
d*e^2*(-a^3*c^7)^(1/2))/(a^2*c^7))^(1/3) + 3^(1/2)*a^2*c^4*(-(a*c^5*e^3 +
a*d^3*(-a^3*c^7)^(1/2) - 3*a^2*c^4*d^2*e - 3*c*d*e^2*(-a^3*c^7)^(1/2))/(a^
2*c^7))^(1/3)*1i + 2*a^2*c^3*d*x)*((3^(1/2)*1i)/2 + 1/2)*(-(a*c^5*e^3 + a*
d^3*(-a^3*c^7)^(1/2) - 3*a^2*c^4*d^2*e - 3*c*d*e^2*(-a^3*c^7)^(1/2))/(216*
a^2*c^7))^(1/3) - log(a^2*c^4*(-(a*c^5*e^3 - a*d^3*(-a^3*c^7)^(1/2) - 3*a^
2*c^4*d^2*e + 3*c*d*e^2*(-a^3*c^7)^(1/2))/(a^2*c^7))^(1/3) - 2*e*x*(-a^...
```

3.38. $\int \frac{d+\frac{e}{x^3}}{c+\frac{a}{x^6}}\,dx$

3.39 $\int \frac{d+\frac{e}{x^3}}{c+\frac{a}{x^6}+\frac{b}{x^3}} \, dx$

3.39.1 Optimal result

Integrand size = 22, antiderivative size = 716

$$\int \frac{d+\frac{e}{x^3}}{c+\frac{a}{x^6}+\frac{b}{x^3}}\,dx = \frac{dx}{c} + \frac{\left(bd-ce-\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$
$$+\frac{\left(bd-ce+\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}c^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$
$$-\frac{\left(bd-ce-\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$
$$-\frac{\left(bd-ce+\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$
$$+\frac{\left(bd-ce-\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$
$$+\frac{\left(bd-ce+\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}c^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

output
```
d*x/c-1/6*ln(2^(1/3)*c^(1/3)*x+(b-(-4*a*c+b^2)^(1/2))^(1/3))*(b*d-c*e+(2*a
*c*d-b^2*d+b*c*e)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4/3)/(b-(-4*a*c+b^2)^(1/2
))^(2/3)+1/12*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b-(-4*a*c+b^2)^(1/
2))^(1/3)+(b-(-4*a*c+b^2)^(1/2))^(2/3))*(b*d-c*e+(2*a*c*d-b^2*d+b*c*e)/(-4
*a*c+b^2)^(1/2))*2^(2/3)/c^(4/3)/(b-(-4*a*c+b^2)^(1/2))^(2/3)+1/6*arctan(1
/3*(1-2*2^(1/3)*c^(1/3)*x/(b-(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*(b*d-c*e+
(2*a*c*d-b^2*d+b*c*e)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4/3)*3^(1/2)/(b-(-4*a
*c+b^2)^(1/2))^(2/3)-1/6*ln(2^(1/3)*c^(1/3)*x+(b+(-4*a*c+b^2)^(1/2))^(1/3)
)*(b*d-c*e+(-2*a*c*d+b^2*d-b*c*e)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4/3)/(b+(
-4*a*c+b^2)^(1/2))^(2/3)+1/12*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b+
(-4*a*c+b^2)^(1/2))^(1/3)+(b+(-4*a*c+b^2)^(1/2))^(2/3))*(b*d-c*e+(-2*a*c*d
+b^2*d-b*c*e)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4/3)/(b+(-4*a*c+b^2)^(1/2))^(
2/3)+1/6*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b+(-4*a*c+b^2)^(1/2))^(1/3))*3
^(1/2))*(b*d-c*e+(-2*a*c*d+b^2*d-b*c*e)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4/3
)*3^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(2/3)
```

3.39.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.12

$$\int \frac{d+\frac{e}{x^3}}{c+\frac{a}{x^6}+\frac{b}{x^3}}\,dx = \frac{dx}{c}-\frac{\text{RootSum}\left[a+b\#1^3+c\#1^6\&,\frac{ad\log(x-\#1)+bd\log(x-\#1)\#1^3-ce\log(x-\#1)\#1^3}{b\#1^2+2c\#1^5}\&\right]}{3c}$$

input
```
Integrate[(d + e/x^3)/(c + a/x^6 + b/x^3),x]
```

output
```
(d*x)/c - RootSum[a + b*#1^3 + c*#1^6 & , (a*d*Log[x - #1] + b*d*Log[x - #
1]*#1^3 - c*e*Log[x - #1]*#1^3)/(b*#1^2 + 2*c*#1^5) & ]/(3*c)
```

3.39.3 Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 550, normalized size of antiderivative = 0.77, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {1727, 1826, 1752, 750, 16, 27, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+\frac{e}{x^3}}{\frac{a}{x^6}+\frac{b}{x^3}+c}dx$$

$$\Big\downarrow \text{1727}$$

$$\int \frac{x^3\left(dx^3+e\right)}{a+bx^3+cx^6}dx$$

$$\Big\downarrow \text{1826}$$

$$\frac{dx}{c}-\frac{\int \frac{(bd-ce)x^3+ad}{cx^6+bx^3+a}dx}{c}$$

$$\Big\downarrow \text{1752}$$

$$\frac{dx}{c}-\frac{\frac{1}{2}\left(-\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)\int \frac{1}{cx^3+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}\left(\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)\int \frac{1}{cx^3+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{c}$$

$$\Big\downarrow \text{750}$$

$$\frac{dx}{c}-\frac{\frac{1}{2}\left(-\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)\left(\frac{2^{2/3}\int \frac{2\left(2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}x\right)}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}\int \frac{1}{\sqrt[3]{c}x+\frac{\sqrt[3]{b-\sqrt{b\ldots}}}{\sqrt[3]{\ldots}}}\ldots}{3\left(b-\sqrt{b^2-4ac}\right)\ldots}\ldots\right)\ldots}{\ldots}$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2}\left(-\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)\left(\frac{2^{2/3}\int\frac{2\left(2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}x\right)}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-\ldots}}\right.}{3\sqrt[3]{c}\left(b-\sqrt{b^2-\ldots}\right.}\right.$$

$\downarrow$ 27

$$\frac{1}{2}\left(-\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)\left(\frac{2\ 2^{2/3}\int\frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}x}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2\ldots}}\right.}{3\sqrt[3]{c}\left(b-\sqrt{b^2\ldots}\right.}\right.$$

$\downarrow$ 1142

$$\frac{1}{2}\left(-\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)\left(\frac{2\ 2^{2/3}\left(\frac{3\sqrt[3]{b-\sqrt{b^2-4ac}}\int\frac{1}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{2\sqrt[3]{2}}-\ldots\int\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\right.$$

$\downarrow$ 25

3.39. $\int\frac{d+\frac{e}{x^3}}{c+\frac{a}{x^6}+\frac{b}{x^3}}\,dx$

$$\frac{1}{2}\left(-\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)\left(\frac{2\,2^{2/3}\left(\frac{3\sqrt[3]{b-\sqrt{b^2-4ac}}\int \frac{1}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{2\sqrt[3]{2}}+\cdots\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\cdots\right.$$

$$\frac{dx}{c}-$$

$\downarrow$ 27

$$\frac{1}{2}\left(-\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)\left(\frac{2\,2^{2/3}\left(\frac{3\sqrt[3]{b-\sqrt{b^2-4ac}}\int \frac{1}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{2\sqrt[3]{2}}+\frac{1}{4}\cdots\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\cdots\right.$$

$$\frac{dx}{c}-$$

$\downarrow$ 1082

3.39. $\int \frac{d+\frac{e}{x^3}}{c+\frac{a}{x^6}+\frac{b}{x^3}}\,dx$

$$\frac{dx}{c}-\frac{\frac{1}{2}\left(-\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)\left(\frac{2\,2^{2/3}\left(\frac{1}{4}\int\frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-4\sqrt[3]{c}x}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{2}\sqrt[3]{\cdots}}{\sqrt[3]{b-\sqrt{b^2\cdots}}}\right.\cdots}\cdots}{\cdots}\right.\cdots}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\cdots\right.}{\cdots}$$

$\downarrow$ 217

$$\frac{dx}{c}-\frac{\frac{1}{2}\left(-\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)\left(\frac{2\,2^{2/3}\left(\frac{1}{4}\int\frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-4\sqrt[3]{c}x}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{\cdots}}{\sqrt[3]{b-\cdots}}}{\cdots}\right.\cdots}{2\sqrt[3]{c}\cdots}\right.\cdots}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\cdots\right.}{\cdots}$$

$\downarrow$ 1103

3.39. $\displaystyle\int\frac{d+\frac{e}{x^3}}{c+\frac{a}{x^6}+\frac{b}{x^3}}\,dx$

$$\frac{dx}{c} - \frac{\frac{1}{2}\left(-\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)\left(\frac{2\,2^{2/3}\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2\sqrt[3]{c}}-\frac{\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)\dots\right)}{4\sqrt[3]{c}}\dots\right)}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\dots\right)}{\dots}$$

input
```
Int[(d + e/x^3)/(c + a/x^6 + b/x^3),x]
```

output
```
(d*x)/c - (((b*d - c*e - (b^2*d - 2*a*c*d - b*c*e)/Sqrt[b^2 - 4*a*c])*((2^
(2/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*c^(1/3)*(
b - Sqrt[b^2 - 4*a*c])^(2/3)) + (2*2^(2/3)*(-1/2*(Sqrt[3]*ArcTan[(1 - (2*2
^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/c^(1/3) - Log[(
b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/
3)*x + 2^(2/3)*c^(2/3)*x^2]/(4*c^(1/3))))/(3*(b - Sqrt[b^2 - 4*a*c])^(2/3)
)))/2 + ((b*d - c*e + (b^2*d - 2*a*c*d - b*c*e)/Sqrt[b^2 - 4*a*c])*((2^(2/
3)*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*c^(1/3)*(b +
 Sqrt[b^2 - 4*a*c])^(2/3)) + (2*2^(2/3)*(-1/2*(Sqrt[3]*ArcTan[(1 - (2*2^(1
/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/c^(1/3) - Log[(b +
 Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*
x + 2^(2/3)*c^(2/3)*x^2]/(4*c^(1/3))))/(3*(b + Sqrt[b^2 - 4*a*c])^(2/3))))
/2)/c
```

3.39. $\int \frac{d+\frac{e}{x^3}}{c+\frac{a}{x^6}+\frac{b}{x^3}}\,dx$

3.39.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.39. $\int \frac{d+\frac{e}{x^3}}{c+\frac{a}{x^6}+\frac{b}{x^3}}\,dx$

rule 1727
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (e_.)*(x_)^(
n_))^(q_.), x_Symbol] :> Int[x^(n*(2*p + q))*(e + d/x^n)^q*(c + b/x^n + a/x
^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && IntegersQ[
p, q] && NegQ[n]
```

rule 1752
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2
 - 4*a*c] ||  !IGtQ[n/2, 0])
```

rule 1826
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[e*f^(n - 1)*(f*x)^(m - n + 1)*((a
+ b*x^n + c*x^(2*n))^(p + 1)/(c*(m + n*(2*p + 1) + 1))), x] - Simp[f^n/(c*(
m + n*(2*p + 1) + 1))   Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*
e*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x]
, x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Intege
rQ[p]
```

3.39.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.62 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.09

method	result	size
default	$\frac{dx}{c}+\frac{\sum_{\_R=\operatorname{RootOf}\left(\_Z^{6}c+\_Z^{3}b+a\right)}\frac{\left(\left(-bd+ec\right)\_R^{3}-da\right)\ln\left(x-\_R\right)}{2\_R^{5}c+\_R^{2}b}}{3c}$	67
risch	$\frac{dx}{c}+\frac{\sum_{\_R=\operatorname{RootOf}\left(\_Z^{6}c+\_Z^{3}b+a\right)}\frac{\left(\left(-bd+ec\right)\_R^{3}-da\right)\ln\left(x-\_R\right)}{2\_R^{5}c+\_R^{2}b}}{3c}$	67

input
```
int((d+e/x^3)/(c+a/x^6+b/x^3),x,method=_RETURNVERBOSE)
```

output
```
d*x/c+1/3/c*sum(((-b*d+c*e)*_R^3-d*a)/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf
(_Z^6*c+_Z^3*b+a))
```

3.39.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8707 vs. 2(580) = 1160.

Time = 3.35 (sec) , antiderivative size = 8707, normalized size of antiderivative = 12.16

$$\int \frac{d+\frac{e}{x^3}}{c+\frac{a}{x^6}+\frac{b}{x^3}}\,dx = \text{Too large to display}$$

input
```
integrate((d+e/x^3)/(c+a/x^6+b/x^3),x, algorithm="fricas")
```

output
```
Too large to include
```

3.39.6 Sympy [F(-1)]

Timed out.

$$\int \frac{d+\frac{e}{x^3}}{c+\frac{a}{x^6}+\frac{b}{x^3}}\,dx = \text{Timed out}$$

input
```
integrate((d+e/x**3)/(c+a/x**6+b/x**3),x)
```

output
```
Timed out
```

3.39.7 Maxima [F]

$$\int \frac{d+\frac{e}{x^3}}{c+\frac{a}{x^6}+\frac{b}{x^3}}\,dx = \int \frac{d+\frac{e}{x^3}}{c+\frac{b}{x^3}+\frac{a}{x^6}}\,dx$$

input
```
integrate((d+e/x^3)/(c+a/x^6+b/x^3),x, algorithm="maxima")
```

output
```
d*x/c + integrate(-((b*d - c*e)*x^3 + a*d)/(c*x^6 + b*x^3 + a), x)/c
```

3.39.8 Giac [F]

$$\int \frac{d+\frac{e}{x^3}}{c+\frac{a}{x^6}+\frac{b}{x^3}}\,dx=\int \frac{d+\frac{e}{x^3}}{c+\frac{b}{x^3}+\frac{a}{x^6}}\,dx$$

input
```
integrate((d+e/x^3)/(c+a/x^6+b/x^3),x, algorithm="giac")
```

output
```
integrate((d + e/x^3)/(c + b/x^3 + a/x^6), x)
```

3.39.9 Mupad [B] (verification not implemented)

Time = 25.62 (sec) , antiderivative size = 11453, normalized size of antiderivative = 16.00

$$\int \frac{d+\frac{e}{x^3}}{c+\frac{a}{x^6}+\frac{b}{x^3}}\,dx=\text{Too large to display}$$

input
```
int((d + e/x^3)/(c + a/x^6 + b/x^3),x)
```

output
```
log((3*a*x*(a*b^4*d^4 - 2*a*c^4*e^4 - b^5*d^3*e + 2*a^3*c^2*d^4 + b^2*c^3*
e^4 - 4*a^2*b^2*c*d^4 - 3*b^3*c^2*d*e^3 + 3*b^4*c*d^2*e^2 + 8*a*b*c^3*d*e^
3 + 2*a*b^3*c*d^3*e + 4*a^2*b*c^2*d^3*e - 9*a*b^2*c^2*d^2*e^2))/c - (2^(2/
3)*((2^(1/3)*(81*a*c^3*e*x*(4*a*c - b^2)^2 - (81*2^(2/3)*a*b*c^3*(4*a*c -
b^2)^2*((b^7*d^3 + b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^5*e^3 - b^4
*c^3*e^3 - 32*a^3*b*c^3*d^3 + 8*a*b^2*c^4*e^3 - b*c^3*e^3*(-(4*a*c - b^2)^
3)^(1/2) + 48*a^3*c^4*d^2*e + 3*b^5*c^2*d*e^2 + 32*a^2*b^3*c^2*d^3 + 2*a^2
*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*b^6*c*d^2*e - 4*a*b
^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d*e^2 + 27*a*b^4*c^2*d^2*
e + 48*a^2*b*c^4*d*e^2 - 6*a*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c*
d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d^2*e + 3*b^2*c^2*d*e^2*(-
(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4
*a*c - b^2)^3))^(1/3))/2)*((b^7*d^3 + b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 1
6*a^2*c^5*e^3 - b^4*c^3*e^3 - 32*a^3*b*c^3*d^3 + 8*a*b^2*c^4*e^3 - b*c^3*e
^3*(-(4*a*c - b^2)^3)^(1/2) + 48*a^3*c^4*d^2*e + 3*b^5*c^2*d*e^2 + 32*a^2*
b^3*c^2*d^3 + 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*
b^6*c*d^2*e - 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d*e^2
+ 27*a*b^4*c^2*d^2*e + 48*a^2*b*c^4*d*e^2 - 6*a*c^3*d*e^2*(-(4*a*c - b^2)^
3)^(1/2) - 3*b^3*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d^2*e +
 3*b^2*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^2*d^2*e*(-(4*a*c - ...
```

3.40 $\int \frac{d+\frac{e}{x^4}}{c+\frac{a}{x^8}} \, dx$

3.40.1 Optimal result

Integrand size = 17, antiderivative size = 753

$$\int \frac{d+\frac{e}{x^4}}{c+\frac{a}{x^8}}\,dx = \frac{dx}{c}+\frac{\sqrt{2-\sqrt{2}}\left(\left(1+\sqrt{2}\right)\sqrt{a}d+\sqrt{c}e\right)\arctan\left(\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}-2\sqrt[8]{c}x}{\sqrt{2+\sqrt{2}}\sqrt[8]{a}}\right)}{8a^{3/8}c^{9/8}}$$
$$-\frac{\sqrt{2+\sqrt{2}}\left(\sqrt{a}\left(d-\sqrt{2}d\right)+\sqrt{c}e\right)\arctan\left(\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}-2\sqrt[8]{c}x}{\sqrt{2-\sqrt{2}}\sqrt[8]{a}}\right)}{8a^{3/8}c^{9/8}}$$
$$-\frac{\sqrt{2-\sqrt{2}}\left(\left(1+\sqrt{2}\right)\sqrt{a}d+\sqrt{c}e\right)\arctan\left(\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}+2\sqrt[8]{c}x}{\sqrt{2+\sqrt{2}}\sqrt[8]{a}}\right)}{8a^{3/8}c^{9/8}}$$
$$+\frac{\sqrt{2+\sqrt{2}}\left(\sqrt{a}\left(d-\sqrt{2}d\right)+\sqrt{c}e\right)\arctan\left(\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}+2\sqrt[8]{c}x}{\sqrt{2-\sqrt{2}}\sqrt[8]{a}}\right)}{8a^{3/8}c^{9/8}}$$
$$-\frac{\left(\sqrt{a}\left(d-\sqrt{2}d\right)+\sqrt{c}e\right)\log\left(\sqrt[4]{a}-\sqrt{2-\sqrt{2}}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{c}x^2\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}a^{3/8}c^{9/8}}$$
$$+\frac{\left(\sqrt{a}\left(d-\sqrt{2}d\right)+\sqrt{c}e\right)\log\left(\sqrt[4]{a}+\sqrt{2-\sqrt{2}}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{c}x^2\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}a^{3/8}c^{9/8}}$$
$$+\frac{\left(\left(1+\sqrt{2}\right)\sqrt{a}d+\sqrt{c}e\right)\log\left(\sqrt[4]{a}-\sqrt{2+\sqrt{2}}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{c}x^2\right)}{8\sqrt{2\left(2+\sqrt{2}\right)}a^{3/8}c^{9/8}}$$
$$-\frac{\left(\left(1+\sqrt{2}\right)\sqrt{a}d+\sqrt{c}e\right)\log\left(\sqrt[4]{a}+\sqrt{2+\sqrt{2}}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{c}x^2\right)}{8\sqrt{2\left(2+\sqrt{2}\right)}a^{3/8}c^{9/8}}$$

output
```
d*x/c+1/8*arctan((-2*c^(1/8)*x+a^(1/8)*(2-2^(1/2))^(1/2))/a^(1/8)/(2+2^(1/
2))^(1/2))*(d*(1+2^(1/2))*a^(1/2)+e*c^(1/2))*(2-2^(1/2))^(1/2)/a^(3/8)/c^(
9/8)-1/8*arctan((2*c^(1/8)*x+a^(1/8)*(2-2^(1/2))^(1/2))/a^(1/8)/(2+2^(1/2)
)^(1/2))*(d*(1+2^(1/2))*a^(1/2)+e*c^(1/2))*(2-2^(1/2))^(1/2)/a^(3/8)/c^(9/
8)-1/8*ln(a^(1/4)+c^(1/4)*x^2-a^(1/8)*c^(1/8)*x*(2-2^(1/2))^(1/2))*((d-d*2
^(1/2))*a^(1/2)+e*c^(1/2))/a^(3/8)/c^(9/8)/(4-2*2^(1/2))^(1/2)+1/8*ln(a^(1
/4)+c^(1/4)*x^2+a^(1/8)*c^(1/8)*x*(2-2^(1/2))^(1/2))*((d-d*2^(1/2))*a^(1/2
)+e*c^(1/2))/a^(3/8)/c^(9/8)/(4-2*2^(1/2))^(1/2)-1/8*arctan((-2*c^(1/8)*x+
a^(1/8)*(2+2^(1/2))^(1/2))/a^(1/8)/(2-2^(1/2))^(1/2))*((d-d*2^(1/2))*a^(1/
2)+e*c^(1/2))*(2+2^(1/2))^(1/2)/a^(3/8)/c^(9/8)+1/8*arctan((2*c^(1/8)*x+a^
(1/8)*(2+2^(1/2))^(1/2))/a^(1/8)/(2-2^(1/2))^(1/2))*((d-d*2^(1/2))*a^(1/2)
+e*c^(1/2))*(2+2^(1/2))^(1/2)/a^(3/8)/c^(9/8)+1/8*ln(a^(1/4)+c^(1/4)*x^2-a
^(1/8)*c^(1/8)*x*(2+2^(1/2))^(1/2))*(d*(1+2^(1/2))*a^(1/2)+e*c^(1/2))/a^(3
/8)/c^(9/8)/(4+2*2^(1/2))^(1/2)-1/8*ln(a^(1/4)+c^(1/4)*x^2+a^(1/8)*c^(1/8)
*x*(2+2^(1/2))^(1/2))*(d*(1+2^(1/2))*a^(1/2)+e*c^(1/2))/a^(3/8)/c^(9/8)/(4
+2*2^(1/2))^(1/2)
```

3.40.2 Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 551, normalized size of antiderivative = 0.73

$$\int \frac{d+\frac{e}{x^4}}{c+\frac{a}{x^8}}\,dx$$

$$= \frac{8ac^{5/8}dx + 2\arctan\left(\cot\left(\frac{\pi}{8}\right)+\frac{\sqrt[8]{c}x\csc\left(\frac{\pi}{8}\right)}{\sqrt[8]{a}}\right)\left(a^{5/8}ce\cos\left(\frac{\pi}{8}\right)-a^{9/8}\sqrt{c}d\sin\left(\frac{\pi}{8}\right)\right)+\log\left(\sqrt[4]{a}+\sqrt[4]{c}x^2+2\sqrt[8]{a}\right.}{}$$

input
```
Integrate[(d + e/x^4)/(c + a/x^8),x]
```

output
```
(8*a*c^(5/8)*d*x + 2*ArcTan[Cot[Pi/8] + (c^(1/8)*x*Csc[Pi/8])/a^(1/8)]*(a^
(5/8)*c*e*Cos[Pi/8] - a^(9/8)*Sqrt[c]*d*Sin[Pi/8]) + Log[a^(1/4) + c^(1/4)
*x^2 + 2*a^(1/8)*c^(1/8)*x*Sin[Pi/8]]*(a^(5/8)*c*e*Cos[Pi/8] - a^(9/8)*Sqr
t[c]*d*Sin[Pi/8]) + 2*ArcTan[Cot[Pi/8] - (c^(1/8)*x*Csc[Pi/8])/a^(1/8)]*(-
(a^(5/8)*c*e*Cos[Pi/8]) + a^(9/8)*Sqrt[c]*d*Sin[Pi/8]) + Log[a^(1/4) + c^(
1/4)*x^2 - 2*a^(1/8)*c^(1/8)*x*Sin[Pi/8]]*(-(a^(5/8)*c*e*Cos[Pi/8]) + a^(9
/8)*Sqrt[c]*d*Sin[Pi/8]) - 2*ArcTan[(c^(1/8)*x*Sec[Pi/8])/a^(1/8) - Tan[Pi
/8]]*(a^(9/8)*Sqrt[c]*d*Cos[Pi/8] + a^(5/8)*c*e*Sin[Pi/8]) - 2*ArcTan[(c^(
1/8)*x*Sec[Pi/8])/a^(1/8) + Tan[Pi/8]]*(a^(9/8)*Sqrt[c]*d*Cos[Pi/8] + a^(5
/8)*c*e*Sin[Pi/8]) + Log[a^(1/4) + c^(1/4)*x^2 - 2*a^(1/8)*c^(1/8)*x*Cos[P
i/8]]*(a^(9/8)*Sqrt[c]*d*Cos[Pi/8] + a^(5/8)*c*e*Sin[Pi/8]) - Log[a^(1/4)
+ c^(1/4)*x^2 + 2*a^(1/8)*c^(1/8)*x*Cos[Pi/8]]*(a^(9/8)*Sqrt[c]*d*Cos[Pi/8
] + a^(5/8)*c*e*Sin[Pi/8]))/(8*a*c^(13/8))
```

3.40.3 Rubi [A] (verified)

Time = 1.39 (sec) , antiderivative size = 867, normalized size of antiderivative = 1.15, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.706$, Rules used = {1728, 1827, 1745, 27, 1483, 27, 1142, 25, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+\frac{e}{x^4}}{\frac{a}{x^8}+c}dx$$

$$\Big\downarrow \ 1728$$

$$\int \frac{x^4\left(dx^4+e\right)}{a+cx^8}dx$$

$$\Big\downarrow \ 1827$$

$$\frac{dx}{c}-\frac{\int \frac{ad-cex^4}{cx^8+a}dx}{c}$$

$$\Big\downarrow \ 1745$$

$$\frac{dx}{c}-\frac{\frac{\int \frac{\sqrt{a}\left(\sqrt{2}a^{3/4}d-\sqrt[4]{c}(\sqrt{a}d+\sqrt{c}e)x^2\right)}{\sqrt[4]{c}\left(x^4-\frac{\sqrt{2}\sqrt[4]{a}x^2}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}a^{3/4}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{a}\left(\sqrt[4]{c}(\sqrt{a}d+\sqrt{c}e)x^2+\sqrt{2}a^{3/4}d\right)}{\sqrt[4]{c}\left(x^4+\frac{\sqrt{2}\sqrt[4]{a}x^2}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}a^{3/4}\sqrt[4]{c}}}{c}$$

$$\Big\downarrow \ 27$$

$$\frac{dx}{c}-\frac{\frac{\int \frac{\sqrt{2}a^{3/4}d-\sqrt[4]{c}(\sqrt{a}d+\sqrt{c}e)x^2}{x^4-\frac{\sqrt{2}\sqrt[4]{a}x^2}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{c}}+\frac{\int \frac{\sqrt[4]{c}(\sqrt{a}d+\sqrt{c}e)x^2+\sqrt{2}a^{3/4}d}{x^4+\frac{\sqrt{2}\sqrt[4]{a}x^2}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{c}}}{c}$$

$\downarrow$ 1483

$$\frac{dx}{c}-\frac{\frac{\frac{c^{3/8}\int \frac{\sqrt[4]{a}\left(\sqrt{2\left(2-\sqrt{2}\right)}a^{5/8}d+\sqrt[8]{c}\left(\sqrt{a}\left(d-\sqrt{2}d\right)+\sqrt{c}e\right)x\right)}{\sqrt[8]{c}\left(x^2-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}}+\frac{\sqrt[4]{a}}{\sqrt[4]{c}}\right)}dx}{2\sqrt{2-\sqrt{2}}a^{3/8}}+\frac{c^{3/8}\int \frac{\sqrt[4]{a}\left(\sqrt{2\left(2-\sqrt{2}\right)}a^{5/8}d-\sqrt[8]{c}\left(\left(1-\sqrt{2}\right)\sqrt{a}d+\sqrt{c}e\right)x\right)}{\sqrt[8]{c}\left(x^2+\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}}+\frac{\sqrt[4]{a}}{\sqrt[4]{c}}\right)}dx}{2\sqrt{2-\sqrt{2}}a^{3/8}}}{2\sqrt{2}\sqrt[4]{a}\sqrt{c}}+\frac{c^{3/8}\int \frac{\sqrt[4]{a}\left(\sqrt{2\left(2+\sqrt{2}\cdots\right.}\right.}{\sqrt[8]{c}\left(\cdots\right.}\cdots}{\cdots}}{c}$$

$\downarrow$ 27

$$\frac{dx}{c}-\frac{\frac{\frac{\sqrt[4]{c}\int \frac{\sqrt{2\left(2-\sqrt{2}\right)}a^{5/8}d+\sqrt[8]{c}\left(\sqrt{a}\left(d-\sqrt{2}d\right)+\sqrt{c}e\right)x}{x^2-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}}+\frac{\sqrt[4]{a}}{\sqrt[4]{c}}}dx}{2\sqrt{2-\sqrt{2}}\sqrt[8]{a}}+\frac{\sqrt[4]{c}\int \frac{\sqrt{2\left(2-\sqrt{2}\right)}a^{5/8}d-\sqrt[8]{c}\left(\sqrt{a}\left(d-\sqrt{2}d\right)+\sqrt{c}e\right)x}{x^2+\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}}+\frac{\sqrt[4]{a}}{\sqrt[4]{c}}}dx}{2\sqrt{2-\sqrt{2}}\sqrt[8]{a}}}{2\sqrt{2}\sqrt[4]{a}\sqrt{c}}+\frac{\frac{\sqrt[4]{c}\int \frac{\sqrt{2\left(2+\sqrt{2}\right)}a^{5/8}d-\sqrt[8]{c}\left(\left(1+\sqrt{2}\cdots\right.\right.}{x^2-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}}+\cdots}}{2\sqrt{2+\sqrt{2}}\sqrt[8]{a}}\cdots}{\cdots}}{c}$$

$\downarrow$ 1142

$$\frac{dx}{c}-\frac{\frac{\sqrt[4]{c}\left(\frac{1}{2}\sqrt{2-\sqrt{2}}\sqrt[8]{a}\left(\left(1+\sqrt{2}\right)\sqrt{a}d+\sqrt{c}e\right)\int \frac{1}{x^2-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}}+\frac{\sqrt[4]{a}}{\sqrt[4]{c}}}dx+\frac{1}{2}\sqrt[8]{c}\left(\sqrt{a}\left(d-\sqrt{2}d\right)+\sqrt{c}e\right)\int -\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}-2\sqrt[8]{c}x}{\sqrt[8]{c}\left(x^2-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}}+\frac{\sqrt[4]{a}}{\sqrt[4]{c}}\right)}dx\right)}{2\sqrt{2-\sqrt{2}}\sqrt[8]{a}}+\frac{\sqrt[4]{c}\left(\frac{1}{2}\sqrt{\cdots}\right.}{\cdots}}{2\sqrt{2}\sqrt[4]{a}\sqrt{c}}\cdots$$

$\downarrow$ 25

$$\frac{dx}{c}-\frac{\frac{\sqrt[4]{c}\left(\frac{1}{2}\sqrt{2-\sqrt{2}}\sqrt[8]{a}\left(\left(1+\sqrt{2}\right)\sqrt{a}d+\sqrt{c}e\right)\int \frac{1}{x^2-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}}+\frac{\sqrt[4]{a}}{\sqrt[4]{c}}}dx-\frac{1}{2}\sqrt[8]{c}\left(\sqrt{a}\left(d-\sqrt{2}d\right)+\sqrt{c}e\right)\int \frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}-2\sqrt[8]{c}x}{\sqrt[8]{c}\left(x^2-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}}+\frac{\sqrt[4]{a}}{\sqrt[4]{c}}\right)}dx\right)}{2\sqrt{2-\sqrt{2}}\sqrt[8]{a}}+\frac{\sqrt[4]{c}\left(\frac{1}{2}\sqrt{2\cdots}\right.}{\cdots}}{2\sqrt{2}\sqrt[4]{a}\sqrt{c}}\cdots$$

$\downarrow$ 27

3.40. $\int \frac{d+\frac{e}{x^4}}{c+\frac{a}{x^8}}\,dx$

$$\frac{dx}{c}-\frac{\frac{\sqrt[4]{c}\left(\frac{1}{2}\sqrt{2-\sqrt{2}}\sqrt[8]{a}\left(\left(1+\sqrt{2}\right)\sqrt{a}d+\sqrt{c}e\right)\int\frac{1}{x^2-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}}+\frac{\sqrt[4]{a}}{\sqrt[4]{c}}}dx-\frac{1}{2}\left(\sqrt{a}\left(d-\sqrt{2}d\right)+\sqrt{c}e\right)\int\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}-2\sqrt[8]{c}x}{x^2-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}}+\frac{\sqrt[4]{a}}{\sqrt[4]{c}}}dx\right)}{2\sqrt{2-\sqrt{2}}\sqrt[8]{a}}+\frac{\sqrt[4]{c}\left(\frac{1}{2}\sqrt{2-\sqrt{2}}\sqrt[8]{a}\left(\left(1\cdots\right.\right.\right.}{\cdots}}{2\sqrt{2}\sqrt[4]{a}\sqrt{c}}$$

$$\Big\downarrow \; 1083$$

$$\frac{dx}{c}-\frac{\frac{\sqrt[4]{c}\left(-\frac{1}{2}\left(\sqrt{a}\left(d-\sqrt{2}d\right)+\sqrt{c}e\right)\int\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}-2\sqrt[8]{c}x}{x^2-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}}+\frac{\sqrt[4]{a}}{\sqrt[4]{c}}}dx-\sqrt{2-\sqrt{2}}\sqrt[8]{a}\left(\left(1+\sqrt{2}\right)\sqrt{a}d+\sqrt{c}e\right)\int\frac{1}{-\left(2x-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}}{\sqrt[8]{c}}\right)^2-\frac{\left(2+\sqrt{2}\right)\sqrt[4]{a}}{\sqrt[4]{c}}}d\left(2x-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{\cdots}}{\sqrt[8]{c}}\cdots\right.\right.}{2\sqrt{2-\sqrt{2}}\sqrt[8]{a}}\cdots}{\cdots}$$

$$\Big\downarrow \; 217$$

$$\frac{dx}{c}-\frac{\frac{\sqrt[4]{c}\left(\sqrt{\frac{2-\sqrt{2}}{2+\sqrt{2}}}\sqrt[8]{c}\left(\left(1+\sqrt{2}\right)\sqrt{a}d+\sqrt{c}e\right)\arctan\left(\frac{\sqrt[8]{c}\left(2x-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}}{\sqrt[8]{c}}\right)}{\sqrt{2+\sqrt{2}}\sqrt[8]{a}}\right)-\frac{1}{2}\left(\sqrt{a}\left(d-\sqrt{2}d\right)+\sqrt{c}e\right)\int\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}-2\sqrt[8]{c}x}{x^2-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}}+\frac{\sqrt[4]{a}}{\sqrt[4]{c}}}dx\right)}{2\sqrt{2-\sqrt{2}}\sqrt[8]{a}}+\frac{\sqrt[4]{c}\left(\sqrt{\frac{2-\sqrt{2}}{2+\sqrt{2}}}\sqrt[8]{c}\left(\cdots\right.\right.}{\cdots}}{2\sqrt{2}\sqrt[4]{a}\sqrt{c}}$$

$$\Big\downarrow \; 1103$$

$$\frac{dx}{c}-\frac{\frac{\sqrt[4]{c}\left(\sqrt{\frac{2-\sqrt{2}}{2+\sqrt{2}}}\sqrt[8]{c}\left(\left(1+\sqrt{2}\right)\sqrt{a}d+\sqrt{c}e\right)\arctan\left(\frac{\sqrt[8]{c}\left(2x-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}}{\sqrt[8]{c}}\right)}{\sqrt{2+\sqrt{2}}\sqrt[8]{a}}\right)+\frac{1}{2}\sqrt[8]{c}\left(\sqrt{a}\left(d-\sqrt{2}d\right)+\sqrt{c}e\right)\log\left(\sqrt[4]{c}x^2-\sqrt{2-\sqrt{2}}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{a}\right)\right)}{2\sqrt{2-\sqrt{2}}\sqrt[8]{a}}+\frac{\sqrt[4]{c}\left(\cdots\right.}{\cdots}}{2\sqrt{2}\sqrt[4]{a}\sqrt{c}}$$

input
```
Int[(d + e/x^4)/(c + a/x^8),x]
```

3.40. $\int \frac{d+\frac{e}{x^4}}{c+\frac{a}{x^8}}\,dx$

output
```
(d*x)/c - (((c^(1/4)*(Sqrt[(2 - Sqrt[2])/(2 + Sqrt[2])]*c^(1/8)*((1 + Sqrt
[2])*Sqrt[a]*d + Sqrt[c]*e)*ArcTan[(c^(1/8)*(-((Sqrt[2 - Sqrt[2]]*a^(1/8))
/c^(1/8)) + 2*x))/(Sqrt[2 + Sqrt[2]]*a^(1/8))] + (c^(1/8)*(Sqrt[a]*(d - Sq
rt[2]*d) + Sqrt[c]*e)*Log[a^(1/4) - Sqrt[2 - Sqrt[2]]*a^(1/8)*c^(1/8)*x +
c^(1/4)*x^2])/2))/(2*Sqrt[2 - Sqrt[2]]*a^(1/8)) + (c^(1/4)*(Sqrt[(2 - Sqrt
[2])/(2 + Sqrt[2])]*c^(1/8)*((1 + Sqrt[2])*Sqrt[a]*d + Sqrt[c]*e)*ArcTan[(
c^(1/8)*((Sqrt[2 - Sqrt[2]]*a^(1/8))/c^(1/8) + 2*x))/(Sqrt[2 + Sqrt[2]]*a^
(1/8))] - (c^(1/8)*(Sqrt[a]*(d - Sqrt[2]*d) + Sqrt[c]*e)*Log[a^(1/4) + Sqr
t[2 - Sqrt[2]]*a^(1/8)*c^(1/8)*x + c^(1/4)*x^2])/2))/(2*Sqrt[2 - Sqrt[2]]*
a^(1/8)))/(2*Sqrt[2]*a^(1/4)*Sqrt[c]) + ((c^(1/4)*(-(Sqrt[(2 + Sqrt[2])/(2
 - Sqrt[2])]*c^(1/8)*((1 - Sqrt[2])*Sqrt[a]*d + Sqrt[c]*e)*ArcTan[(c^(1/8)
*(-((Sqrt[2 + Sqrt[2]]*a^(1/8))/c^(1/8)) + 2*x))/(Sqrt[2 - Sqrt[2]]*a^(1/8
))]) - (c^(1/8)*((1 + Sqrt[2])*Sqrt[a]*d + Sqrt[c]*e)*Log[a^(1/4) - Sqrt[2
 + Sqrt[2]]*a^(1/8)*c^(1/8)*x + c^(1/4)*x^2])/2))/(2*Sqrt[2 + Sqrt[2]]*a^(
1/8)) + (c^(1/4)*(-(Sqrt[(2 + Sqrt[2])/(2 - Sqrt[2])]*c^(1/8)*((1 - Sqrt[2
])*Sqrt[a]*d + Sqrt[c]*e)*ArcTan[(c^(1/8)*((Sqrt[2 + Sqrt[2]]*a^(1/8))/c^(
1/8) + 2*x))/(Sqrt[2 - Sqrt[2]]*a^(1/8))]) + (c^(1/8)*((1 + Sqrt[2])*Sqrt[
a]*d + Sqrt[c]*e)*Log[a^(1/4) + Sqrt[2 + Sqrt[2]]*a^(1/8)*c^(1/8)*x + c^(1
/4)*x^2])/2))/(2*Sqrt[2 + Sqrt[2]]*a^(1/8)))/(2*Sqrt[2]*a^(1/4)*Sqrt[c]))/
c
```

3.40.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 1728
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> Int[x^(n*(2*p + q))*(e + d/x^n)^q*(c + a/x^(2*n))^p, x] /; FreeQ[{a,
 c, d, e, n}, x] && EqQ[n2, 2*n] && IntegersQ[p, q] && NegQ[n]
```

rule 1745
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> With[{
q = Rt[a/c, 4]}, Simp[1/(2*Sqrt[2]*c*q^3)   Int[(Sqrt[2]*d*q - (d - e*q^2)*
x^(n/2))/(q^2 - Sqrt[2]*q*x^(n/2) + x^n), x], x] + Simp[1/(2*Sqrt[2]*c*q^3)
   Int[(Sqrt[2]*d*q + (d - e*q^2)*x^(n/2))/(q^2 + Sqrt[2]*q*x^(n/2) + x^n),
 x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0]
 && NeQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && PosQ[a*c]
```

rule 1827
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (c_.)*(x_)^(n2_))^(
p_), x_Symbol] :> Simp[e*f^(n - 1)*(f*x)^(m - n + 1)*((a + c*x^(2*n))^(p +
1)/(c*(m + n*(2*p + 1) + 1))), x] - Simp[f^n/(c*(m + n*(2*p + 1) + 1))   In
t[(f*x)^(m - n)*(a + c*x^(2*n))^p*(a*e*(m - n + 1) - c*d*(m + n*(2*p + 1) +
 1)*x^n), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && EqQ[n2, 2*n] && IGtQ[n,
 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && IntegerQ[p]
```

3.40. $\int \frac{d+\frac{e}{x^4}}{c+\frac{a}{x^8}}\,dx$

3.40.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.06

method	result	size
default	$\frac{dx}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{8}+a\right)}\frac{\left(\_R^{4}ce-da\right)\ln\left(x-\_R\right)}{\_R^{7}}}{8c^{2}}$	45
risch	$\frac{dx}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{8}+a\right)}\frac{\left(\_R^{4}ce-da\right)\ln\left(x-\_R\right)}{\_R^{7}}}{8c^{2}}$	45

input
```
int((d+e/x^4)/(c+a/x^8),x,method=_RETURNVERBOSE)
```

output
```
d*x/c+1/8/c^2*sum((_R^4*c*e-a*d)/_R^7*ln(x-_R),_R=RootOf(_Z^8*c+a))
```

3.40.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2730 vs. 2(529) = 1058.

Time = 0.66 (sec) , antiderivative size = 2730, normalized size of antiderivative = 3.63

$$\int \frac{d+\frac{e}{x^4}}{c+\frac{a}{x^8}}\,dx=\text{Too large to display}$$

input
```
integrate((d+e/x^4)/(c+a/x^8),x, algorithm="fricas")
```

output
```
-1/8*(c*sqrt(-sqrt((a*c^4*sqrt(-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d
^4*e^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)/(a^3*c^9)) + 4*a*d^3*e - 4*c*d*e^3)/(
a*c^4)))*log((a^3*d^6 - 5*a^2*c*d^4*e^2 - 5*a*c^2*d^2*e^4 + c^3*e^6)*x + (
a^2*c^6*e*sqrt(-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a*c^
3*d^2*e^6 + c^4*e^8)/(a^3*c^9)) + a^3*c*d^5 - 6*a^2*c^2*d^3*e^2 + a*c^3*d*
e^4)*sqrt(-sqrt((a*c^4*sqrt(-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d^4*
e^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)/(a^3*c^9)) + 4*a*d^3*e - 4*c*d*e^3)/(a*c
^4)))) - c*sqrt(-sqrt((a*c^4*sqrt(-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^
2*d^4*e^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)/(a^3*c^9)) + 4*a*d^3*e - 4*c*d*e^3
)/(a*c^4)))*log((a^3*d^6 - 5*a^2*c*d^4*e^2 - 5*a*c^2*d^2*e^4 + c^3*e^6)*x
- (a^2*c^6*e*sqrt(-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a
*c^3*d^2*e^6 + c^4*e^8)/(a^3*c^9)) + a^3*c*d^5 - 6*a^2*c^2*d^3*e^2 + a*c^3
*d*e^4)*sqrt(-sqrt((a*c^4*sqrt(-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d
^4*e^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)/(a^3*c^9)) + 4*a*d^3*e - 4*c*d*e^3)/(
a*c^4)))) - c*sqrt(-sqrt(-(a*c^4*sqrt(-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^
2*c^2*d^4*e^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)/(a^3*c^9)) - 4*a*d^3*e + 4*c*d
*e^3)/(a*c^4)))*log((a^3*d^6 - 5*a^2*c*d^4*e^2 - 5*a*c^2*d^2*e^4 + c^3*e^6
)*x + (a^2*c^6*e*sqrt(-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d^4*e^4 -
12*a*c^3*d^2*e^6 + c^4*e^8)/(a^3*c^9)) - a^3*c*d^5 + 6*a^2*c^2*d^3*e^2 - a
*c^3*d*e^4)*sqrt(-sqrt(-(a*c^4*sqrt(-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a...
```

3.40.6 Sympy [F(-1)]

Timed out.

$$\int \frac{d+\frac{e}{x^4}}{c+\frac{a}{x^8}}\,dx = \text{Timed out}$$

input
```
integrate((d+e/x**4)/(c+a/x**8),x)
```

output
```
Timed out
```

3.40.7 Maxima [F]

$$\int \frac{d+\frac{e}{x^4}}{c+\frac{a}{x^8}}\,dx=\int \frac{d+\frac{e}{x^4}}{c+\frac{a}{x^8}}\,dx$$

input
```
integrate((d+e/x^4)/(c+a/x^8),x, algorithm="maxima")
```

output
```
d*x/c + integrate((c*e*x^4 - a*d)/(c*x^8 + a), x)/c
```

3.40.8 Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 639, normalized size of antiderivative = 0.85

$$\begin{aligned}
&\int \frac{d+\frac{e}{x^4}}{c+\frac{a}{x^8}}\,dx \\
&= \frac{dx}{c} - \frac{\left(ce\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{5}{8}}+ad\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}\right)\arctan\left(\frac{2\,x+\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{8\,ac} \\
&\quad - \frac{\left(ce\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{5}{8}}+ad\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}\right)\arctan\left(\frac{2\,x-\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{8\,ac} \\
&\quad + \frac{\left(ce\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{5}{8}}-ad\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}\right)\arctan\left(\frac{2\,x+\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{8\,ac} \\
&\quad + \frac{\left(ce\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{5}{8}}-ad\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}\right)\arctan\left(\frac{2\,x-\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{8\,ac} \\
&\quad - \frac{\left(ce\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{5}{8}}+ad\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}\right)\log\left(x^2+x\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{16\,ac} \\
&\quad + \frac{\left(ce\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{5}{8}}+ad\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}\right)\log\left(x^2-x\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{16\,ac} \\
&\quad + \frac{\left(ce\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{5}{8}}-ad\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}\right)\log\left(x^2+x\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{16\,ac} \\
&\quad - \frac{\left(ce\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{5}{8}}-ad\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}\right)\log\left(x^2-x\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{16\,ac}
\end{aligned}$$

input
```
integrate((d+e/x^4)/(c+a/x^8),x, algorithm="giac")
```

output
```
d*x/c - 1/8*(c*e*sqrt(-sqrt(2) + 2)*(a/c)^(5/8) + a*d*sqrt(sqrt(2) + 2)*(a
/c)^(1/8))*arctan((2*x + sqrt(-sqrt(2) + 2)*(a/c)^(1/8))/(sqrt(sqrt(2) + 2
)*(a/c)^(1/8)))/(a*c) - 1/8*(c*e*sqrt(-sqrt(2) + 2)*(a/c)^(5/8) + a*d*sqrt
(sqrt(2) + 2)*(a/c)^(1/8))*arctan((2*x - sqrt(-sqrt(2) + 2)*(a/c)^(1/8))/(
sqrt(sqrt(2) + 2)*(a/c)^(1/8)))/(a*c) + 1/8*(c*e*sqrt(sqrt(2) + 2)*(a/c)^(
5/8) - a*d*sqrt(-sqrt(2) + 2)*(a/c)^(1/8))*arctan((2*x + sqrt(sqrt(2) + 2)
*(a/c)^(1/8))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(a*c) + 1/8*(c*e*sqrt(sqrt
(2) + 2)*(a/c)^(5/8) - a*d*sqrt(-sqrt(2) + 2)*(a/c)^(1/8))*arctan((2*x - s
qrt(sqrt(2) + 2)*(a/c)^(1/8))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(a*c) - 1/
16*(c*e*sqrt(-sqrt(2) + 2)*(a/c)^(5/8) + a*d*sqrt(sqrt(2) + 2)*(a/c)^(1/8)
)*log(x^2 + x*sqrt(sqrt(2) + 2)*(a/c)^(1/8) + (a/c)^(1/4))/(a*c) + 1/16*(c
*e*sqrt(-sqrt(2) + 2)*(a/c)^(5/8) + a*d*sqrt(sqrt(2) + 2)*(a/c)^(1/8))*log
(x^2 - x*sqrt(sqrt(2) + 2)*(a/c)^(1/8) + (a/c)^(1/4))/(a*c) + 1/16*(c*e*sq
rt(sqrt(2) + 2)*(a/c)^(5/8) - a*d*sqrt(-sqrt(2) + 2)*(a/c)^(1/8))*log(x^2
+ x*sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + (a/c)^(1/4))/(a*c) - 1/16*(c*e*sqrt(s
qrt(2) + 2)*(a/c)^(5/8) - a*d*sqrt(-sqrt(2) + 2)*(a/c)^(1/8))*log(x^2 - x*
sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + (a/c)^(1/4))/(a*c)
```

3.40.9 Mupad [B] (verification not implemented)

Time = 9.10 (sec) , antiderivative size = 2520, normalized size of antiderivative = 3.35

$$\int \frac{d+\frac{e}{x^4}}{c+\frac{a}{x^8}}\,dx = \text{Too large to display}$$

input
```
int((d + e/x^4)/(c + a/x^8),x)
```

output
```
(atan((a^3*d^6*x - c^3*e^6*x - a*c^2*d^2*e^4*x + a^2*c*d^4*e^2*x + (2*d*e*
x*(a^2*d^4*(-a^3*c^9)^(1/2) + c^2*e^4*(-a^3*c^9)^(1/2) + 4*a^2*c^6*d*e^3 -
 4*a^3*c^5*d^3*e - 6*a*c*d^2*e^2*(-a^3*c^9)^(1/2)))/(a*c^4))/(a^2*c^6*e*(-
(a^2*d^4*(-a^3*c^9)^(1/2) + c^2*e^4*(-a^3*c^9)^(1/2) + 4*a^2*c^6*d*e^3 - 4
*a^3*c^5*d^3*e - 6*a*c*d^2*e^2*(-a^3*c^9)^(1/2))/(a^3*c^9))^(5/4) - a^3*c*
d^5*(-(a^2*d^4*(-a^3*c^9)^(1/2) + c^2*e^4*(-a^3*c^9)^(1/2) + 4*a^2*c^6*d*e
^3 - 4*a^3*c^5*d^3*e - 6*a*c*d^2*e^2*(-a^3*c^9)^(1/2))/(a^3*c^9))^(1/4) +
2*a^2*c^2*d^3*e^2*(-(a^2*d^4*(-a^3*c^9)^(1/2) + c^2*e^4*(-a^3*c^9)^(1/2) +
 4*a^2*c^6*d*e^3 - 4*a^3*c^5*d^3*e - 6*a*c*d^2*e^2*(-a^3*c^9)^(1/2))/(a^3*
c^9))^(1/4) + 3*a*c^3*d*e^4*(-(a^2*d^4*(-a^3*c^9)^(1/2) + c^2*e^4*(-a^3*c^
9)^(1/2) + 4*a^2*c^6*d*e^3 - 4*a^3*c^5*d^3*e - 6*a*c*d^2*e^2*(-a^3*c^9)^(1
/2))/(a^3*c^9))^(1/4)))*(-(a^2*d^4*(-a^3*c^9)^(1/2) + c^2*e^4*(-a^3*c^9)^(
1/2) + 4*a^2*c^6*d*e^3 - 4*a^3*c^5*d^3*e - 6*a*c*d^2*e^2*(-a^3*c^9)^(1/2))
/(a^3*c^9))^(1/4))/4 - (atan((c^3*e^6*x - a^3*d^6*x + a*c^2*d^2*e^4*x - a^
2*c*d^4*e^2*x + (2*d*e*x*(a^2*d^4*(-a^3*c^9)^(1/2) + c^2*e^4*(-a^3*c^9)^(1
/2) - 4*a^2*c^6*d*e^3 + 4*a^3*c^5*d^3*e - 6*a*c*d^2*e^2*(-a^3*c^9)^(1/2)))
/(a*c^4))/(a^2*c^6*e*((a^2*d^4*(-a^3*c^9)^(1/2) + c^2*e^4*(-a^3*c^9)^(1/2)
 - 4*a^2*c^6*d*e^3 + 4*a^3*c^5*d^3*e - 6*a*c*d^2*e^2*(-a^3*c^9)^(1/2))/(a^
3*c^9))^(5/4) - a^3*c*d^5*((a^2*d^4*(-a^3*c^9)^(1/2) + c^2*e^4*(-a^3*c^9)^
(1/2) - 4*a^2*c^6*d*e^3 + 4*a^3*c^5*d^3*e - 6*a*c*d^2*e^2*(-a^3*c^9)^(1...
```

3.40. $\int \frac{d+\frac{e}{x^4}}{c+\frac{a}{x^8}}\,dx$

3.41 $\int \frac{d+\frac{e}{x^4}}{c+\frac{a}{x^8}+\frac{b}{x^4}}\,dx$

3.41.1 Optimal result

Integrand size = 22, antiderivative size = 433

$$\int \frac{d+\frac{e}{x^4}}{c+\frac{a}{x^8}+\frac{b}{x^4}}\,dx = \frac{dx}{c} + \frac{\left(bd-ce+\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} + \frac{\left(bd-ce-\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}} + \frac{\left(bd-ce+\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} + \frac{\left(bd-ce-\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}$$

output
```
d*x/c+1/4*arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(b*d-c*e
+(-2*a*c*d+b^2*d-b*c*e)/(-4*a*c+b^2)^(1/2))*2^(3/4)/c^(5/4)/(-b-(-4*a*c+b^
2)^(1/2))^(3/4)+1/4*arctanh(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4
))*(b*d-c*e+(-2*a*c*d+b^2*d-b*c*e)/(-4*a*c+b^2)^(1/2))*2^(3/4)/c^(5/4)/(-b
-(-4*a*c+b^2)^(1/2))^(3/4)+1/4*arctan(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(
1/2))^(1/4))*(b*d-c*e+(2*a*c*d-b^2*d+b*c*e)/(-4*a*c+b^2)^(1/2))*2^(3/4)/c^
(5/4)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)+1/4*arctanh(2^(1/4)*c^(1/4)*x/(-b+(-4*
a*c+b^2)^(1/2))^(1/4))*(b*d-c*e+(2*a*c*d-b^2*d+b*c*e)/(-4*a*c+b^2)^(1/2))*
2^(3/4)/c^(5/4)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)
```

3.41.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.20

$$\int \frac{d+\frac{e}{x^4}}{c+\frac{a}{x^8}+\frac{b}{x^4}}\,dx = \frac{dx}{c}-\frac{\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{ad\log(x-\#1)+bd\log(x-\#1)\#1^4-ce\log(x-\#1)\#1^4}{b\#1^3+2c\#1^7}\&\right]}{4c}$$

input
```
Integrate[(d + e/x^4)/(c + a/x^8 + b/x^4),x]
```

output
```
(d*x)/c - RootSum[a + b*#1^4 + c*#1^8 & , (a*d*Log[x - #1] + b*d*Log[x - #
1]*#1^4 - c*e*Log[x - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ]/(4*c)
```

3.41.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 366, normalized size of antiderivative = 0.85, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1727, 1826, 1752, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+\frac{e}{x^4}}{\frac{a}{x^8}+\frac{b}{x^4}+c}\,dx$$

$$\Big\downarrow \text{1727}$$

$$\int \frac{x^4\left(dx^4+e\right)}{a+bx^4+cx^8}dx$$

$\Big\downarrow$ 1826

$$\frac{dx}{c}-\frac{\int \frac{(bd-ce)x^4+ad}{cx^8+bx^4+a}dx}{c}$$

$\Big\downarrow$ 1752

$$\frac{dx}{c}-\frac{\frac{1}{2}\left(-\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)\int \frac{1}{cx^4+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}\left(\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)\int \frac{1}{cx^4+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{c}$$

$\Big\downarrow$ 756

$$\frac{dx}{c}-\frac{\frac{1}{2}\left(\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)\left(-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}dx}{\sqrt{-\sqrt{b^2-4ac}-b}}-\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x^2+\sqrt{-b-\sqrt{b^2-4ac}}}dx}{\sqrt{-\sqrt{b^2-4ac}-b}}\right)+\frac{1}{2}\left(-\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce \cdots\right.}{c}$$

$\Big\downarrow$ 218

$$\frac{dx}{c}-\frac{\frac{1}{2}\left(\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)\left(-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}dx}{\sqrt{-\sqrt{b^2-4ac}-b}}-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}\right)+\frac{1}{2}\left(-\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+b \cdots\right.}{c}$$

$\Big\downarrow$ 221

$$\frac{dx}{c}-\frac{\frac{1}{2}\left(\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)\left(-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}\right)+\frac{1}{2}\left(-\frac{-2acd+b^2d \cdots}{\sqrt{b^2-4a \cdots}}\right.}{c}$$

input
```
Int[(d + e/x^4)/(c + a/x^8 + b/x^4),x]
```

3.41. $\int \frac{d+\frac{e}{x^4}}{c+\frac{a}{x^8}+\frac{b}{x^4}}\,dx$

output
```
(d*x)/c - (((b*d - c*e + (b^2*d - 2*a*c*d - b*c*e)/Sqrt[b^2 - 4*a*c])*(-(A
rcTan[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)
*(-b - Sqrt[b^2 - 4*a*c])^(3/4))) - ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt
[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4))))/2
 + ((b*d - c*e - (b^2*d - 2*a*c*d - b*c*e)/Sqrt[b^2 - 4*a*c])*(-(ArcTan[(2
^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b + S
qrt[b^2 - 4*a*c])^(3/4))) - ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4
*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))))/2)/c
```

3.41.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 1727
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (e_.)*(x_)^(
n_))^(q_.), x_Symbol] :> Int[x^(n*(2*p + q))*(e + d/x^n)^q*(c + b/x^n + a/x
^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && IntegersQ[
p, q] && NegQ[n]
```

rule 1752
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2
 - 4*a*c] ||  !IGtQ[n/2, 0])
```

3.41. $\int \frac{d+\frac{e}{x^4}}{c+\frac{a}{x^8}+\frac{b}{x^4}}\,dx$

rule 1826
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[e*f^(n - 1)*(f*x)^(m - n + 1)*((a
+ b*x^n + c*x^(2*n))^(p + 1)/(c*(m + n*(2*p + 1) + 1))), x] - Simp[f^n/(c*(
m + n*(2*p + 1) + 1))   Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*
e*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x]
, x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Intege
rQ[p]
```

3.41.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.15

method	result	size
default	$\frac{dx}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+\_Z^4b+a\right)}\frac{\left((-bd+ec)\_R^4-da\right)\ln\left(x-\_R\right)}{2\_R^7c+\_R^3b}}{4c}$	67
risch	$\frac{dx}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+\_Z^4b+a\right)}\frac{\left((-bd+ec)\_R^4-da\right)\ln\left(x-\_R\right)}{2\_R^7c+\_R^3b}}{4c}$	67

input
```
int((d+e/x^4)/(c+a/x^8+b/x^4),x,method=_RETURNVERBOSE)
```

output
```
d*x/c+1/4/c*sum(((-b*d+c*e)*_R^4-d*a)/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf
(_Z^8*c+_Z^4*b+a))
```

3.41.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12946 vs. 2(353) = 706.

Time = 5.10 (sec) , antiderivative size = 12946, normalized size of antiderivative = 29.90

$$\int \frac{d+\frac{e}{x^4}}{c+\frac{a}{x^8}+\frac{b}{x^4}}\,dx=\text{Too large to display}$$

input
```
integrate((d+e/x^4)/(c+a/x^8+b/x^4),x, algorithm="fricas")
```

output
```
Too large to include
```

3.41.6 Sympy [F(-1)]

Timed out.

$$\int \frac{d+\frac{e}{x^4}}{c+\frac{a}{x^8}+\frac{b}{x^4}}\,dx = \text{Timed out}$$

input
```
integrate((d+e/x**4)/(c+a/x**8+b/x**4),x)
```

output
```
Timed out
```

3.41.7 Maxima [F]

$$\int \frac{d+\frac{e}{x^4}}{c+\frac{a}{x^8}+\frac{b}{x^4}}\,dx = \int \frac{d+\frac{e}{x^4}}{c+\frac{b}{x^4}+\frac{a}{x^8}}\,dx$$

input
```
integrate((d+e/x^4)/(c+a/x^8+b/x^4),x, algorithm="maxima")
```

output
```
d*x/c + integrate(-((b*d - c*e)*x^4 + a*d)/(c*x^8 + b*x^4 + a), x)/c
```

3.41.8 Giac [F(-1)]

Timed out.

$$\int \frac{d+\frac{e}{x^4}}{c+\frac{a}{x^8}+\frac{b}{x^4}}\,dx = \text{Timed out}$$

input
```
integrate((d+e/x^4)/(c+a/x^8+b/x^4),x, algorithm="giac")
```

output
```
Timed out
```

3.41.9 Mupad [B] (verification not implemented)

Time = 13.00 (sec) , antiderivative size = 50213, normalized size of antiderivative = 115.97

$$\int \frac{d+\frac{e}{x^4}}{c+\frac{a}{x^8}+\frac{b}{x^4}}\,dx = \text{Too large to display}$$

input
```
int((d + e/x^4)/(c + a/x^8 + b/x^4),x)
```

output
```
atan((((((4*x*(4096*a^5*b*c^6*d^2 + 4096*a^4*b*c^7*e^2 + 256*a^3*b^5*c^4*d
^2 - 2048*a^4*b^3*c^5*d^2 + 256*a^2*b^5*c^5*e^2 - 2048*a^3*b^3*c^6*e^2 - 1
6384*a^5*c^7*d*e - 1024*a^3*b^4*c^5*d*e + 8192*a^4*b^2*c^6*d*e))/c - (16*(
-(b^9*d^4 + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 + c^4*e^4*(-(4*
a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^
4 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c
^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^
7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 +
 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*d^4*(-(4*a*c - b^2
)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e - 4*b*c^3*d*e^3*(-(4*
a*c - b^2)^5)^(1/2) - 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^
3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*
d^2*e^2 + 320*a^3*b^2*c^4*d^3*e - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2)
 + 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 -
 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*(16384*a^5*c^8*e
 - 256*a^2*b^6*c^5*e + 3072*a^3*b^4*c^6*e - 12288*a^4*b^2*c^7*e))/c)*(-(b^
9*d^4 + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 + c^4*e^4*(-(4*a*c
- b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 +
128*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d
^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7...
```

3.42 $\int \frac{(d+ex^n)^3}{a+cx^{2n}}\,dx$

3.42.1 Optimal result

Integrand size = 21, antiderivative size = 141

$$\int \frac{(d+ex^n)^3}{a+cx^{2n}}\,dx = \frac{3de^2x}{c} + \frac{e^3x^{1+n}}{c(1+n)} + \frac{d(cd^2-3ae^2)\,x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{ac} + \frac{e(3cd^2-ae^2)\,x^{1+n}\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{ac(1+n)}$$

output
```
3*d*e^2*x/c+e^3*x^(1+n)/c/(1+n)+d*(-3*a*e^2+c*d^2)*x*hypergeom([1, 1/2/n],
[1+1/2/n],-c*x^(2*n)/a)/a/c+e*(-a*e^2+3*c*d^2)*x^(1+n)*hypergeom([1, 1/2*(
1+n)/n],[3/2+1/2/n],-c*x^(2*n)/a)/a/c/(1+n)
```

3.42.2 Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.90

$$\int \frac{(d+ex^n)^3}{a+cx^{2n}}\,dx = \frac{x\left(d(cd^2-3ae^2)(1+n)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)+e\left(ae(3d(1+n)+ex^n)+(3cd^2\right.\right.}{ac(1+n)}$$

input
```
Integrate[(d + e*x^n)^3/(a + c*x^(2*n)),x]
```

output
```
(x*(d*(c*d^2 - 3*a*e^2)*(1 + n)*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))
/2, -((c*x^(2*n))/a)] + e*(a*e*(3*d*(1 + n) + e*x^n) + (3*c*d^2 - a*e^2)*x
^n*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)]))
)/(a*c*(1 + n))
```

3.42.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1755, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^n)^3}{a+cx^{2n}}\,dx$$
$$\Big\downarrow \text{1755}$$
$$\int \left(\frac{x^n\left(3cd^2e-ae^3\right)-3ade^2+cd^3}{c\left(a+cx^{2n}\right)}+\frac{3de^2}{c}+\frac{e^3x^n}{c}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{ex^{n+1}\left(3cd^2-ae^2\right)\text{Hypergeometric2F1}\left(1,\frac{n+1}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{ac(n+1)}+\frac{dx\left(cd^2-3ae^2\right)\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{ac}+\frac{3de^2x}{c}+\frac{e^3x^{n+1}}{c(n+1)}$$

input
```
Int[(d + e*x^n)^3/(a + c*x^(2*n)),x]
```

output
```
(3*d*e^2*x)/c + (e^3*x^(1 + n))/(c*(1 + n)) + (d*(c*d^2 - 3*a*e^2)*x*Hyper
geometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/(a*c) + (e*(3*
c*d^2 - a*e^2)*x^(1 + n)*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/
2, -((c*x^(2*n))/a)])/(a*c*(1 + n))
```

3.42.3.1 Defintions of rubi rules used

rule 1755
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^n)^q/(a + c*x^(2*n)), x], x] /; FreeQ[{a, c, d,
 e, n}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.42.4 Maple [F]

$$\int \frac{(d + e\,x^n)^3}{a + c\,x^{2n}}dx$$

input
```
int((d+e*x^n)^3/(a+c*x^(2*n)),x)
```

output
```
int((d+e*x^n)^3/(a+c*x^(2*n)),x)
```

3.42.5 Fricas [F]

$$\int \frac{(d + ex^n)^3}{a + cx^{2n}}\,dx = \int \frac{(ex^n + d)^3}{cx^{2\,n} + a}\,dx$$

input
```
integrate((d+e*x^n)^3/(a+c*x^(2*n)),x, algorithm="fricas")
```

output
```
integral((e^3*x^(3*n) + 3*d*e^2*x^(2*n) + 3*d^2*e*x^n + d^3)/(c*x^(2*n) +
a), x)
```

3.42.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.01 (sec) , antiderivative size = 466, normalized size of antiderivative = 3.30

$$\int \frac{(d+ex^n)^3}{a+cx^{2n}}\,dx = \frac{a^{\frac{1}{2n}}a^{-1-\frac{1}{2n}}d^3x\Phi\left(\frac{cx^{2n}e^{i\pi}}{a},1,\frac{1}{2n}\right)\Gamma\left(\frac{1}{2n}\right)}{4n^2\Gamma\left(1+\frac{1}{2n}\right)} + \frac{3a^{-\frac{5}{2}-\frac{1}{2n}}a^{\frac{3}{2}+\frac{1}{2n}}e^3x^{3n+1}\Phi\left(\frac{cx^{2n}e^{i\pi}}{a},1,\frac{3}{2}+\frac{1}{2n}\right)\Gamma\left(\frac{3}{2}+\frac{1}{2n}\right)}{4n\Gamma\left(\frac{5}{2}+\frac{1}{2n}\right)} + \frac{a^{-\frac{5}{2}-\frac{1}{2n}}a^{\frac{3}{2}+\frac{1}{2n}}e^3x^{3n+1}\Phi\left(\frac{cx^{2n}e^{i\pi}}{a},1,\frac{3}{2}+\frac{1}{2n}\right)\Gamma\left(\frac{3}{2}+\frac{1}{2n}\right)}{4n^2\Gamma\left(\frac{5}{2}+\frac{1}{2n}\right)} + \frac{3a^{-\frac{3}{2}-\frac{1}{2n}}a^{\frac{1}{2}+\frac{1}{2n}}d^2ex^{n+1}\Phi\left(\frac{cx^{2n}e^{i\pi}}{a},1,\frac{1}{2}+\frac{1}{2n}\right)\Gamma\left(\frac{1}{2}+\frac{1}{2n}\right)}{4n\Gamma\left(\frac{3}{2}+\frac{1}{2n}\right)} + \frac{3a^{-\frac{3}{2}-\frac{1}{2n}}a^{\frac{1}{2}+\frac{1}{2n}}d^2ex^{n+1}\Phi\left(\frac{cx^{2n}e^{i\pi}}{a},1,\frac{1}{2}+\frac{1}{2n}\right)\Gamma\left(\frac{1}{2}+\frac{1}{2n}\right)}{4n^2\Gamma\left(\frac{3}{2}+\frac{1}{2n}\right)} - \frac{3a^{-\frac{1}{2n}}a^{1+\frac{1}{2n}}c^{\frac{1}{2n}}c^{-1-\frac{1}{2n}}de^2x\Phi\left(\frac{ax^{-2n}e^{i\pi}}{c},1,\frac{e^{i\pi}}{2n}\right)\Gamma\left(\frac{1}{2n}\right)}{4an^2\Gamma\left(1+\frac{1}{2n}\right)}$$

input
```
integrate((d+e*x**n)**3/(a+c*x**(2*n)),x)
```

output
```
a**(1/(2*n))*a**(-1 - 1/(2*n))*d**3*x*lerchphi(c*x**(2*n)*exp_polar(I*pi)/
a, 1, 1/(2*n))*gamma(1/(2*n))/(4*n**2*gamma(1 + 1/(2*n))) + 3*a**(-5/2 - 1
/(2*n))*a**(3/2 + 1/(2*n))*e**3*x**(3*n + 1)*lerchphi(c*x**(2*n)*exp_polar
(I*pi)/a, 1, 3/2 + 1/(2*n))*gamma(3/2 + 1/(2*n))/(4*n*gamma(5/2 + 1/(2*n))
) + a**(-5/2 - 1/(2*n))*a**(3/2 + 1/(2*n))*e**3*x**(3*n + 1)*lerchphi(c*x*
*(2*n)*exp_polar(I*pi)/a, 1, 3/2 + 1/(2*n))*gamma(3/2 + 1/(2*n))/(4*n**2*g
amma(5/2 + 1/(2*n))) + 3*a**(-3/2 - 1/(2*n))*a**(1/2 + 1/(2*n))*d**2*e*x**
(n + 1)*lerchphi(c*x**(2*n)*exp_polar(I*pi)/a, 1, 1/2 + 1/(2*n))*gamma(1/2
 + 1/(2*n))/(4*n*gamma(3/2 + 1/(2*n))) + 3*a**(-3/2 - 1/(2*n))*a**(1/2 + 1
/(2*n))*d**2*e*x**(n + 1)*lerchphi(c*x**(2*n)*exp_polar(I*pi)/a, 1, 1/2 +
1/(2*n))*gamma(1/2 + 1/(2*n))/(4*n**2*gamma(3/2 + 1/(2*n))) - 3*a**(1 + 1/
(2*n))*c**(1/(2*n))*c**(-1 - 1/(2*n))*d*e**2*x*lerchphi(a*exp_polar(I*pi)/
(c*x**(2*n)), 1, exp_polar(I*pi)/(2*n))*gamma(1/(2*n))/(4*a*a**(1/(2*n))*n
**2*gamma(1 + 1/(2*n)))
```

3.42.7 Maxima [F]

$$\int \frac{(d+ex^n)^3}{a+cx^{2n}}\,dx = \int \frac{(ex^n+d)^3}{cx^{2\,n}+a}\,dx$$

input
```
integrate((d+e*x^n)^3/(a+c*x^(2*n)),x, algorithm="maxima")
```

output
```
(3*d*e^2*(n + 1)*x + e^3*x*x^n)/(c*(n + 1)) - integrate(-(c*d^3 - 3*a*d*e^
2 + (3*c*d^2*e - a*e^3)*x^n)/(c^2*x^(2*n) + a*c), x)
```

3.42.8 Giac [F]

$$\int \frac{(d+ex^n)^3}{a+cx^{2n}}\,dx = \int \frac{(ex^n+d)^3}{cx^{2\,n}+a}\,dx$$

input
```
integrate((d+e*x^n)^3/(a+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate((e*x^n + d)^3/(c*x^(2*n) + a), x)
```

3.42.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^n)^3}{a+cx^{2n}}\,dx = \int \frac{(d+e\,x^n)^3}{a+c\,x^{2\,n}}\,dx$$

input
```
int((d + e*x^n)^3/(a + c*x^(2*n)),x)
```

output
```
int((d + e*x^n)^3/(a + c*x^(2*n)), x)
```

3.43 $\int \frac{(d+ex^n)^2}{a+cx^{2n}}\,dx$

3.43.1 Optimal result

Integrand size = 21, antiderivative size = 107

$$\int \frac{(d+ex^n)^2}{a+cx^{2n}}\,dx = \frac{e^2x}{c} + \frac{(cd^2-ae^2)\,x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{ac} + \frac{2dex^{1+n}\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a(1+n)}$$

output
```
e^2*x/c+(-a*e^2+c*d^2)*x*hypergeom([1, 1/2/n],[1+1/2/n],-c*x^(2*n)/a)/a/c+
2*d*e*x^(1+n)*hypergeom([1, 1/2*(1+n)/n],[3/2+1/2/n],-c*x^(2*n)/a)/a/(1+n)
```

3.43.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^n)^2}{a+cx^{2n}}\,dx = \frac{e^2x}{c} + \frac{(cd^2-ae^2)\,x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{ac} + \frac{2dex^{1+n}\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a(1+n)}$$

input
```
Integrate[(d + e*x^n)^2/(a + c*x^(2*n)),x]
```

output
```
(e^2*x)/c + ((c*d^2 - a*e^2)*x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/
2, -((c*x^(2*n))/a)])/(a*c) + (2*d*e*x^(1 + n)*Hypergeometric2F1[1, (1 + n
)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a*(1 + n))
```

3.43.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1755, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^n)^2}{a+cx^{2n}}\,dx$$

$$\Big\downarrow \text{ 1755}$$

$$\int \left(\frac{-ae^2+cd^2+2cdex^n}{c\,(a+cx^{2n})}+\frac{e^2}{c}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x\left(cd^2-ae^2\right)\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{ac}+\frac{2dex^{n+1}\text{Hypergeometric2F1}\left(1,\frac{n+1}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a(n+1)}+\frac{e^2x}{c}$$

input
```
Int[(d + e*x^n)^2/(a + c*x^(2*n)),x]
```

output
```
(e^2*x)/c + ((c*d^2 - a*e^2)*x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/
2, -((c*x^(2*n))/a)])/(a*c) + (2*d*e*x^(1 + n)*Hypergeometric2F1[1, (1 + n
)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a*(1 + n))
```

3.43.3.1 Defintions of rubi rules used

rule 1755
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^n)^q/(a + c*x^(2*n)), x], x] /; FreeQ[{a, c, d,
 e, n}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.43.4 Maple [F]

$$\int \frac{(d+e\,x^{n})^{2}}{a+c\,x^{2n}}dx$$

input
```
int((d+e*x^n)^2/(a+c*x^(2*n)),x)
```

output
```
int((d+e*x^n)^2/(a+c*x^(2*n)),x)
```

3.43.5 Fricas [F]

$$\int \frac{(d+ex^n)^2}{a+cx^{2n}}\,dx=\int \frac{(ex^n+d)^2}{cx^{2\,n}+a}\,dx$$

input
```
integrate((d+e*x^n)^2/(a+c*x^(2*n)),x, algorithm="fricas")
```

output
```
integral((e^2*x^(2*n) + 2*d*e*x^n + d^2)/(c*x^(2*n) + a), x)
```

3.43.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.82 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.77

$$\int \frac{(d+ex^n)^2}{a+cx^{2n}}\,dx = \frac{a^{\frac{1}{2n}}a^{-1-\frac{1}{2n}}d^2x\Phi\left(\frac{cx^{2n}e^{i\pi}}{a},1,\frac{1}{2n}\right)\Gamma\left(\frac{1}{2n}\right)}{4n^2\Gamma\left(1+\frac{1}{2n}\right)} + \frac{a^{-\frac{3}{2}-\frac{1}{2n}}a^{\frac{1}{2}+\frac{1}{2n}}dex^{n+1}\Phi\left(\frac{cx^{2n}e^{i\pi}}{a},1,\frac{1}{2}+\frac{1}{2n}\right)\Gamma\left(\frac{1}{2}+\frac{1}{2n}\right)}{2n\Gamma\left(\frac{3}{2}+\frac{1}{2n}\right)} + \frac{a^{-\frac{3}{2}-\frac{1}{2n}}a^{\frac{1}{2}+\frac{1}{2n}}dex^{n+1}\Phi\left(\frac{cx^{2n}e^{i\pi}}{a},1,\frac{1}{2}+\frac{1}{2n}\right)\Gamma\left(\frac{1}{2}+\frac{1}{2n}\right)}{2n^2\Gamma\left(\frac{3}{2}+\frac{1}{2n}\right)} - \frac{a^{-\frac{1}{2n}}a^{1+\frac{1}{2n}}c^{\frac{1}{2n}}c^{-1-\frac{1}{2n}}e^2x\Phi\left(\frac{ax^{-2n}e^{i\pi}}{c},1,\frac{e^{i\pi}}{2n}\right)\Gamma\left(\frac{1}{2n}\right)}{4an^2\Gamma\left(1+\frac{1}{2n}\right)}$$

input
```
integrate((d+e*x**n)**2/(a+c*x**(2*n)),x)
```

output
```
a**(1/(2*n))*a**(-1 - 1/(2*n))*d**2*x*lerchphi(c*x**(2*n)*exp_polar(I*pi)/
a, 1, 1/(2*n))*gamma(1/(2*n))/(4*n**2*gamma(1 + 1/(2*n))) + a**(-3/2 - 1/(
2*n))*a**(1/2 + 1/(2*n))*d*e*x**(n + 1)*lerchphi(c*x**(2*n)*exp_polar(I*pi
)/a, 1, 1/2 + 1/(2*n))*gamma(1/2 + 1/(2*n))/(2*n*gamma(3/2 + 1/(2*n))) + a
**(-3/2 - 1/(2*n))*a**(1/2 + 1/(2*n))*d*e*x**(n + 1)*lerchphi(c*x**(2*n)*e
xp_polar(I*pi)/a, 1, 1/2 + 1/(2*n))*gamma(1/2 + 1/(2*n))/(2*n**2*gamma(3/2
 + 1/(2*n))) - a**(1 + 1/(2*n))*c**(1/(2*n))*c**(-1 - 1/(2*n))*e**2*x*lerc
hphi(a*exp_polar(I*pi)/(c*x**(2*n)), 1, exp_polar(I*pi)/(2*n))*gamma(1/(2*
n))/(4*a*a**(1/(2*n))*n**2*gamma(1 + 1/(2*n)))
```

3.43.7 Maxima [F]

$$\int \frac{(d+ex^n)^2}{a+cx^{2n}}\,dx = \int \frac{(ex^n+d)^2}{cx^{2n}+a}\,dx$$

input
```
integrate((d+e*x^n)^2/(a+c*x^(2*n)),x, algorithm="maxima")
```

output
```
e^2*x/c + integrate((2*c*d*e*x^n + c*d^2 - a*e^2)/(c^2*x^(2*n) + a*c), x)
```

3.43.8 Giac [F]

$$\int \frac{(d+ex^n)^2}{a+cx^{2n}}\,dx = \int \frac{(ex^n+d)^2}{cx^{2n}+a}\,dx$$

input
```
integrate((d+e*x^n)^2/(a+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate((e*x^n + d)^2/(c*x^(2*n) + a), x)
```

3.43.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^n)^2}{a+cx^{2n}}\,dx = \int \frac{(d+e\,x^n)^2}{a+c\,x^{2\,n}}\,dx$$

input
```
int((d + e*x^n)^2/(a + c*x^(2*n)),x)
```

output
```
int((d + e*x^n)^2/(a + c*x^(2*n)), x)
```

3.44 $\int \frac{d+ex^n}{a+cx^{2n}}\,dx$

3.44.1 Optimal result

Integrand size = 19, antiderivative size = 83

$$\int \frac{d+ex^n}{a+cx^{2n}}\,dx = \frac{dx\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a} + \frac{ex^{1+n}\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a(1+n)}$$

output
```
d*x*hypergeom([1, 1/2/n],[1+1/2/n],-c*x^(2*n)/a)/a+e*x^(1+n)*hypergeom([1,
 1/2*(1+n)/n],[3/2+1/2/n],-c*x^(2*n)/a)/a/(1+n)
```

3.44.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00

$$\int \frac{d+ex^n}{a+cx^{2n}}\,dx = \frac{dx\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a} + \frac{ex^{1+n}\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a(1+n)}$$

input
```
Integrate[(d + e*x^n)/(a + c*x^(2*n)),x]
```

output
```
(d*x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/a +
(e*x^(1 + n)*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2
*n))/a)])/(a*(1 + n))
```

3.44.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1748, 778, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^n}{a+cx^{2n}}\,dx$$

$$\Big\downarrow \ 1748$$

$$d\int \frac{1}{cx^{2n}+a}dx+e\int \frac{x^n}{cx^{2n}+a}dx$$

$$\Big\downarrow \ 778$$

$$e\int \frac{x^n}{cx^{2n}+a}dx+\frac{dx\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a}$$

$$\Big\downarrow \ 888$$

$$\frac{dx\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a}+\frac{ex^{n+1}\,\text{Hypergeometric2F1}\left(1,\frac{n+1}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a(n+1)}$$

input
```
Int[(d + e*x^n)/(a + c*x^(2*n)),x]
```

output
```
(d*x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/a +
(e*x^(1 + n)*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2
*n))/a)])/(a*(1 + n))
```

3.44.3.1 Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1748
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> Simp[d
   Int[1/(a + c*x^(2*n)), x], x] + Simp[e   Int[x^n/(a + c*x^(2*n)), x], x]
 /; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && (
PosQ[a*c] ||  !IntegerQ[n])
```

3.44.4 Maple [F]

$$\int \frac{d+e\,x^{n}}{a+c\,x^{2n}}dx$$

input
```
int((d+e*x^n)/(a+c*x^(2*n)),x)
```

output
```
int((d+e*x^n)/(a+c*x^(2*n)),x)
```

3.44.5 Fricas [F]

$$\int \frac{d+ex^n}{a+cx^{2n}}\,dx = \int \frac{ex^n+d}{cx^{2\,n}+a}\,dx$$

input
```
integrate((d+e*x^n)/(a+c*x^(2*n)),x, algorithm="fricas")
```

output
```
integral((e*x^n + d)/(c*x^(2*n) + a), x)
```

3.44.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.16 (sec) , antiderivative size = 209, normalized size of antiderivative = 2.52

$$\int \frac{d+ex^n}{a+cx^{2n}}\,dx = \frac{a^{\frac{1}{2n}}a^{-1-\frac{1}{2n}}dx\Phi\left(\frac{cx^{2n}e^{i\pi}}{a},1,\frac{1}{2n}\right)\Gamma\left(\frac{1}{2n}\right)}{4n^2\Gamma\left(1+\frac{1}{2n}\right)} + \frac{a^{-\frac{3}{2}-\frac{1}{2n}}a^{\frac{1}{2}+\frac{1}{2n}}ex^{n+1}\Phi\left(\frac{cx^{2n}e^{i\pi}}{a},1,\frac{1}{2}+\frac{1}{2n}\right)\Gamma\left(\frac{1}{2}+\frac{1}{2n}\right)}{4n\Gamma\left(\frac{3}{2}+\frac{1}{2n}\right)} + \frac{a^{-\frac{3}{2}-\frac{1}{2n}}a^{\frac{1}{2}+\frac{1}{2n}}ex^{n+1}\Phi\left(\frac{cx^{2n}e^{i\pi}}{a},1,\frac{1}{2}+\frac{1}{2n}\right)\Gamma\left(\frac{1}{2}+\frac{1}{2n}\right)}{4n^2\Gamma\left(\frac{3}{2}+\frac{1}{2n}\right)}$$

input
```
integrate((d+e*x**n)/(a+c*x**(2*n)),x)
```

output
```
a**(1/(2*n))*a**(-1 - 1/(2*n))*d*x*lerchphi(c*x**(2*n)*exp_polar(I*pi)/a,
1, 1/(2*n))*gamma(1/(2*n))/(4*n**2*gamma(1 + 1/(2*n))) + a**(-3/2 - 1/(2*n
))*a**(1/2 + 1/(2*n))*e*x**(n + 1)*lerchphi(c*x**(2*n)*exp_polar(I*pi)/a,
1, 1/2 + 1/(2*n))*gamma(1/2 + 1/(2*n))/(4*n*gamma(3/2 + 1/(2*n))) + a**(-3
/2 - 1/(2*n))*a**(1/2 + 1/(2*n))*e*x**(n + 1)*lerchphi(c*x**(2*n)*exp_pola
r(I*pi)/a, 1, 1/2 + 1/(2*n))*gamma(1/2 + 1/(2*n))/(4*n**2*gamma(3/2 + 1/(2
*n)))
```

3.44.7 Maxima [F]

$$\int \frac{d+ex^n}{a+cx^{2n}}\,dx = \int \frac{ex^n+d}{cx^{2n}+a}\,dx$$

input
```
integrate((d+e*x^n)/(a+c*x^(2*n)),x, algorithm="maxima")
```

output
```
integrate((e*x^n + d)/(c*x^(2*n) + a), x)
```

3.44.8 Giac [F]

$$\int \frac{d+ex^n}{a+cx^{2n}}\,dx = \int \frac{ex^n+d}{cx^{2\,n}+a}\,dx$$

input
```
integrate((d+e*x^n)/(a+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate((e*x^n + d)/(c*x^(2*n) + a), x)
```

3.44.9 Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^n}{a+cx^{2n}}\,dx = \int \frac{d+e\,x^n}{a+c\,x^{2\,n}}\,dx$$

input
```
int((d + e*x^n)/(a + c*x^(2*n)),x)
```

output
```
int((d + e*x^n)/(a + c*x^(2*n)), x)
```

3.45 $\int \frac{1}{(d+ex^n)(a+cx^{2n})}\,dx$

3.45.1 Optimal result

Integrand size = 21, antiderivative size = 152

$$\int \frac{1}{(d+ex^n)(a+cx^{2n})}\,dx = \frac{cdx\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a(cd^2+ae^2)} + \frac{e^2x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)}{d(cd^2+ae^2)} - \frac{cex^{1+n}\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a(cd^2+ae^2)(1+n)}$$

output
```
c*d*x*hypergeom([1, 1/2/n],[1+1/2/n],-c*x^(2*n)/a)/a/(a*e^2+c*d^2)+e^2*x*h
ypergeom([1, 1/n],[1+1/n],-e*x^n/d)/d/(a*e^2+c*d^2)-c*e*x^(1+n)*hypergeom(
[1, 1/2*(1+n)/n],[3/2+1/2/n],-c*x^(2*n)/a)/a/(a*e^2+c*d^2)/(1+n)
```

3.45.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.86

$$\int \frac{1}{(d+ex^n)(a+cx^{2n})}\,dx = \frac{x\Big(cd^2(1+n)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)+e\Big(ae(1+n)\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\right.}{ad(cd^2+ae^2)(1+n)}$$

input
```
Integrate[1/((d + e*x^n)*(a + c*x^(2*n))),x]
```

output
```
(x*(c*d^2*(1 + n)*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*
n))/a)] + e*(a*e*(1 + n)*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((e*x^n
)/d)] - c*d*x^n*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x
^(2*n))/a)])))/(a*d*(c*d^2 + a*e^2)*(1 + n))
```

3.45.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1755, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^{2n})(d+ex^n)}dx$$

$$\Big\downarrow \text{1755}$$

$$\int \left(\frac{e^2}{(ae^2+cd^2)(d+ex^n)}-\frac{c(ex^n-d)}{(ae^2+cd^2)(a+cx^{2n})}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{cex^{n+1}\text{Hypergeometric2F1}\left(1,\frac{n+1}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a(n+1)(ae^2+cd^2)}+\frac{cdx\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a(ae^2+cd^2)}+\frac{e^2x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)}{d(ae^2+cd^2)}$$

input
```
Int[1/((d + e*x^n)*(a + c*x^(2*n))),x]
```

output
```
(c*d*x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/(a
*(c*d^2 + a*e^2)) + (e^2*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((e*x
^n)/d)])/(d*(c*d^2 + a*e^2)) - (c*e*x^(1 + n)*Hypergeometric2F1[1, (1 + n)
/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a*(c*d^2 + a*e^2)*(1 + n))
```

3.45.3.1 Defintions of rubi rules used

rule 1755
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^n)^q/(a + c*x^(2*n)), x], x] /; FreeQ[{a, c, d,
 e, n}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.45.4 Maple [F]

$$\int \frac{1}{(d+e\,x^{n})\,(a+c\,x^{2n})}dx$$

input
```
int(1/(d+e*x^n)/(a+c*x^(2*n)),x)
```

output
```
int(1/(d+e*x^n)/(a+c*x^(2*n)),x)
```

3.45.5 Fricas [F]

$$\int \frac{1}{(d+ex^{n})\,(a+cx^{2n})}\,dx = \int \frac{1}{(cx^{2\,n}+a)(ex^{n}+d)}\,dx$$

input
```
integrate(1/(d+e*x^n)/(a+c*x^(2*n)),x, algorithm="fricas")
```

output
```
integral(1/(a*e*x^n + a*d + (c*e*x^n + c*d)*x^(2*n)), x)
```

3.45.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex^{n})\,(a+cx^{2n})}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(1/(d+e*x**n)/(a+c*x**(2*n)),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.45.7 Maxima [F]

$$\int \frac{1}{(d+ex^n)(a+cx^{2n})}\,dx = \int \frac{1}{(cx^{2n}+a)(ex^n+d)}\,dx$$

input
```
integrate(1/(d+e*x^n)/(a+c*x^(2*n)),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^(2*n) + a)*(e*x^n + d)), x)
```

3.45.8 Giac [F]

$$\int \frac{1}{(d+ex^n)(a+cx^{2n})}\,dx = \int \frac{1}{(cx^{2n}+a)(ex^n+d)}\,dx$$

input
```
integrate(1/(d+e*x^n)/(a+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate(1/((c*x^(2*n) + a)*(e*x^n + d)), x)
```

3.45.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^n)(a+cx^{2n})}\,dx = \int \frac{1}{(a+c\,x^{2\,n})\,(d+e\,x^n)}\,dx$$

input
```
int(1/((a + c*x^(2*n))*(d + e*x^n)),x)
```

output
```
int(1/((a + c*x^(2*n))*(d + e*x^n)), x)
```

3.46 $\int \frac{1}{(d+ex^n)^2(a+cx^{2n})}\,dx$

3.46.1 Optimal result

Integrand size = 21, antiderivative size = 205

$$\int \frac{1}{(d+ex^n)^2(a+cx^{2n})}\,dx = \frac{c(cd^2-ae^2)\,x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a\,(cd^2+ae^2)^2} + \frac{2ce^2x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)}{(cd^2+ae^2)^2} - \frac{2c^2dex^{1+n}\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a\,(cd^2+ae^2)^2\,(1+n)} + \frac{e^2x\,\text{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)}{d^2\,(cd^2+ae^2)}$$

output
```
c*(-a*e^2+c*d^2)*x*hypergeom([1, 1/2/n],[1+1/2/n],-c*x^(2*n)/a)/a/(a*e^2+c
*d^2)^2+2*c*e^2*x*hypergeom([1, 1/n],[1+1/n],-e*x^n/d)/(a*e^2+c*d^2)^2-2*c
^2*d*e*x^(1+n)*hypergeom([1, 1/2*(1+n)/n],[3/2+1/2/n],-c*x^(2*n)/a)/a/(a*e
^2+c*d^2)^2/(1+n)+e^2*x*hypergeom([2, 1/n],[1+1/n],-e*x^n/d)/d^2/(a*e^2+c*
d^2)
```

3.46.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.91

$$\int \frac{1}{(d+ex^n)^2(a+cx^{2n})}\,dx$$
$$= \frac{x\left(cd^2(cd^2-ae^2)(1+n)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)+e\left(2acd^2e(1+n)\,\text{Hypergeometr}\right.\right.}{}$$

input
```
Integrate[1/((d + e*x^n)^2*(a + c*x^(2*n))),x]
```

output
```
(x*(c*d^2*(c*d^2 - a*e^2)*(1 + n)*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1
))/2, -((c*x^(2*n))/a)] + e*(2*a*c*d^2*e*(1 + n)*Hypergeometric2F1[1, n^(-
1), 1 + n^(-1), -((e*x^n)/d)] - 2*c^2*d^3*x^n*Hypergeometric2F1[1, (1 + n)
/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)] + a*e*(c*d^2 + a*e^2)*(1 + n)*Hy
pergeometric2F1[2, n^(-1), 1 + n^(-1), -((e*x^n)/d)])))/(a*(c*d^3 + a*d*e^
2)^2*(1 + n))
```

3.46.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1755, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^{2n})(d+ex^n)^2}\,dx$$
$$\Big\downarrow \text{1755}$$
$$\int \left(\frac{2cde^2}{(ae^2+cd^2)^2(d+ex^n)}+\frac{e^2}{(ae^2+cd^2)(d+ex^n)^2}-\frac{c(ae^2-cd^2+2cdex^n)}{(ae^2+cd^2)^2(a+cx^{2n})}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{2c^2dex^{n+1}\,\text{Hypergeometric2F1}\left(1,\frac{n+1}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a(n+1)\left(ae^2+cd^2\right)^2}+\frac{cx\left(cd^2-ae^2\right)\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a\left(ae^2+cd^2\right)^2}+\frac{2ce^2x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)}{\left(ae^2+cd^2\right)^2}+\frac{e^2x\,\text{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)}{d^2\left(ae^2+cd^2\right)}$$

input
```
Int[1/((d + e*x^n)^2*(a + c*x^(2*n))),x]
```

output
```
(c*(c*d^2 - a*e^2)*x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^
(2*n))/a)])/(a*(c*d^2 + a*e^2)^2) + (2*c*e^2*x*Hypergeometric2F1[1, n^(-1)
, 1 + n^(-1), -((e*x^n)/d)])/(c*d^2 + a*e^2)^2 - (2*c^2*d*e*x^(1 + n)*Hype
rgeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a*(c*d
^2 + a*e^2)^2*(1 + n)) + (e^2*x*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -
((e*x^n)/d)])/(d^2*(c*d^2 + a*e^2))
```

3.46.3.1 Defintions of rubi rules used

rule 1755
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^n)^q/(a + c*x^(2*n)), x], x] /; FreeQ[{a, c, d,
 e, n}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.46.4 Maple [F]

$$\int \frac{1}{\left(d+e\,x^n\right)^2\left(a+c\,x^{2n}\right)}dx$$

input
```
int(1/(d+e*x^n)^2/(a+c*x^(2*n)),x)
```

output
```
int(1/(d+e*x^n)^2/(a+c*x^(2*n)),x)
```

3.46.5 Fricas [F]

$$\int \frac{1}{(d+ex^n)^2 (a+cx^{2n})}\,dx = \int \frac{1}{(cx^{2n}+a)(ex^n+d)^2}\,dx$$

input
```
integrate(1/(d+e*x^n)^2/(a+c*x^(2*n)),x, algorithm="fricas")
```

output
```
integral(1/(a*e^2*x^(2*n) + 2*a*d*e*x^n + a*d^2 + (c*e^2*x^(2*n) + 2*c*d*e
*x^n + c*d^2)*x^(2*n)), x)
```

3.46.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex^n)^2 (a+cx^{2n})}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(1/(d+e*x**n)**2/(a+c*x**(2*n)),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.46.7 Maxima [F]

$$\int \frac{1}{(d+ex^n)^2 (a+cx^{2n})}\,dx = \int \frac{1}{(cx^{2n}+a)(ex^n+d)^2}\,dx$$

input
```
integrate(1/(d+e*x^n)^2/(a+c*x^(2*n)),x, algorithm="maxima")
```

output
```
e^2*x/(c*d^4*n + a*d^2*e^2*n + (c*d^3*e*n + a*d*e^3*n)*x^n) + (c*d^2*e^2*(
3*n - 1) + a*e^4*(n - 1))*integrate(1/(c^2*d^6*n + 2*a*c*d^4*e^2*n + a^2*d
^2*e^4*n + (c^2*d^5*e*n + 2*a*c*d^3*e^3*n + a^2*d*e^5*n)*x^n), x) - integr
ate((2*c^2*d*e*x^n - c^2*d^2 + a*c*e^2)/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3
*e^4 + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^(2*n)), x)
```

3.46.8 Giac [F]

$$\int \frac{1}{(d+ex^n)^2 (a+cx^{2n})} \, dx = \int \frac{1}{(cx^{2n}+a)(ex^n+d)^2} \, dx$$

input
```
integrate(1/(d+e*x^n)^2/(a+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate(1/((c*x^(2*n) + a)*(e*x^n + d)^2), x)
```

3.46.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^n)^2 (a+cx^{2n})} \, dx = \int \frac{1}{(a+c\,x^{2\,n})\,(d+e\,x^n)^2} \, dx$$

input
```
int(1/((a + c*x^(2*n))*(d + e*x^n)^2),x)
```

output
```
int(1/((a + c*x^(2*n))*(d + e*x^n)^2), x)
```

3.47 $\int \frac{d+ex^n}{a-cx^{2n}}\,dx$

3.47.1 Optimal result

Integrand size = 20, antiderivative size = 81

$$\int \frac{d+ex^n}{a-cx^{2n}}\,dx = \frac{dx\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),\frac{cx^{2n}}{a}\right)}{a} + \frac{ex^{1+n}\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),\frac{cx^{2n}}{a}\right)}{a(1+n)}$$

output
```
d*x*hypergeom([1, 1/2/n],[1+1/2/n],c*x^(2*n)/a)/a+e*x^(1+n)*hypergeom([1,
1/2*(1+n)/n],[3/2+1/2/n],c*x^(2*n)/a)/a/(1+n)
```

3.47.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00

$$\int \frac{d+ex^n}{a-cx^{2n}}\,dx = \frac{dx\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),\frac{cx^{2n}}{a}\right)}{a} + \frac{ex^{1+n}\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),\frac{cx^{2n}}{a}\right)}{a(1+n)}$$

input
```
Integrate[(d + e*x^n)/(a - c*x^(2*n)),x]
```

output
```
(d*x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, (c*x^(2*n))/a])/a + (e*
x^(1 + n)*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, (c*x^(2*n))/
a])/(a*(1 + n))
```

3.47.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1748, 778, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^n}{a-cx^{2n}}\,dx$$

$$\Big\downarrow \text{1748}$$

$$d\int \frac{1}{a-cx^{2n}}dx+e\int \frac{x^n}{a-cx^{2n}}dx$$

$$\Big\downarrow \text{778}$$

$$e\int \frac{x^n}{a-cx^{2n}}dx+\frac{dx\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),\frac{cx^{2n}}{a}\right)}{a}$$

$$\Big\downarrow \text{888}$$

$$\frac{dx\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),\frac{cx^{2n}}{a}\right)}{a}+\frac{ex^{n+1}\,\text{Hypergeometric2F1}\left(1,\frac{n+1}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),\frac{cx^{2n}}{a}\right)}{a(n+1)}$$

input
```
Int[(d + e*x^n)/(a - c*x^(2*n)),x]
```

output
```
(d*x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, (c*x^(2*n))/a])/a + (e*
x^(1 + n)*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, (c*x^(2*n))/
a])/(a*(1 + n))
```

3.47.3.1 Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1748
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> Simp[d
   Int[1/(a + c*x^(2*n)), x], x] + Simp[e   Int[x^n/(a + c*x^(2*n)), x], x]
 /; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && (
PosQ[a*c] ||  !IntegerQ[n])
```

3.47.4 Maple [F]

$$\int \frac{d+e\,x^{n}}{a-c\,x^{2 n}}dx$$

input
```
int((d+e*x^n)/(a-c*x^(2*n)),x)
```

output
```
int((d+e*x^n)/(a-c*x^(2*n)),x)
```

3.47.5 Fricas [F]

$$\int \frac{d+ex^n}{a-cx^{2n}}\,dx=\int -\frac{ex^{n}+d}{cx^{2\,n}-a}\,dx$$

input
```
integrate((d+e*x^n)/(a-c*x^(2*n)),x, algorithm="fricas")
```

output
```
integral(-(e*x^n + d)/(c*x^(2*n) - a), x)
```

3.47.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.25 (sec) , antiderivative size = 214, normalized size of antiderivative = 2.64

$$\int \frac{d+ex^n}{a-cx^{2n}}\,dx = \frac{a^{\frac{1}{2n}}a^{-1-\frac{1}{2n}}dx\Phi\left(\frac{cx^{2n}e^{2i\pi}}{a},1,\frac{1}{2n}\right)\Gamma\left(\frac{1}{2n}\right)}{4n^2\Gamma\left(1+\frac{1}{2n}\right)} + \frac{a^{-\frac{3}{2}-\frac{1}{2n}}a^{\frac{1}{2}+\frac{1}{2n}}ex^{n+1}\Phi\left(\frac{cx^{2n}e^{2i\pi}}{a},1,\frac{1}{2}+\frac{1}{2n}\right)\Gamma\left(\frac{1}{2}+\frac{1}{2n}\right)}{4n\Gamma\left(\frac{3}{2}+\frac{1}{2n}\right)} + \frac{a^{-\frac{3}{2}-\frac{1}{2n}}a^{\frac{1}{2}+\frac{1}{2n}}ex^{n+1}\Phi\left(\frac{cx^{2n}e^{2i\pi}}{a},1,\frac{1}{2}+\frac{1}{2n}\right)\Gamma\left(\frac{1}{2}+\frac{1}{2n}\right)}{4n^2\Gamma\left(\frac{3}{2}+\frac{1}{2n}\right)}$$

input
```
integrate((d+e*x**n)/(a-c*x**(2*n)),x)
```

output
```
a**(1/(2*n))*a**(-1 - 1/(2*n))*d*x*lerchphi(c*x**(2*n)*exp_polar(2*I*pi)/a
, 1, 1/(2*n))*gamma(1/(2*n))/(4*n**2*gamma(1 + 1/(2*n))) + a**(-3/2 - 1/(2
*n))*a**(1/2 + 1/(2*n))*e*x**(n + 1)*lerchphi(c*x**(2*n)*exp_polar(2*I*pi)
/a, 1, 1/2 + 1/(2*n))*gamma(1/2 + 1/(2*n))/(4*n*gamma(3/2 + 1/(2*n))) + a*
*(-3/2 - 1/(2*n))*a**(1/2 + 1/(2*n))*e*x**(n + 1)*lerchphi(c*x**(2*n)*exp_
polar(2*I*pi)/a, 1, 1/2 + 1/(2*n))*gamma(1/2 + 1/(2*n))/(4*n**2*gamma(3/2
+ 1/(2*n)))
```

3.47.7 Maxima [F]

$$\int \frac{d+ex^n}{a-cx^{2n}}\,dx = \int -\frac{ex^n+d}{cx^{2n}-a}\,dx$$

input
```
integrate((d+e*x^n)/(a-c*x^(2*n)),x, algorithm="maxima")
```

output
```
-integrate((e*x^n + d)/(c*x^(2*n) - a), x)
```

3.47.8 Giac [F]

$$\int \frac{d+ex^n}{a-cx^{2n}}\,dx=\int -\frac{ex^n+d}{cx^{2\,n}-a}\,dx$$

input
```
integrate((d+e*x^n)/(a-c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate(-(e*x^n + d)/(c*x^(2*n) - a), x)
```

3.47.9 Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^n}{a-cx^{2n}}\,dx=\int \frac{d+e\,x^n}{a-c\,x^{2\,n}}\,dx$$

input
```
int((d + e*x^n)/(a - c*x^(2*n)),x)
```

output
```
int((d + e*x^n)/(a - c*x^(2*n)), x)
```

3.48 $\int \frac{(d+ex^n)^3}{(a+cx^{2n})^2}\,dx$

3.48.1 Optimal result

Integrand size = 21, antiderivative size = 288

$$\begin{aligned}\int \frac{(d+ex^n)^3}{(a+cx^{2n})^2}\,dx &= \frac{x(d(cd^2-3ae^2)+e(3cd^2-ae^2)\,x^n)}{2acn\,(a+cx^{2n})} \\ &+ \frac{3de^2x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{ac} \\ &- \frac{d(cd^2-3ae^2)\,(1-2n)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{2a^2cn} \\ &+ \frac{e^3x^{1+n}\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{ac(1+n)} \\ &- \frac{e(3cd^2-ae^2)\,(1-n)x^{1+n}\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{2a^2cn(1+n)}\end{aligned}$$

output
```
1/2*x*(d*(-3*a*e^2+c*d^2)+e*(-a*e^2+3*c*d^2)*x^n)/a/c/n/(a+c*x^(2*n))+3*d*
e^2*x*hypergeom([1, 1/2/n],[1+1/2/n],-c*x^(2*n)/a)/a/c-1/2*d*(-3*a*e^2+c*d
^2)*(1-2*n)*x*hypergeom([1, 1/2/n],[1+1/2/n],-c*x^(2*n)/a)/a^2/c/n+e^3*x^(
1+n)*hypergeom([1, 1/2*(1+n)/n],[3/2+1/2/n],-c*x^(2*n)/a)/a/c/(1+n)-1/2*e*
(-a*e^2+3*c*d^2)*(1-n)*x^(1+n)*hypergeom([1, 1/2*(1+n)/n],[3/2+1/2/n],-c*x
^(2*n)/a)/a^2/c/n/(1+n)
```

3.48.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.65

$$\int \frac{(d+ex^n)^3}{(a+cx^{2n})^2}\,dx$$
$$= \frac{x\left(3ade^2\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)+\frac{ae^3x^n\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{1+n}+d(cd^2\right.}{a^2}$$

input
```
Integrate[(d + e*x^n)^3/(a + c*x^(2*n))^2,x]
```

output
```
(x*(3*a*d*e^2*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/
a)] + (a*e^3*x^n*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*
x^(2*n))/a)])/(1 + n) + d*(c*d^2 - 3*a*e^2)*Hypergeometric2F1[2, 1/(2*n),
(2 + n^(-1))/2, -((c*x^(2*n))/a)] + (e*(3*c*d^2 - a*e^2)*x^n*Hypergeometri
c2F1[2, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(1 + n)))/(a^2*c
)
```

3.48.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1767, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^n)^3}{(a+cx^{2n})^2}\,dx$$
$$\Big\downarrow \text{ 1767}$$
$$\int \left(\frac{x^n\left(3cd^2e-ae^3\right)-3ade^2+cd^3}{c\left(a+cx^{2n}\right)^2}+\frac{e^2(3d+ex^n)}{c\left(a+cx^{2n}\right)}\right)dx$$
$$\Big\downarrow \text{ 2009}$$

$$-\frac{e(1-n)x^{n+1}(3cd^2-ae^2)\,\text{Hypergeometric2F1}\left(1,\frac{n+1}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{2a^2cn(n+1)}-\frac{d(1-2n)x(cd^2-3ae^2)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{2a^2cn}+\frac{x(ex^n(3cd^2-ae^2)+d(cd^2-3ae^2))}{2acn(a+cx^{2n})}+\frac{3de^2x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{ac}+\frac{e^3x^{n+1}\,\text{Hypergeometric2F1}\left(1,\frac{n+1}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{ac(n+1)}$$

input
```
Int[(d + e*x^n)^3/(a + c*x^(2*n))^2,x]
```

output
```
(x*(d*(c*d^2 - 3*a*e^2) + e*(3*c*d^2 - a*e^2)*x^n))/(2*a*c*n*(a + c*x^(2*n
))) + (3*d*e^2*x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n
))/a)])/(a*c) - (d*(c*d^2 - 3*a*e^2)*(1 - 2*n)*x*Hypergeometric2F1[1, 1/(2
*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/(2*a^2*c*n) + (e^3*x^(1 + n)*Hyper
geometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a*c*(1
+ n)) - (e*(3*c*d^2 - a*e^2)*(1 - n)*x^(1 + n)*Hypergeometric2F1[1, (1 + n
)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(2*a^2*c*n*(1 + n))
```

3.48.3.1 Defintions of rubi rules used

rule 1767
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + c*x^(2*n))^p, x], x] /; FreeQ[{a
, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && ((Integ
ersQ[p, q] &&  !IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] &&  !IntegerQ[n])
)
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.48.4 Maple [F]

$$\int \frac{(d+e\,x^{n})^{3}}{(a+c\,x^{2n})^{2}}dx$$

input
```
int((d+e*x^n)^3/(a+c*x^(2*n))^2,x)
```

output
```
int((d+e*x^n)^3/(a+c*x^(2*n))^2,x)
```

3.48.5 Fricas [F]

$$\int \frac{(d+ex^n)^3}{(a+cx^{2n})^2}\,dx = \int \frac{(ex^n+d)^3}{(cx^{2\,n}+a)^2}\,dx$$

input
```
integrate((d+e*x^n)^3/(a+c*x^(2*n))^2,x, algorithm="fricas")
```

output
```
integral((e^3*x^(3*n) + 3*d*e^2*x^(2*n) + 3*d^2*e*x^n + d^3)/(c^2*x^(4*n)
+ 2*a*c*x^(2*n) + a^2), x)
```

3.48.6 Sympy [F]

$$\int \frac{(d+ex^n)^3}{(a+cx^{2n})^2}\,dx = \int \frac{(d+ex^n)^3}{(a+cx^{2n})^2}\,dx$$

input
```
integrate((d+e*x**n)**3/(a+c*x**(2*n))**2,x)
```

output
```
Integral((d + e*x**n)**3/(a + c*x**(2*n))**2, x)
```

3.48.7 Maxima [F]

$$\int \frac{(d+ex^n)^3}{(a+cx^{2n})^2}\,dx = \int \frac{(ex^n+d)^3}{(cx^{2n}+a)^2}\,dx$$

input
```
integrate((d+e*x^n)^3/(a+c*x^(2*n))^2,x, algorithm="maxima")
```

output
```
1/2*((3*c*d^2*e - a*e^3)*x*x^n + (c*d^3 - 3*a*d*e^2)*x)/(a*c^2*n*x^(2*n) +
 a^2*c*n) + integrate(1/2*(c*d^3*(2*n - 1) + 3*a*d*e^2 + (a*e^3*(n + 1) +
3*c*d^2*e*(n - 1))*x^n)/(a*c^2*n*x^(2*n) + a^2*c*n), x)
```

3.48.8 Giac [F]

$$\int \frac{(d+ex^n)^3}{(a+cx^{2n})^2}\,dx = \int \frac{(ex^n+d)^3}{(cx^{2n}+a)^2}\,dx$$

input
```
integrate((d+e*x^n)^3/(a+c*x^(2*n))^2,x, algorithm="giac")
```

output
```
integrate((e*x^n + d)^3/(c*x^(2*n) + a)^2, x)
```

3.48.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^n)^3}{(a+cx^{2n})^2}\,dx = \int \frac{(d+e\,x^n)^3}{(a+c\,x^{2\,n})^2}\,dx$$

input
```
int((d + e*x^n)^3/(a + c*x^(2*n))^2,x)
```

output
```
int((d + e*x^n)^3/(a + c*x^(2*n))^2, x)
```

3.49 $\int \frac{(d+ex^n)^2}{(a+cx^{2n})^2} dx$

3.49.1 Optimal result

Integrand size = 21, antiderivative size = 203

$$\int \frac{(d+ex^n)^2}{(a+cx^{2n})^2} dx = \frac{x(cd^2-ae^2+2cdex^n)}{2acn(a+cx^{2n})} + \frac{e^2x \operatorname{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{ac} - \frac{(cd^2-ae^2)(1-2n)x \operatorname{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{2a^2cn} - \frac{de(1-n)x^{1+n} \operatorname{Hypergeometric2F1}\left(1,\frac{1+n}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a^2n(1+n)}$$

output
```
1/2*x*(c*d^2-a*e^2+2*c*d*e*x^n)/a/c/n/(a+c*x^(2*n))+e^2*x*hypergeom([1, 1/
2/n],[1+1/2/n],-c*x^(2*n)/a)/a/c-1/2*(-a*e^2+c*d^2)*(1-2*n)*x*hypergeom([1
, 1/2/n],[1+1/2/n],-c*x^(2*n)/a)/a^2/c/n-d*e*(1-n)*x^(1+n)*hypergeom([1, 1
/2*(1+n)/n],[3/2+1/2/n],-c*x^(2*n)/a)/a^2/n/(1+n)
```

3.49.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.67

$$\int \frac{(d+ex^n)^2}{(a+cx^{2n})^2}\,dx = \frac{x\left(ae^2(1+n)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)+(cd^2-ae^2)\,(1+n)\,\text{Hypergeometric2F1}\left(2\right.\right.}{a^2c(1+n)}$$

input
```
Integrate[(d + e*x^n)^2/(a + c*x^(2*n))^2,x]
```

output
```
(x*(a*e^2*(1 + n)*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*
n))/a)] + (c*d^2 - a*e^2)*(1 + n)*Hypergeometric2F1[2, 1/(2*n), (2 + n^(-1
))/2, -((c*x^(2*n))/a)] + 2*c*d*e*x^n*Hypergeometric2F1[2, (1 + n)/(2*n),
(3 + n^(-1))/2, -((c*x^(2*n))/a)]))/(a^2*c*(1 + n))
```

3.49.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1767, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^n)^2}{(a+cx^{2n})^2}\,dx$$

$$\Big\downarrow \ 1767$$

$$\int \left(\frac{-ae^2+cd^2+2cdex^n}{c\,(a+cx^{2n})^2}+\frac{e^2}{c\,(a+cx^{2n})}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{(1-2n)x\,(cd^2-ae^2)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{2a^2cn}-\frac{de(1-n)x^{n+1}\,\text{Hypergeometric2F1}\left(1,\frac{n+1}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a^2n(n+1)}+\frac{x\,(-ae^2+cd^2+2cdex^n)}{2acn\,(a+cx^{2n})}+\frac{e^2x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{ac}$$

input
```
Int[(d + e*x^n)^2/(a + c*x^(2*n))^2,x]
```

output
```
(x*(c*d^2 - a*e^2 + 2*c*d*e*x^n))/(2*a*c*n*(a + c*x^(2*n))) + (e^2*x*Hyper
geometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/(a*c) - ((c*d^
2 - a*e^2)*(1 - 2*n)*x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*
x^(2*n))/a)])/(2*a^2*c*n) - (d*e*(1 - n)*x^(1 + n)*Hypergeometric2F1[1, (1
 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a^2*n*(1 + n))
```

3.49.3.1 Defintions of rubi rules used

rule 1767
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + c*x^(2*n))^p, x], x] /; FreeQ[{a
, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && ((Integ
ersQ[p, q] &&  !IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] &&  !IntegerQ[n])
)
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.49.4 Maple [F]

$$\int \frac{(d+e\,x^n)^2}{(a+c\,x^{2n})^2}dx$$

input
```
int((d+e*x^n)^2/(a+c*x^(2*n))^2,x)
```

output
```
int((d+e*x^n)^2/(a+c*x^(2*n))^2,x)
```

3.49.5 Fricas [F]

$$\int \frac{(d+ex^n)^2}{(a+cx^{2n})^2}\,dx = \int \frac{(ex^n+d)^2}{(cx^{2\,n}+a)^2}\,dx$$

input
```
integrate((d+e*x^n)^2/(a+c*x^(2*n))^2,x, algorithm="fricas")
```

output
```
integral((e^2*x^(2*n) + 2*d*e*x^n + d^2)/(c^2*x^(4*n) + 2*a*c*x^(2*n) + a^
2), x)
```

3.49.6 Sympy [F]

$$\int \frac{(d+ex^n)^2}{(a+cx^{2n})^2}\,dx=\int \frac{(d+ex^n)^2}{(a+cx^{2n})^2}\,dx$$

input
```
integrate((d+e*x**n)**2/(a+c*x**(2*n))**2,x)
```

output
```
Integral((d + e*x**n)**2/(a + c*x**(2*n))**2, x)
```

3.49.7 Maxima [F]

$$\int \frac{(d+ex^n)^2}{(a+cx^{2n})^2}\,dx=\int \frac{(ex^n+d)^2}{(cx^{2n}+a)^2}\,dx$$

input
```
integrate((d+e*x^n)^2/(a+c*x^(2*n))^2,x, algorithm="maxima")
```

output
```
1/2*(2*c*d*e*x*x^n + (c*d^2 - a*e^2)*x)/(a*c^2*n*x^(2*n) + a^2*c*n) + inte
grate(1/2*(2*c*d*e*(n - 1)*x^n + c*d^2*(2*n - 1) + a*e^2)/(a*c^2*n*x^(2*n)
 + a^2*c*n), x)
```

3.49.8 Giac [F]

$$\int \frac{(d+ex^n)^2}{(a+cx^{2n})^2}\,dx=\int \frac{(ex^n+d)^2}{(cx^{2n}+a)^2}\,dx$$

input
```
integrate((d+e*x^n)^2/(a+c*x^(2*n))^2,x, algorithm="giac")
```

output
```
integrate((e*x^n + d)^2/(c*x^(2*n) + a)^2, x)
```

3.49.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^n)^2}{(a+cx^{2n})^2}\,dx=\int \frac{{\left(d+e\,x^n\right)}^2}{{\left(a+c\,x^{2\,n}\right)}^2}\,dx$$

input
```
int((d + e*x^n)^2/(a + c*x^(2*n))^2,x)
```

output
```
int((d + e*x^n)^2/(a + c*x^(2*n))^2, x)
```

3.50 $\int \frac{d+ex^n}{(a+cx^{2n})^2}\,dx$

3.50.1 Optimal result

Integrand size = 19, antiderivative size = 134

$$\int \frac{d+ex^n}{(a+cx^{2n})^2}\,dx = \frac{x(d+ex^n)}{2an(a+cx^{2n})} - \frac{d(1-2n)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{2a^2n} - \frac{e(1-n)x^{1+n}\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{2a^2n(1+n)}$$

output
```
1/2*x*(d+e*x^n)/a/n/(a+c*x^(2*n))-1/2*d*(1-2*n)*x*hypergeom([1, 1/2/n],[1+
1/2/n],-c*x^(2*n)/a)/a^2/n-1/2*e*(1-n)*x^(1+n)*hypergeom([1, 1/2*(1+n)/n],
[3/2+1/2/n],-c*x^(2*n)/a)/a^2/n/(1+n)
```

3.50.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.62

$$\int \frac{d+ex^n}{(a+cx^{2n})^2}\,dx = \frac{dx\,\text{Hypergeometric2F1}\left(2,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a^2} + \frac{ex^{1+n}\,\text{Hypergeometric2F1}\left(2,\frac{1+n}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a^2(1+n)}$$

input
```
Integrate[(d + e*x^n)/(a + c*x^(2*n))^2,x]
```

output
```
(d*x*Hypergeometric2F1[2, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/a^2
+ (e*x^(1 + n)*Hypergeometric2F1[2, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^
(2*n))/a)])/(a^2*(1 + n))
```

3.50.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {1761, 1748, 778, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^n}{(a+cx^{2n})^2}dx$$

$$\Big\downarrow \text{1761}$$

$$\frac{x(d+ex^n)}{2an(a+cx^{2n})}-\frac{\int \frac{e(1-n)x^n+d(1-2n)}{cx^{2n}+a}dx}{2an}$$

$$\Big\downarrow \text{1748}$$

$$\frac{x(d+ex^n)}{2an(a+cx^{2n})}-\frac{d(1-2n)\int \frac{1}{cx^{2n}+a}dx+e(1-n)\int \frac{x^n}{cx^{2n}+a}dx}{2an}$$

$$\Big\downarrow \text{778}$$

$$\frac{x(d+ex^n)}{2an(a+cx^{2n})}-\frac{e(1-n)\int \frac{x^n}{cx^{2n}+a}dx+\frac{d(1-2n)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a}}{2an}$$

$$\Big\downarrow \text{888}$$

$$\frac{x(d+ex^n)}{2an(a+cx^{2n})}-\frac{\frac{d(1-2n)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a}+\frac{e(1-n)x^{n+1}\,\text{Hypergeometric2F1}\left(1,\frac{n+1}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a(n+1)}}{2an}$$

input
```
Int[(d + e*x^n)/(a + c*x^(2*n))^2,x]
```

output
```
(x*(d + e*x^n))/(2*a*n*(a + c*x^(2*n))) - ((d*(1 - 2*n)*x*Hypergeometric2F
1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/a + (e*(1 - n)*x^(1 + n)*
Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a*
(1 + n)))/(2*a*n)
```

3.50.3.1 Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1748
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> Simp[d
   Int[1/(a + c*x^(2*n)), x], x] + Simp[e   Int[x^n/(a + c*x^(2*n)), x], x]
 /; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && (
PosQ[a*c] ||  !IntegerQ[n])
```

rule 1761
```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> S
imp[(-x)*(d + e*x^n)*((a + c*x^(2*n))^(p + 1)/(2*a*n*(p + 1))), x] + Simp[1
/(2*a*n*(p + 1))   Int[(d*(2*n*p + 2*n + 1) + e*(2*n*p + 3*n + 1)*x^n)*(a +
 c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] &&
 ILtQ[p, -1]
```

3.50.4 Maple [F]

$$\int \frac{d + e\,x^n}{\left(a + c\,x^{2n}\right)^2} dx$$

input
```
int((d+e*x^n)/(a+c*x^(2*n))^2,x)
```

output
```
int((d+e*x^n)/(a+c*x^(2*n))^2,x)
```

3.50.5 Fricas [F]

$$\int \frac{d+ex^n}{(a+cx^{2n})^2}\,dx = \int \frac{ex^n+d}{(cx^{2n}+a)^2}\,dx$$

input
```
integrate((d+e*x^n)/(a+c*x^(2*n))^2,x, algorithm="fricas")
```

output
```
integral((e*x^n + d)/(c^2*x^(4*n) + 2*a*c*x^(2*n) + a^2), x)
```

3.50.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 168.68 (sec) , antiderivative size = 994, normalized size of antiderivative = 7.42

$$\int \frac{d+ex^n}{(a+cx^{2n})^2}\,dx = \text{Too large to display}$$

input
```
integrate((d+e*x**n)/(a+c*x**(2*n))**2,x)
```

output
```
d*(2*a*a**(1/(2*n))*a**(-2 - 1/(2*n))*n*x*lerchphi(c*x**(2*n)*exp_polar(I*
pi)/a, 1, 1/(2*n))*gamma(1/(2*n))/(8*a*n**3*gamma(1 + 1/(2*n)) + 8*c*n**3*
x**(2*n)*gamma(1 + 1/(2*n))) + 2*a*a**(1/(2*n))*a**(-2 - 1/(2*n))*n*x*gamm
a(1/(2*n))/(8*a*n**3*gamma(1 + 1/(2*n)) + 8*c*n**3*x**(2*n)*gamma(1 + 1/(2
*n))) - a*a**(1/(2*n))*a**(-2 - 1/(2*n))*x*lerchphi(c*x**(2*n)*exp_polar(I
*pi)/a, 1, 1/(2*n))*gamma(1/(2*n))/(8*a*n**3*gamma(1 + 1/(2*n)) + 8*c*n**3
*x**(2*n)*gamma(1 + 1/(2*n))) + 2*a**(1/(2*n))*a**(-2 - 1/(2*n))*c*n*x*x**
(2*n)*lerchphi(c*x**(2*n)*exp_polar(I*pi)/a, 1, 1/(2*n))*gamma(1/(2*n))/(8
*a*n**3*gamma(1 + 1/(2*n)) + 8*c*n**3*x**(2*n)*gamma(1 + 1/(2*n))) - a**(1
/(2*n))*a**(-2 - 1/(2*n))*c*x*x**(2*n)*lerchphi(c*x**(2*n)*exp_polar(I*pi)
/a, 1, 1/(2*n))*gamma(1/(2*n))/(8*a*n**3*gamma(1 + 1/(2*n)) + 8*c*n**3*x**
(2*n)*gamma(1 + 1/(2*n)))) + e*(a*a**(-5/2 - 1/(2*n))*a**(1/2 + 1/(2*n))*n
**2*x**(n + 1)*lerchphi(c*x**(2*n)*exp_polar(I*pi)/a, 1, 1/2 + 1/(2*n))*ga
mma(1/2 + 1/(2*n))/(8*a*n**3*gamma(3/2 + 1/(2*n)) + 8*c*n**3*x**(2*n)*gamm
a(3/2 + 1/(2*n))) + 2*a*a**(-5/2 - 1/(2*n))*a**(1/2 + 1/(2*n))*n**2*x**(n
+ 1)*gamma(1/2 + 1/(2*n))/(8*a*n**3*gamma(3/2 + 1/(2*n)) + 8*c*n**3*x**(2*
n)*gamma(3/2 + 1/(2*n))) + 2*a*a**(-5/2 - 1/(2*n))*a**(1/2 + 1/(2*n))*n*x*
*(n + 1)*gamma(1/2 + 1/(2*n))/(8*a*n**3*gamma(3/2 + 1/(2*n)) + 8*c*n**3*x*
*(2*n)*gamma(3/2 + 1/(2*n))) - a*a**(-5/2 - 1/(2*n))*a**(1/2 + 1/(2*n))*x*
*(n + 1)*lerchphi(c*x**(2*n)*exp_polar(I*pi)/a, 1, 1/2 + 1/(2*n))*gamma...
```

3.50.7 Maxima [F]

$$\int \frac{d+ex^n}{(a+cx^{2n})^2}\,dx = \int \frac{ex^n+d}{(cx^{2n}+a)^2}\,dx$$

input
```
integrate((d+e*x^n)/(a+c*x^(2*n))^2,x, algorithm="maxima")
```

output
```
1/2*(e*x*x^n + d*x)/(a*c*n*x^(2*n) + a^2*n) + integrate(1/2*(e*(n - 1)*x^n
 + d*(2*n - 1))/(a*c*n*x^(2*n) + a^2*n), x)
```

3.50.8 Giac [F]

$$\int \frac{d+ex^n}{(a+cx^{2n})^2}\,dx = \int \frac{ex^n+d}{(cx^{2n}+a)^2}\,dx$$

input
```
integrate((d+e*x^n)/(a+c*x^(2*n))^2,x, algorithm="giac")
```

output
```
integrate((e*x^n + d)/(c*x^(2*n) + a)^2, x)
```

3.50.9 Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^n}{(a+cx^{2n})^2}\,dx = \int \frac{d+e\,x^n}{(a+c\,x^{2\,n})^2}\,dx$$

input
```
int((d + e*x^n)/(a + c*x^(2*n))^2,x)
```

output
```
int((d + e*x^n)/(a + c*x^(2*n))^2, x)
```

3.51 $\int \frac{1}{(d+ex^n)\left(a+cx^{2n}\right)^2}\,dx$

3.51.1 Optimal result

Integrand size = 21, antiderivative size = 333

$$\begin{aligned}\int \frac{1}{(d+ex^n)\left(a+cx^{2n}\right)^2}\,dx &= \frac{cx(d-ex^n)}{2a\left(cd^2+ae^2\right)n\left(a+cx^{2n}\right)} + \frac{cde^2x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a\left(cd^2+ae^2\right)^2} \\ &\quad - \frac{cd(1-2n)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{2a^2\left(cd^2+ae^2\right)n} \\ &\quad + \frac{e^4x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)}{d\left(cd^2+ae^2\right)^2} \\ &\quad - \frac{ce^3x^{1+n}\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a\left(cd^2+ae^2\right)^2(1+n)} \\ &\quad + \frac{ce(1-n)x^{1+n}\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{2a^2\left(cd^2+ae^2\right)n(1+n)}\end{aligned}$$

output
```
1/2*c*x*(d-e*x^n)/a/(a*e^2+c*d^2)/n/(a+c*x^(2*n))+c*d*e^2*x*hypergeom([1,
1/2/n],[1+1/2/n],-c*x^(2*n)/a)/a/(a*e^2+c*d^2)^2-1/2*c*d*(1-2*n)*x*hyperge
om([1, 1/2/n],[1+1/2/n],-c*x^(2*n)/a)/a^2/(a*e^2+c*d^2)/n+e^4*x*hypergeom(
[1, 1/n],[1+1/n],-e*x^n/d)/d/(a*e^2+c*d^2)^2-c*e^3*x^(1+n)*hypergeom([1, 1
/2*(1+n)/n],[3/2+1/2/n],-c*x^(2*n)/a)/a/(a*e^2+c*d^2)^2/(1+n)+1/2*c*e*(1-n
)*x^(1+n)*hypergeom([1, 1/2*(1+n)/n],[3/2+1/2/n],-c*x^(2*n)/a)/a^2/(a*e^2+
c*d^2)/n/(1+n)
```

3.51.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.68

$$\int \frac{1}{(d+ex^n)(a+cx^{2n})^2}\,dx$$
$$= \frac{x\left(acd^2e^2(1+n)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)+a^2e^4(1+n)\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1\right.\right.}{}$$

input
```
Integrate[1/((d + e*x^n)*(a + c*x^(2*n))^2),x]
```

output
```
(x*(a*c*d^2*e^2*(1 + n)*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c
*x^(2*n))/a)] + a^2*e^4*(1 + n)*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -
((e*x^n)/d)] + c*d*(-(a*e^3*x^n*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n
^(-1))/2, -((c*x^(2*n))/a)]) + (c*d^2 + a*e^2)*(d*(1 + n)*Hypergeometric2F
1[2, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)] - e*x^n*Hypergeometric2F1[
2, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)]))))/(a^2*d*(c*d^2 + a*
e^2)^2*(1 + n))
```

3.51.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1767, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^{2n})^2(d+ex^n)}\,dx$$
$$\Big\downarrow \text{1767}$$
$$\int \left(-\frac{ce^2(ex^n-d)}{(ae^2+cd^2)^2(a+cx^{2n})}-\frac{c(ex^n-d)}{(ae^2+cd^2)(a+cx^{2n})^2}+\frac{e^4}{(ae^2+cd^2)^2(d+ex^n)}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{ce(1-n)x^{n+1}\,\text{Hypergeometric2F1}\left(1,\frac{n+1}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{2a^2n(n+1)\left(ae^2+cd^2\right)}-\frac{cd(1-2n)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{2a^2n\left(ae^2+cd^2\right)}+\frac{cde^2x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a\left(ae^2+cd^2\right)^2}+\frac{cx(d-ex^n)}{2an\left(ae^2+cd^2\right)\left(a+cx^{2n}\right)}+\frac{e^4x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)}{d\left(ae^2+cd^2\right)^2}-\frac{ce^3x^{n+1}\,\text{Hypergeometric2F1}\left(1,\frac{n+1}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a(n+1)\left(ae^2+cd^2\right)^2}$$

input
```
Int[1/((d + e*x^n)*(a + c*x^(2*n))^2),x]
```

output
```
(c*x*(d - e*x^n))/(2*a*(c*d^2 + a*e^2)*n*(a + c*x^(2*n))) + (c*d*e^2*x*Hyp
ergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/(a*(c*d^2 +
a*e^2)^2) - (c*d*(1 - 2*n)*x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2,
 -((c*x^(2*n))/a)])/(2*a^2*(c*d^2 + a*e^2)*n) + (e^4*x*Hypergeometric2F1[1
, n^(-1), 1 + n^(-1), -((e*x^n)/d)])/(d*(c*d^2 + a*e^2)^2) - (c*e^3*x^(1 +
 n)*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])
/(a*(c*d^2 + a*e^2)^2*(1 + n)) + (c*e*(1 - n)*x^(1 + n)*Hypergeometric2F1[
1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(2*a^2*(c*d^2 + a*e^2
)*n*(1 + n))
```

3.51.3.1 Defintions of rubi rules used

rule 1767
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + c*x^(2*n))^p, x], x] /; FreeQ[{a
, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && ((Integ
ersQ[p, q] &&  !IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] &&  !IntegerQ[n])
)
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.51.4 Maple [F]

$$\int \frac{1}{(d+e\,x^{n})\,(a+c\,x^{2n})^{2}}dx$$

input
```
int(1/(d+e*x^n)/(a+c*x^(2*n))^2,x)
```

output
```
int(1/(d+e*x^n)/(a+c*x^(2*n))^2,x)
```

3.51.5 Fricas [F]

$$\int \frac{1}{(d+ex^{n})\,(a+cx^{2n})^{2}}\,dx = \int \frac{1}{(cx^{2\,n}+a)^{2}(ex^{n}+d)}\,dx$$

input
```
integrate(1/(d+e*x^n)/(a+c*x^(2*n))^2,x, algorithm="fricas")
```

output
```
integral(1/(a^2*e*x^n + a^2*d + (c^2*e*x^n + c^2*d)*x^(4*n) + 2*(a*c*e*x^n
 + a*c*d)*x^(2*n)), x)
```

3.51.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^{n})\,(a+cx^{2n})^{2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(d+e*x**n)/(a+c*x**(2*n))**2,x)
```

output
```
Timed out
```

3.51.7 Maxima [F]

$$\int \frac{1}{(d+ex^n)\left(a+cx^{2n}\right)^2}\,dx = \int \frac{1}{(cx^{2n}+a)^2(ex^n+d)}\,dx$$

input
```
integrate(1/(d+e*x^n)/(a+c*x^(2*n))^2,x, algorithm="maxima")
```

output
```
e^4*integrate(1/(c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4 + (c^2*d^4*e + 2*a*c*
d^2*e^3 + a^2*e^5)*x^n), x) - 1/2*(c*e*x*x^n - c*d*x)/(a^2*c*d^2*n + a^3*e
^2*n + (a*c^2*d^2*n + a^2*c*e^2*n)*x^(2*n)) - integrate(-1/2*(a*c*d*e^2*(4
*n - 1) + c^2*d^3*(2*n - 1) - (a*c*e^3*(3*n - 1) + c^2*d^2*e*(n - 1))*x^n)
/(a^2*c^2*d^4*n + 2*a^3*c*d^2*e^2*n + a^4*e^4*n + (a*c^3*d^4*n + 2*a^2*c^2
*d^2*e^2*n + a^3*c*e^4*n)*x^(2*n)), x)
```

3.51.8 Giac [F]

$$\int \frac{1}{(d+ex^n)\left(a+cx^{2n}\right)^2}\,dx = \int \frac{1}{(cx^{2n}+a)^2(ex^n+d)}\,dx$$

input
```
integrate(1/(d+e*x^n)/(a+c*x^(2*n))^2,x, algorithm="giac")
```

output
```
integrate(1/((c*x^(2*n) + a)^2*(e*x^n + d)), x)
```

3.51.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^n)\left(a+cx^{2n}\right)^2}\,dx = \int \frac{1}{\left(a+c\,x^{2\,n}\right)^2\,\left(d+e\,x^n\right)}\,dx$$

input
```
int(1/((a + c*x^(2*n))^2*(d + e*x^n)),x)
```

output
```
int(1/((a + c*x^(2*n))^2*(d + e*x^n)), x)
```

3.52 $\int \frac{1}{(d+ex^n)^2(a+cx^{2n})^2}\,dx$

3.52.1 Optimal result

Integrand size = 21, antiderivative size = 410

$$\begin{aligned}
&\int \frac{1}{(d+ex^n)^2\,(a+cx^{2n})^2}\,dx \\
&= \frac{cx(cd^2-ae^2-2cdex^n)}{2a\,(cd^2+ae^2)^2\,n\,(a+cx^{2n})} \\
&\quad + \frac{ce^2(3cd^2-ae^2)\,x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a\,(cd^2+ae^2)^3} \\
&\quad - \frac{c(cd^2-ae^2)\,(1-2n)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{2a^2\,(cd^2+ae^2)^2\,n} \\
&\quad + \frac{4ce^4x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)}{(cd^2+ae^2)^3} \\
&\quad - \frac{4c^2de^3x^{1+n}\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a\,(cd^2+ae^2)^3\,(1+n)} \\
&\quad + \frac{c^2de(1-n)x^{1+n}\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a^2\,(cd^2+ae^2)^2\,n(1+n)} \\
&\quad + \frac{e^4x\,\text{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)}{d^2\,(cd^2+ae^2)^2}
\end{aligned}$$

output
```
1/2*c*x*(c*d^2-a*e^2-2*c*d*e*x^n)/a/(a*e^2+c*d^2)^2/n/(a+c*x^(2*n))+c*e^2*
(-a*e^2+3*c*d^2)*x*hypergeom([1, 1/2/n],[1+1/2/n],-c*x^(2*n)/a)/a/(a*e^2+c
*d^2)^3-1/2*c*(-a*e^2+c*d^2)*(1-2*n)*x*hypergeom([1, 1/2/n],[1+1/2/n],-c*x
^(2*n)/a)/a^2/(a*e^2+c*d^2)^2/n+4*c*e^4*x*hypergeom([1, 1/n],[1+1/n],-e*x^
n/d)/(a*e^2+c*d^2)^3-4*c^2*d*e^3*x^(1+n)*hypergeom([1, 1/2*(1+n)/n],[3/2+1
/2/n],-c*x^(2*n)/a)/a/(a*e^2+c*d^2)^3/(1+n)+c^2*d*e*(1-n)*x^(1+n)*hypergeo
m([1, 1/2*(1+n)/n],[3/2+1/2/n],-c*x^(2*n)/a)/a^2/(a*e^2+c*d^2)^2/n/(1+n)+e
^4*x*hypergeom([2, 1/n],[1+1/n],-e*x^n/d)/d^2/(a*e^2+c*d^2)^2
```

3.52.2 Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 298, normalized size of antiderivative = 0.73

$$\int \frac{1}{(d+ex^n)^2(a+cx^{2n})^2}\,dx = \frac{x\left(\frac{ce^2(3cd^2-ae^2)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a}+4ce^4\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)-\frac{4c^2de^3x^n}{\ldots}\right.}{\ldots}$$

input
```
Integrate[1/((d + e*x^n)^2*(a + c*x^(2*n))^2),x]
```

output
```
(x*((c*e^2*(3*c*d^2 - a*e^2)*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2,
 -((c*x^(2*n))/a)])/a + 4*c*e^4*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -
((e*x^n)/d)] - (4*c^2*d*e^3*x^n*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n
^(-1))/2, -((c*x^(2*n))/a)])/(a*(1 + n)) + (c*(c*d^2 - a*e^2)*(c*d^2 + a*e
^2)*Hypergeometric2F1[2, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/a^2 +
 (e^4*(c*d^2 + a*e^2)*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((e*x^n)/d
)])/d^2 - (2*c^2*d*e*(c*d^2 + a*e^2)*x^n*Hypergeometric2F1[2, (1 + n)/(2*n
), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a^2*(1 + n))))/(c*d^2 + a*e^2)^3
```

3.52.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1767, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^{2n})^2(d+ex^n)^2}\,dx$$

$$\Big\downarrow \text{1767}$$

$$\int \left(-\frac{ce^2(ae^2-3cd^2+4cdex^n)}{(ae^2+cd^2)^3(a+cx^{2n})}-\frac{c(ae^2-cd^2+2cdex^n)}{(ae^2+cd^2)^2(a+cx^{2n})^2}+\frac{4cde^4}{(ae^2+cd^2)^3(d+ex^n)}+\frac{e^4}{(ae^2+cd^2)^2(d+ex^n)^2}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{c^2de(1-n)x^{n+1}\,\text{Hypergeometric2F1}\left(1,\frac{n+1}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a^2n(n+1)(ae^2+cd^2)^2}-\frac{c(1-2n)x(cd^2-ae^2)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{2a^2n(ae^2+cd^2)^2}-\frac{4c^2de^3x^{n+1}\,\text{Hypergeometric2F1}\left(1,\frac{n+1}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a(n+1)(ae^2+cd^2)^3}+\frac{ce^2x(3cd^2-ae^2)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a(ae^2+cd^2)^3}+\frac{cx(-ae^2+cd^2-2cdex^n)}{2an(ae^2+cd^2)^2(a+cx^{2n})}+\frac{4ce^4x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)}{(ae^2+cd^2)^3}+\frac{e^4x\,\text{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)}{d^2(ae^2+cd^2)^2}$$

input
```
Int[1/((d + e*x^n)^2*(a + c*x^(2*n))^2),x]
```

output
```
(c*x*(c*d^2 - a*e^2 - 2*c*d*e*x^n))/(2*a*(c*d^2 + a*e^2)^2*n*(a + c*x^(2*n
))) + (c*e^2*(3*c*d^2 - a*e^2)*x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1)
)/2, -((c*x^(2*n))/a)])/(a*(c*d^2 + a*e^2)^3) - (c*(c*d^2 - a*e^2)*(1 - 2*
n)*x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/(2*a
^2*(c*d^2 + a*e^2)^2*n) + (4*c*e^4*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-
1), -((e*x^n)/d)])/(c*d^2 + a*e^2)^3 - (4*c^2*d*e^3*x^(1 + n)*Hypergeometr
ic2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a*(c*d^2 + a*e
^2)^3*(1 + n)) + (c^2*d*e*(1 - n)*x^(1 + n)*Hypergeometric2F1[1, (1 + n)/(
2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a^2*(c*d^2 + a*e^2)^2*n*(1 + n))
 + (e^4*x*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((e*x^n)/d)])/(d^2*(c*
d^2 + a*e^2)^2)
```

3.52. $\int \frac{1}{(d+ex^n)^2(a+cx^{2n})^2}\,dx$

3.52.3.1 Defintions of rubi rules used

rule 1767
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + c*x^(2*n))^p, x], x] /; FreeQ[{a
, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && ((Integ
ersQ[p, q] && !IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] && !IntegerQ[n])
)
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.52.4 Maple [F]

$$\int \frac{1}{(d+e\,x^{n})^{2}\,(a+c\,x^{2n})^{2}}dx$$

input
```
int(1/(d+e*x^n)^2/(a+c*x^(2*n))^2,x)
```

output
```
int(1/(d+e*x^n)^2/(a+c*x^(2*n))^2,x)
```

3.52.5 Fricas [F]

$$\int \frac{1}{(d+ex^{n})^{2}\,(a+cx^{2n})^{2}}\,dx = \int \frac{1}{(cx^{2\,n}+a)^{2}(ex^{n}+d)^{2}}\,dx$$

input
```
integrate(1/(d+e*x^n)^2/(a+c*x^(2*n))^2,x, algorithm="fricas")
```

output
```
integral(1/(a^2*e^2*x^(2*n) + 2*a^2*d*e*x^n + a^2*d^2 + (c^2*e^2*x^(2*n) +
 2*c^2*d*e*x^n + c^2*d^2)*x^(4*n) + 2*(a*c*e^2*x^(2*n) + 2*a*c*d*e*x^n + a
*c*d^2)*x^(2*n)), x)
```

3.52.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^n)^2(a+cx^{2n})^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(d+e*x**n)**2/(a+c*x**(2*n))**2,x)
```

output
```
Timed out
```

3.52.7 Maxima [F]

$$\int \frac{1}{(d+ex^n)^2(a+cx^{2n})^2}\,dx = \int \frac{1}{(cx^{2n}+a)^2(ex^n+d)^2}\,dx$$

input
```
integrate(1/(d+e*x^n)^2/(a+c*x^(2*n))^2,x, algorithm="maxima")
```

output
```
(c*d^2*e^4*(5*n - 1) + a*e^6*(n - 1))*integrate(1/(c^3*d^8*n + 3*a*c^2*d^6
*e^2*n + 3*a^2*c*d^4*e^4*n + a^3*d^2*e^6*n + (c^3*d^7*e*n + 3*a*c^2*d^5*e^
3*n + 3*a^2*c*d^3*e^5*n + a^3*d*e^7*n)*x^n), x) - 1/2*(2*(c^2*d^2*e^2 - a*
c*e^4)*x*x^(2*n) + (c^2*d^3*e + a*c*d*e^3)*x*x^n - (c^2*d^4 - a*c*d^2*e^2
+ 2*a^2*e^4)*x)/(a^2*c^2*d^6*n + 2*a^3*c*d^4*e^2*n + a^4*d^2*e^4*n + (a*c^
3*d^5*e*n + 2*a^2*c^2*d^3*e^3*n + a^3*c*d*e^5*n)*x^(3*n) + (a*c^3*d^6*n +
2*a^2*c^2*d^4*e^2*n + a^3*c*d^2*e^4*n)*x^(2*n) + (a^2*c^2*d^5*e*n + 2*a^3*
c*d^3*e^3*n + a^4*d*e^5*n)*x^n) - integrate(1/2*(a^2*c*e^4*(4*n - 1) - c^3
*d^4*(2*n - 1) - 6*a*c^2*d^2*e^2*n + 2*(a*c^2*d*e^3*(5*n - 1) + c^3*d^3*e*
(n - 1))*x^n)/(a^2*c^3*d^6*n + 3*a^3*c^2*d^4*e^2*n + 3*a^4*c*d^2*e^4*n + a
^5*e^6*n + (a*c^4*d^6*n + 3*a^2*c^3*d^4*e^2*n + 3*a^3*c^2*d^2*e^4*n + a^4*
c*e^6*n)*x^(2*n)), x)
```

3.52.8 Giac [F]

$$\int \frac{1}{(d+ex^n)^2 (a+cx^{2n})^2} dx = \int \frac{1}{(cx^{2n}+a)^2 (ex^n+d)^2} dx$$

input
```
integrate(1/(d+e*x^n)^2/(a+c*x^(2*n))^2,x, algorithm="giac")
```

output
```
integrate(1/((c*x^(2*n) + a)^2*(e*x^n + d)^2), x)
```

3.52.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^n)^2 (a+cx^{2n})^2} dx = \int \frac{1}{(a+c x^{2n})^2 (d+e x^n)^2} dx$$

input
```
int(1/((a + c*x^(2*n))^2*(d + e*x^n)^2),x)
```

output
```
int(1/((a + c*x^(2*n))^2*(d + e*x^n)^2), x)
```

3.53 $\int \frac{(d+ex^n)^3}{(a+cx^{2n})^3}\,dx$

3.53.1 Optimal result

Integrand size = 21, antiderivative size = 424

$$\begin{aligned}
&\int \frac{(d+ex^n)^3}{(a+cx^{2n})^3}\,dx \\
&= \frac{x(d(cd^2-3ae^2)+e(3cd^2-ae^2)\,x^n)}{4acn\,(a+cx^{2n})^2}+\frac{e^2x(3d+ex^n)}{2acn\,(a+cx^{2n})} \\
&\quad -\frac{x(d(cd^2-3ae^2)\,(1-4n)+e(3cd^2-ae^2)\,(1-3n)x^n)}{8a^2cn^2\,(a+cx^{2n})} \\
&\quad +\frac{d(cd^2-3ae^2)\,(1-4n)(1-2n)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{8a^3cn^2} \\
&\quad -\frac{3de^2(1-2n)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{2a^2cn} \\
&\quad +\frac{e(3cd^2-ae^2)\,(1-3n)(1-n)x^{1+n}\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{8a^3cn^2(1+n)} \\
&\quad -\frac{e^3(1-n)x^{1+n}\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{2a^2cn(1+n)}
\end{aligned}$$

output
```
1/4*x*(d*(-3*a*e^2+c*d^2)+e*(-a*e^2+3*c*d^2)*x^n)/a/c/n/(a+c*x^(2*n))^2+1/
2*e^2*x*(3*d+e*x^n)/a/c/n/(a+c*x^(2*n))-1/8*x*(d*(-3*a*e^2+c*d^2)*(1-4*n)+
e*(-a*e^2+3*c*d^2)*(1-3*n)*x^n)/a^2/c/n^2/(a+c*x^(2*n))+1/8*d*(-3*a*e^2+c*
d^2)*(1-4*n)*(1-2*n)*x*hypergeom([1, 1/2/n],[1+1/2/n],-c*x^(2*n)/a)/a^3/c/
n^2-3/2*d*e^2*(1-2*n)*x*hypergeom([1, 1/2/n],[1+1/2/n],-c*x^(2*n)/a)/a^2/c
/n+1/8*e*(-a*e^2+3*c*d^2)*(1-3*n)*(1-n)*x^(1+n)*hypergeom([1, 1/2*(1+n)/n]
,[3/2+1/2/n],-c*x^(2*n)/a)/a^3/c/n^2/(1+n)-1/2*e^3*(1-n)*x^(1+n)*hypergeom
([1, 1/2*(1+n)/n],[3/2+1/2/n],-c*x^(2*n)/a)/a^2/c/n/(1+n)
```

3.53.2 Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.44

$$\int \frac{(d+ex^n)^3}{(a+cx^{2n})^3}\,dx = \frac{x\left(3ade^2\,\text{Hypergeometric2F1}\left(2,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)+\frac{ae^3x^n\,\text{Hypergeometric2F1}\left(2,\frac{1+n}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{1+n}+d(cd^2\right.}{a^3}$$

input
```
Integrate[(d + e*x^n)^3/(a + c*x^(2*n))^3,x]
```

output
```
(x*(3*a*d*e^2*Hypergeometric2F1[2, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/
a)] + (a*e^3*x^n*Hypergeometric2F1[2, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*
x^(2*n))/a)])/(1 + n) + d*(c*d^2 - 3*a*e^2)*Hypergeometric2F1[3, 1/(2*n),
(2 + n^(-1))/2, -((c*x^(2*n))/a)] + (e*(3*c*d^2 - a*e^2)*x^n*Hypergeometri
c2F1[3, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(1 + n)))/(a^3*c
)
```

3.53.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1767, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^n)^3}{(a+cx^{2n})^3}\,dx$$

$$\Big\downarrow \text{1767}$$

$$\int \left(\frac{x^n(3cd^2e-ae^3)-3ade^2+cd^3}{c(a+cx^{2n})^3}+\frac{e^2(3d+ex^n)}{c(a+cx^{2n})^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{e(1-3n)(1-n)x^{n+1}(3cd^2-ae^2)\,\text{Hypergeometric2F1}\left(1,\frac{n+1}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{8a^3cn^2(n+1)}+\frac{d(1-4n)(1-2n)x(cd^2-3ae^2)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{8a^3cn^2}-\frac{x\left(e(1-3n)x^n(3cd^2-ae^2)+d(1-4n)(cd^2-3ae^2)\right)}{8a^2cn^2(a+cx^{2n})}-\frac{3de^2(1-2n)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{2a^2cn}-\frac{e^3(1-n)x^{n+1}\,\text{Hypergeometric2F1}\left(1,\frac{n+1}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{2a^2cn(n+1)}+\frac{x\left(ex^n(3cd^2-ae^2)+d(cd^2-3ae^2)\right)}{4acn(a+cx^{2n})^2}+\frac{e^2x(3d+ex^n)}{2acn(a+cx^{2n})}$$

input
```
Int[(d + e*x^n)^3/(a + c*x^(2*n))^3,x]
```

output
```
(x*(d*(c*d^2 - 3*a*e^2) + e*(3*c*d^2 - a*e^2)*x^n))/(4*a*c*n*(a + c*x^(2*n
))^2) + (e^2*x*(3*d + e*x^n))/(2*a*c*n*(a + c*x^(2*n))) - (x*(d*(c*d^2 - 3
*a*e^2)*(1 - 4*n) + e*(3*c*d^2 - a*e^2)*(1 - 3*n)*x^n))/(8*a^2*c*n^2*(a +
c*x^(2*n))) + (d*(c*d^2 - 3*a*e^2)*(1 - 4*n)*(1 - 2*n)*x*Hypergeometric2F1
[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/(8*a^3*c*n^2) - (3*d*e^2*(
1 - 2*n)*x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)]
)/(2*a^2*c*n) + (e*(3*c*d^2 - a*e^2)*(1 - 3*n)*(1 - n)*x^(1 + n)*Hypergeom
etric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(8*a^3*c*n^2
*(1 + n)) - (e^3*(1 - n)*x^(1 + n)*Hypergeometric2F1[1, (1 + n)/(2*n), (3
+ n^(-1))/2, -((c*x^(2*n))/a)])/(2*a^2*c*n*(1 + n))
```

3.53. $\int \frac{(d+ex^n)^3}{(a+cx^{2n})^3}\,dx$

3.53.3.1 Defintions of rubi rules used

rule 1767
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + c*x^(2*n))^p, x], x] /; FreeQ[{a
, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && ((Integ
ersQ[p, q] && !IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] && !IntegerQ[n])
)
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.53.4 Maple [F]

$$\int \frac{(d+e\,x^{n})^{3}}{(a+c\,x^{2n})^{3}}dx$$

input
```
int((d+e*x^n)^3/(a+c*x^(2*n))^3,x)
```

output
```
int((d+e*x^n)^3/(a+c*x^(2*n))^3,x)
```

3.53.5 Fricas [F]

$$\int \frac{(d+ex^n)^3}{(a+cx^{2n})^3}\,dx = \int \frac{(ex^n+d)^3}{(cx^{2\,n}+a)^3}\,dx$$

input
```
integrate((d+e*x^n)^3/(a+c*x^(2*n))^3,x, algorithm="fricas")
```

output
```
integral((e^3*x^(3*n) + 3*d*e^2*x^(2*n) + 3*d^2*e*x^n + d^3)/(c^3*x^(6*n)
+ 3*a*c^2*x^(4*n) + 3*a^2*c*x^(2*n) + a^3), x)
```

3.53.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^n)^3}{(a+cx^{2n})^3}\,dx = \text{Timed out}$$

input
```
integrate((d+e*x**n)**3/(a+c*x**(2*n))**3,x)
```

output
```
Timed out
```

3.53.7 Maxima [F]

$$\int \frac{(d+ex^n)^3}{(a+cx^{2n})^3}\,dx = \int \frac{(ex^n+d)^3}{(cx^{2n}+a)^3}\,dx$$

input
```
integrate((d+e*x^n)^3/(a+c*x^(2*n))^3,x, algorithm="maxima")
```

output
```
1/8*((3*c^2*d^2*e*(3*n - 1) + a*c*e^3*(n + 1))*x*x^(3*n) + (c^2*d^3*(4*n -
 1) + 3*a*c*d*e^2)*x*x^(2*n) + (3*a*c*d^2*e*(5*n - 1) - a^2*e^3*(n - 1))*x
*x^n + (a*c*d^3*(6*n - 1) - 3*a^2*d*e^2*(2*n - 1))*x)/(a^2*c^3*n^2*x^(4*n)
 + 2*a^3*c^2*n^2*x^(2*n) + a^4*c*n^2) + integrate(1/8*((8*n^2 - 6*n + 1)*c
*d^3 + 3*a*d*e^2*(2*n - 1) + (3*(3*n^2 - 4*n + 1)*c*d^2*e + (n^2 - 1)*a*e^
3)*x^n)/(a^2*c^2*n^2*x^(2*n) + a^3*c*n^2), x)
```

3.53.8 Giac [F]

$$\int \frac{(d+ex^n)^3}{(a+cx^{2n})^3}\,dx = \int \frac{(ex^n+d)^3}{(cx^{2n}+a)^3}\,dx$$

input
```
integrate((d+e*x^n)^3/(a+c*x^(2*n))^3,x, algorithm="giac")
```

output
```
integrate((e*x^n + d)^3/(c*x^(2*n) + a)^3, x)
```

3.53.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^n)^3}{(a+cx^{2n})^3}\,dx = \int \frac{(d+e\,x^n)^3}{(a+c\,x^{2\,n})^3}\,dx$$

input
```
int((d + e*x^n)^3/(a + c*x^(2*n))^3,x)
```

output
```
int((d + e*x^n)^3/(a + c*x^(2*n))^3, x)
```

3.54 $\int \frac{(d+ex^n)^2}{(a+cx^{2n})^3}\,dx$

3.54.1 Optimal result

Integrand size = 21, antiderivative size = 272

$$\begin{aligned}\int \frac{(d+ex^n)^2}{(a+cx^{2n})^3}\,dx &= \frac{x(cd^2-ae^2+2cdex^n)}{4acn\,(a+cx^{2n})^2} - \frac{x((cd^2-ae^2)\,(1-4n)+2cde(1-3n)x^n)}{8a^2cn^2\,(a+cx^{2n})} \\ &+ \frac{(cd^2-ae^2)\,(1-4n)(1-2n)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{8a^3cn^2} \\ &+ \frac{de(1-3n)(1-n)x^{1+n}\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{4a^3n^2(1+n)} \\ &+ \frac{e^2x\,\text{Hypergeometric2F1}\left(2,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a^2c}\end{aligned}$$

output
```
1/4*x*(c*d^2-a*e^2+2*c*d*e*x^n)/a/c/n/(a+c*x^(2*n))^2-1/8*x*((-a*e^2+c*d^2
)*(1-4*n)+2*c*d*e*(1-3*n)*x^n)/a^2/c/n^2/(a+c*x^(2*n))+1/8*(-a*e^2+c*d^2)*
(1-4*n)*(1-2*n)*x*hypergeom([1, 1/2/n],[1+1/2/n],-c*x^(2*n)/a)/a^3/c/n^2+1
/4*d*e*(1-3*n)*(1-n)*x^(1+n)*hypergeom([1, 1/2*(1+n)/n],[3/2+1/2/n],-c*x^(
2*n)/a)/a^3/n^2/(1+n)+e^2*x*hypergeom([2, 1/2/n],[1+1/2/n],-c*x^(2*n)/a)/a
^2/c
```

3.54.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.50

$$\int \frac{(d+ex^n)^2}{(a+cx^{2n})^3}\,dx = \frac{x\left(ae^2(1+n)\,\text{Hypergeometric2F1}\left(2,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)+(cd^2-ae^2)\,(1+n)\,\text{Hypergeometric2F1}\left(3\right.\right.}{a^3c(1+n)}$$

input
```
Integrate[(d + e*x^n)^2/(a + c*x^(2*n))^3,x]
```

output
```
(x*(a*e^2*(1 + n)*Hypergeometric2F1[2, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*
n))/a)] + (c*d^2 - a*e^2)*(1 + n)*Hypergeometric2F1[3, 1/(2*n), (2 + n^(-1
))/2, -((c*x^(2*n))/a)] + 2*c*d*e*x^n*Hypergeometric2F1[3, (1 + n)/(2*n),
(3 + n^(-1))/2, -((c*x^(2*n))/a)]))/(a^3*c*(1 + n))
```

3.54.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1767, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^n)^2}{(a+cx^{2n})^3}\,dx$$

$$\Big\downarrow \text{1767}$$

$$\int \left(\frac{-ae^2+cd^2+2cdex^n}{c\,(a+cx^{2n})^3}+\frac{e^2}{c\,(a+cx^{2n})^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(1-4n)(1-2n)x(cd^2-ae^2)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{8a^3cn^2}+\frac{de(1-3n)(1-n)x^{n+1}\,\text{Hypergeometric2F1}\left(1,\frac{n+1}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{4a^3n^2(n+1)}-\frac{x\left((1-4n)\left(cd^2-ae^2\right)+2cde(1-3n)x^n\right)}{8a^2cn^2\left(a+cx^{2n}\right)}+\frac{e^2x\,\text{Hypergeometric2F1}\left(2,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a^2c}+\frac{x\left(-ae^2+cd^2+2cdex^n\right)}{4acn\left(a+cx^{2n}\right)^2}$$

input
```
Int[(d + e*x^n)^2/(a + c*x^(2*n))^3,x]
```

output
```
(x*(c*d^2 - a*e^2 + 2*c*d*e*x^n))/(4*a*c*n*(a + c*x^(2*n))^2) - (x*((c*d^2
 - a*e^2)*(1 - 4*n) + 2*c*d*e*(1 - 3*n)*x^n))/(8*a^2*c*n^2*(a + c*x^(2*n))
) + ((c*d^2 - a*e^2)*(1 - 4*n)*(1 - 2*n)*x*Hypergeometric2F1[1, 1/(2*n), (
2 + n^(-1))/2, -((c*x^(2*n))/a)])/(8*a^3*c*n^2) + (d*e*(1 - 3*n)*(1 - n)*x
^(1 + n)*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))
/a)])/(4*a^3*n^2*(1 + n)) + (e^2*x*Hypergeometric2F1[2, 1/(2*n), (2 + n^(-
1))/2, -((c*x^(2*n))/a)])/(a^2*c)
```

3.54.3.1 Defintions of rubi rules used

rule 1767
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + c*x^(2*n))^p, x], x] /; FreeQ[{a
, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && ((Integ
ersQ[p, q] && !IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] && !IntegerQ[n])
)
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.54.4 Maple [F]

$$\int \frac{(d+e\,x^n)^2}{(a+c\,x^{2n})^3}dx$$

input
```
int((d+e*x^n)^2/(a+c*x^(2*n))^3,x)
```

output
```
int((d+e*x^n)^2/(a+c*x^(2*n))^3,x)
```

3.54.5 Fricas [F]

$$\int \frac{(d+ex^n)^2}{(a+cx^{2n})^3}\,dx = \int \frac{(ex^n+d)^2}{(cx^{2n}+a)^3}\,dx$$

input
```
integrate((d+e*x^n)^2/(a+c*x^(2*n))^3,x, algorithm="fricas")
```

output
```
integral((e^2*x^(2*n) + 2*d*e*x^n + d^2)/(c^3*x^(6*n) + 3*a*c^2*x^(4*n) +
3*a^2*c*x^(2*n) + a^3), x)
```

3.54.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^n)^2}{(a+cx^{2n})^3}\,dx = \text{Timed out}$$

input
```
integrate((d+e*x**n)**2/(a+c*x**(2*n))**3,x)
```

output
```
Timed out
```

3.54.7 Maxima [F]

$$\int \frac{(d+ex^n)^2}{(a+cx^{2n})^3}\,dx = \int \frac{(ex^n+d)^2}{(cx^{2n}+a)^3}\,dx$$

input
```
integrate((d+e*x^n)^2/(a+c*x^(2*n))^3,x, algorithm="maxima")
```

output
```
1/8*(2*c^2*d*e*(3*n - 1)*x*x^(3*n) + 2*a*c*d*e*(5*n - 1)*x*x^n + (c^2*d^2*
(4*n - 1) + a*c*e^2)*x*x^(2*n) + (a*c*d^2*(6*n - 1) - a^2*e^2*(2*n - 1))*x
)/(a^2*c^3*n^2*x^(4*n) + 2*a^3*c^2*n^2*x^(2*n) + a^4*c*n^2) + integrate(1/
8*(2*(3*n^2 - 4*n + 1)*c*d*e*x^n + (8*n^2 - 6*n + 1)*c*d^2 + a*e^2*(2*n -
1))/(a^2*c^2*n^2*x^(2*n) + a^3*c*n^2), x)
```

3.54.8 Giac [F]

$$\int \frac{(d+ex^n)^2}{(a+cx^{2n})^3}\,dx = \int \frac{(ex^n+d)^2}{(cx^{2n}+a)^3}\,dx$$

input
```
integrate((d+e*x^n)^2/(a+c*x^(2*n))^3,x, algorithm="giac")
```

output
```
integrate((e*x^n + d)^2/(c*x^(2*n) + a)^3, x)
```

3.54.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^n)^2}{(a+cx^{2n})^3}\,dx = \int \frac{(d+e\,x^n)^2}{(a+c\,x^{2\,n})^3}\,dx$$

input
```
int((d + e*x^n)^2/(a + c*x^(2*n))^3,x)
```

output
```
int((d + e*x^n)^2/(a + c*x^(2*n))^3, x)
```

3.55 $\int \frac{d+ex^n}{(a+cx^{2n})^3} dx$

3.55.1 Optimal result

Integrand size = 19, antiderivative size = 184

$$\int \frac{d+ex^n}{(a+cx^{2n})^3} dx = \frac{x(d+ex^n)}{4an(a+cx^{2n})^2} - \frac{x(d(1-4n)+e(1-3n)x^n)}{8a^2n^2(a+cx^{2n})} + \frac{d(1-4n)(1-2n)x \text{ Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{8a^3n^2} + \frac{e(1-3n)(1-n)x^{1+n} \text{ Hypergeometric2F1}\left(1,\frac{1+n}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{8a^3n^2(1+n)}$$

output
```
1/4*x*(d+e*x^n)/a/n/(a+c*x^(2*n))^2-1/8*x*(d*(1-4*n)+e*(1-3*n)*x^n)/a^2/n^
2/(a+c*x^(2*n))+1/8*d*(1-4*n)*(1-2*n)*x*hypergeom([1, 1/2/n],[1+1/2/n],-c*
x^(2*n)/a)/a^3/n^2+1/8*e*(1-3*n)*(1-n)*x^(1+n)*hypergeom([1, 1/2*(1+n)/n],
[3/2+1/2/n],-c*x^(2*n)/a)/a^3/n^2/(1+n)
```

3.55.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.45

$$\int \frac{d+ex^n}{(a+cx^{2n})^3} dx = \frac{dx \text{ Hypergeometric2F1}\left(3,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a^3} + \frac{ex^{1+n} \text{ Hypergeometric2F1}\left(3,\frac{1+n}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a^3(1+n)}$$

input
```
Integrate[(d + e*x^n)/(a + c*x^(2*n))^3,x]
```

output
```
(d*x*Hypergeometric2F1[3, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/a^3
+ (e*x^(1 + n)*Hypergeometric2F1[3, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^
(2*n))/a)])/(a^3*(1 + n))
```

3.55.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1761, 1761, 1748, 778, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^n}{\left(a+cx^{2n}\right)^3}dx$$

$$\Big\downarrow \text{1761}$$

$$\frac{x(d+ex^n)}{4an\left(a+cx^{2n}\right)^2}-\frac{\int \frac{e(1-3n)x^n+d(1-4n)}{\left(cx^{2n}+a\right)^2}dx}{4an}$$

$$\Big\downarrow \text{1761}$$

$$\frac{x(d+ex^n)}{4an\left(a+cx^{2n}\right)^2}-\frac{\frac{x(d(1-4n)+e(1-3n)x^n)}{2an(a+cx^{2n})}-\frac{\int \frac{e(1-3n)(1-n)x^n+d\left(8n^2-6n+1\right)}{cx^{2n}+a}dx}{2an}}{4an}$$

$$\Big\downarrow \text{1748}$$

$$\frac{x(d+ex^n)}{4an\left(a+cx^{2n}\right)^2}-\frac{\frac{x(d(1-4n)+e(1-3n)x^n)}{2an(a+cx^{2n})}-\frac{d\left(8n^2-6n+1\right)\int \frac{1}{cx^{2n}+a}dx+e(1-3n)(1-n)\int \frac{x^n}{cx^{2n}+a}dx}{2an}}{4an}$$

$$\Big\downarrow \text{778}$$

$$\frac{x(d+ex^n)}{4an\left(a+cx^{2n}\right)^2}-\frac{\frac{x(d(1-4n)+e(1-3n)x^n)}{2an(a+cx^{2n})}-\frac{e(1-3n)(1-n)\int \frac{x^n}{cx^{2n}+a}dx+\frac{d\left(8n^2-6n+1\right)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a}}{2an}}{4an}$$

$$\Big\downarrow \text{888}$$

$$\frac{x(d+ex^n)}{4an\left(a+cx^{2n}\right)^2}-\frac{\frac{x(d(1-4n)+e(1-3n)x^n)}{2an(a+cx^{2n})}-\frac{\frac{d\left(8n^2-6n+1\right)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a}+\frac{e(1-3n)(1-n)x^{n+1}\,\text{Hypergeometric2F1}\left(1,\frac{n+1}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a(n+1)}}{2an}}{4an}$$

input
```
Int[(d + e*x^n)/(a + c*x^(2*n))^3,x]
```

output
```
(x*(d + e*x^n))/(4*a*n*(a + c*x^(2*n))^2) - ((x*(d*(1 - 4*n) + e*(1 - 3*n)
*x^n))/(2*a*n*(a + c*x^(2*n))) - ((d*(1 - 6*n + 8*n^2)*x*Hypergeometric2F1
[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/a + (e*(1 - 3*n)*(1 - n)*x
^(1 + n)*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))
/a)])/(a*(1 + n)))/(2*a*n))/(4*a*n)
```

3.55.3.1 Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1748
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> Simp[d
   Int[1/(a + c*x^(2*n)), x], x] + Simp[e   Int[x^n/(a + c*x^(2*n)), x], x]
 /; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && (
PosQ[a*c] ||  !IntegerQ[n])
```

rule 1761
```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> S
imp[(-x)*(d + e*x^n)*((a + c*x^(2*n))^(p + 1)/(2*a*n*(p + 1))), x] + Simp[1
/(2*a*n*(p + 1))   Int[(d*(2*n*p + 2*n + 1) + e*(2*n*p + 3*n + 1)*x^n)*(a +
 c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] &&
 ILtQ[p, -1]
```

3.55.4 Maple [F]

$$\int \frac{d+e\,x^{n}}{\left(a+c\,x^{2n}\right)^{3}}dx$$

input
```
int((d+e*x^n)/(a+c*x^(2*n))^3,x)
```

output
```
int((d+e*x^n)/(a+c*x^(2*n))^3,x)
```

3.55.5 Fricas [F]

$$\int \frac{d+ex^n}{(a+cx^{2n})^3}\,dx = \int \frac{ex^n+d}{(cx^{2\,n}+a)^3}\,dx$$

input
```
integrate((d+e*x^n)/(a+c*x^(2*n))^3,x, algorithm="fricas")
```

output
```
integral((e*x^n + d)/(c^3*x^(6*n) + 3*a*c^2*x^(4*n) + 3*a^2*c*x^(2*n) + a^
3), x)
```

3.55.6 Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^n}{(a+cx^{2n})^3}\,dx = \text{Timed out}$$

input
```
integrate((d+e*x**n)/(a+c*x**(2*n))**3,x)
```

output
```
Timed out
```

3.55.7 Maxima [F]

$$\int \frac{d+ex^n}{(a+cx^{2n})^3}\,dx = \int \frac{ex^n+d}{(cx^{2n}+a)^3}\,dx$$

input
```
integrate((d+e*x^n)/(a+c*x^(2*n))^3,x, algorithm="maxima")
```

output
```
1/8*(c*e*(3*n - 1)*x*x^(3*n) + c*d*(4*n - 1)*x*x^(2*n) + a*e*(5*n - 1)*x*x
^n + a*d*(6*n - 1)*x)/(a^2*c^2*n^2*x^(4*n) + 2*a^3*c*n^2*x^(2*n) + a^4*n^2
) + integrate(1/8*((3*n^2 - 4*n + 1)*e*x^n + (8*n^2 - 6*n + 1)*d)/(a^2*c*n
^2*x^(2*n) + a^3*n^2), x)
```

3.55.8 Giac [F]

$$\int \frac{d+ex^n}{(a+cx^{2n})^3}\,dx = \int \frac{ex^n+d}{(cx^{2n}+a)^3}\,dx$$

input
```
integrate((d+e*x^n)/(a+c*x^(2*n))^3,x, algorithm="giac")
```

output
```
integrate((e*x^n + d)/(c*x^(2*n) + a)^3, x)
```

3.55.9 Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^n}{(a+cx^{2n})^3}\,dx = \int \frac{d+e\,x^n}{(a+c\,x^{2n})^3}\,dx$$

input
```
int((d + e*x^n)/(a + c*x^(2*n))^3,x)
```

output
```
int((d + e*x^n)/(a + c*x^(2*n))^3, x)
```

3.56 $\int \frac{1}{(d+ex^n)(a+cx^{2n})^3}\,dx$

3.56.1 Optimal result

Integrand size = 21, antiderivative size = 582

$$\begin{aligned}\int \frac{1}{(d+ex^n)(a+cx^{2n})^3}\,dx &= \frac{cx(d-ex^n)}{4a(cd^2+ae^2)n(a+cx^{2n})^2}+\frac{ce^2x(d-ex^n)}{2a(cd^2+ae^2)^2n(a+cx^{2n})}\\
&-\frac{cx(d(1-4n)-e(1-3n)x^n)}{8a^2(cd^2+ae^2)n^2(a+cx^{2n})}+\frac{cde^4x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a(cd^2+ae^2)^3}\\
&+\frac{cd(1-4n)(1-2n)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{8a^3(cd^2+ae^2)n^2}\\
&-\frac{cde^2(1-2n)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{2a^2(cd^2+ae^2)^2n}\\
&+\frac{e^6x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)}{d(cd^2+ae^2)^3}\\
&-\frac{ce^5x^{1+n}\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a(cd^2+ae^2)^3(1+n)}\\
&-\frac{ce(1-3n)(1-n)x^{1+n}\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{8a^3(cd^2+ae^2)n^2(1+n)}\\
&+\frac{ce^3(1-n)x^{1+n}\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{2a^2(cd^2+ae^2)^2n(1+n)}\end{aligned}$$

output
```
1/4*c*x*(d-e*x^n)/a/(a*e^2+c*d^2)/n/(a+c*x^(2*n))^2+1/2*c*e^2*x*(d-e*x^n)/
a/(a*e^2+c*d^2)^2/n/(a+c*x^(2*n))-1/8*c*x*(d*(1-4*n)-e*(1-3*n)*x^n)/a^2/(a
*e^2+c*d^2)/n^2/(a+c*x^(2*n))+c*d*e^4*x*hypergeom([1, 1/2/n],[1+1/2/n],-c*
x^(2*n)/a)/a/(a*e^2+c*d^2)^3+1/8*c*d*(1-4*n)*(1-2*n)*x*hypergeom([1, 1/2/n
],[1+1/2/n],-c*x^(2*n)/a)/a^3/(a*e^2+c*d^2)/n^2-1/2*c*d*e^2*(1-2*n)*x*hype
rgeom([1, 1/2/n],[1+1/2/n],-c*x^(2*n)/a)/a^2/(a*e^2+c*d^2)^2/n+e^6*x*hyper
geom([1, 1/n],[1+1/n],-e*x^n/d)/d/(a*e^2+c*d^2)^3-c*e^5*x^(1+n)*hypergeom(
[1, 1/2*(1+n)/n],[3/2+1/2/n],-c*x^(2*n)/a)/a/(a*e^2+c*d^2)^3/(1+n)-1/8*c*e
*(1-3*n)*(1-n)*x^(1+n)*hypergeom([1, 1/2*(1+n)/n],[3/2+1/2/n],-c*x^(2*n)/a
)/a^3/(a*e^2+c*d^2)/n^2/(1+n)+1/2*c*e^3*(1-n)*x^(1+n)*hypergeom([1, 1/2*(1
+n)/n],[3/2+1/2/n],-c*x^(2*n)/a)/a^2/(a*e^2+c*d^2)^2/n/(1+n)
```

3.56.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 346, normalized size of antiderivative = 0.59

$$\int \frac{1}{(d+ex^n)\left(a+cx^{2n}\right)^3}\,dx$$

$$= \frac{x\left(\frac{cde^4\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a}+\frac{e^6\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)}{d}-\frac{ce^5x^n\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2n},\frac{1}{2}(3\cdots\right.}{a(1+n)}\right.}{}$$

input
```
Integrate[1/((d + e*x^n)*(a + c*x^(2*n))^3),x]
```

output
```
(x*((c*d*e^4*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a
)])/a + (e^6*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((e*x^n)/d)])/d - (
c*e^5*x^n*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n)
)/a)])/(a*(1 + n)) + (c*d*e^2*(c*d^2 + a*e^2)*Hypergeometric2F1[2, 1/(2*n)
, (2 + n^(-1))/2, -((c*x^(2*n))/a)])/a^2 - (c*e^3*(c*d^2 + a*e^2)*x^n*Hype
rgeometric2F1[2, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a^2*(1
 + n)) + (c*d*(c*d^2 + a*e^2)^2*Hypergeometric2F1[3, 1/(2*n), (2 + n^(-1))
/2, -((c*x^(2*n))/a)])/a^3 - (c*e*(c*d^2 + a*e^2)^2*x^n*Hypergeometric2F1[
3, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a^3*(1 + n))))/(c*d^
2 + a*e^2)^3
```

3.56.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 582, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1767, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^{2n})^3(d+ex^n)}dx$$

$$\Big\downarrow \text{1767}$$

$$\int \left(-\frac{ce^2(ex^n-d)}{(ae^2+cd^2)^2(a+cx^{2n})^2}-\frac{c(ex^n-d)}{(ae^2+cd^2)(a+cx^{2n})^3}+\frac{e^6}{(ae^2+cd^2)^3(d+ex^n)}-\frac{ce^4(ex^n-d)}{(ae^2+cd^2)^3(a+cx^{2n})}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ce(1-3n)(1-n)x^{n+1}\text{Hypergeometric2F1}\left(1,\frac{n+1}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{8a^3n^2(n+1)(ae^2+cd^2)}+\frac{cd(1-4n)(1-2n)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{8a^3n^2(ae^2+cd^2)}-\frac{cde^2(1-2n)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{2a^2n(ae^2+cd^2)^2}-\frac{cx(d(1-4n)-e(1-3n)x^n)}{8a^2n^2(ae^2+cd^2)(a+cx^{2n})}+\frac{ce^3(1-n)x^{n+1}\text{Hypergeometric2F1}\left(1,\frac{n+1}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{2a^2n(n+1)(ae^2+cd^2)^2}+\frac{ce^2x(d-ex^n)}{2an(ae^2+cd^2)^2(a+cx^{2n})}+\frac{cx(d-ex^n)}{4an(ae^2+cd^2)(a+cx^{2n})^2}+\frac{e^6x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)}{d(ae^2+cd^2)^3}-\frac{ce^5x^{n+1}\text{Hypergeometric2F1}\left(1,\frac{n+1}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a(n+1)(ae^2+cd^2)^3}+\frac{cde^4x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a(ae^2+cd^2)^3}$$

input
```
Int[1/((d + e*x^n)*(a + c*x^(2*n))^3),x]
```

output
```
(c*x*(d - e*x^n))/(4*a*(c*d^2 + a*e^2)*n*(a + c*x^(2*n))^2) + (c*e^2*x*(d
- e*x^n))/(2*a*(c*d^2 + a*e^2)^2*n*(a + c*x^(2*n))) - (c*x*(d*(1 - 4*n) -
e*(1 - 3*n)*x^n))/(8*a^2*(c*d^2 + a*e^2)*n^2*(a + c*x^(2*n))) + (c*d*e^4*x
*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/(a*(c*d^
2 + a*e^2)^3) + (c*d*(1 - 4*n)*(1 - 2*n)*x*Hypergeometric2F1[1, 1/(2*n), (
2 + n^(-1))/2, -((c*x^(2*n))/a)])/(8*a^3*(c*d^2 + a*e^2)*n^2) - (c*d*e^2*(
1 - 2*n)*x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)]
)/(2*a^2*(c*d^2 + a*e^2)^2*n) + (e^6*x*Hypergeometric2F1[1, n^(-1), 1 + n^
(-1), -((e*x^n)/d)])/(d*(c*d^2 + a*e^2)^3) - (c*e^5*x^(1 + n)*Hypergeometr
ic2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a*(c*d^2 + a*e
^2)^3*(1 + n)) - (c*e*(1 - 3*n)*(1 - n)*x^(1 + n)*Hypergeometric2F1[1, (1
+ n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(8*a^3*(c*d^2 + a*e^2)*n^2*
(1 + n)) + (c*e^3*(1 - n)*x^(1 + n)*Hypergeometric2F1[1, (1 + n)/(2*n), (3
 + n^(-1))/2, -((c*x^(2*n))/a)])/(2*a^2*(c*d^2 + a*e^2)^2*n*(1 + n))
```

3.56.3.1 Defintions of rubi rules used

rule 1767
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + c*x^(2*n))^p, x], x] /; FreeQ[{a
, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && ((Integ
ersQ[p, q] && !IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] && !IntegerQ[n])
)
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.56.4 Maple [F]

$$\int \frac{1}{(d + e\, x^n)\,(a + c\, x^{2n})^3}\, dx$$

input
```
int(1/(d+e*x^n)/(a+c*x^(2*n))^3,x)
```

output
```
int(1/(d+e*x^n)/(a+c*x^(2*n))^3,x)
```

3.56.5 Fricas [F]

$$\int \frac{1}{(d+ex^n)\,(a+cx^{2n})^3}\,dx = \int \frac{1}{(cx^{2n}+a)^3(ex^n+d)}\,dx$$

input
```
integrate(1/(d+e*x^n)/(a+c*x^(2*n))^3,x, algorithm="fricas")
```

output
```
integral(1/(a^3*e*x^n + a^3*d + (c^3*e*x^n + c^3*d)*x^(6*n) + 3*(a*c^2*e*x
^n + a*c^2*d)*x^(4*n) + 3*(a^2*c*e*x^n + a^2*c*d)*x^(2*n)), x)
```

3.56.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^n)\,(a+cx^{2n})^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(d+e*x**n)/(a+c*x**(2*n))**3,x)
```

output
```
Timed out
```

3.56.7 Maxima [F]

$$\int \frac{1}{(d+ex^n)\,(a+cx^{2n})^3}\,dx = \int \frac{1}{(cx^{2n}+a)^3(ex^n+d)}\,dx$$

input
```
integrate(1/(d+e*x^n)/(a+c*x^(2*n))^3,x, algorithm="maxima")
```

output
```
e^6*integrate(1/(c^3*d^7 + 3*a*c^2*d^5*e^2 + 3*a^2*c*d^3*e^4 + a^3*d*e^6 +
 (c^3*d^6*e + 3*a*c^2*d^4*e^3 + 3*a^2*c*d^2*e^5 + a^3*e^7)*x^n), x) - 1/8*
((a*c^2*e^3*(7*n - 1) + c^3*d^2*e*(3*n - 1))*x*x^(3*n) - (a*c^2*d*e^2*(8*n
 - 1) + c^3*d^3*(4*n - 1))*x*x^(2*n) + (a^2*c*e^3*(9*n - 1) + a*c^2*d^2*e*
(5*n - 1))*x*x^n - (a^2*c*d*e^2*(10*n - 1) + a*c^2*d^3*(6*n - 1))*x)/(a^4*
c^2*d^4*n^2 + 2*a^5*c*d^2*e^2*n^2 + a^6*e^4*n^2 + (a^2*c^4*d^4*n^2 + 2*a^3
*c^3*d^2*e^2*n^2 + a^4*c^2*e^4*n^2)*x^(4*n) + 2*(a^3*c^3*d^4*n^2 + 2*a^4*c
^2*d^2*e^2*n^2 + a^5*c*e^4*n^2)*x^(2*n)) - integrate(-1/8*((8*n^2 - 6*n +
1)*c^3*d^5 + 2*(12*n^2 - 8*n + 1)*a*c^2*d^3*e^2 + (24*n^2 - 10*n + 1)*a^2*
c*d*e^4 - ((3*n^2 - 4*n + 1)*c^3*d^4*e + 2*(5*n^2 - 6*n + 1)*a*c^2*d^2*e^3
 + (15*n^2 - 8*n + 1)*a^2*c*e^5)*x^n)/(a^3*c^3*d^6*n^2 + 3*a^4*c^2*d^4*e^2
*n^2 + 3*a^5*c*d^2*e^4*n^2 + a^6*e^6*n^2 + (a^2*c^4*d^6*n^2 + 3*a^3*c^3*d^
4*e^2*n^2 + 3*a^4*c^2*d^2*e^4*n^2 + a^5*c*e^6*n^2)*x^(2*n)), x)
```

3.56.8 Giac [F]

$$\int \frac{1}{(d+ex^n)(a+cx^{2n})^3}\,dx = \int \frac{1}{(cx^{2n}+a)^3(ex^n+d)}\,dx$$

input
```
integrate(1/(d+e*x^n)/(a+c*x^(2*n))^3,x, algorithm="giac")
```

output
```
integrate(1/((c*x^(2*n) + a)^3*(e*x^n + d)), x)
```

3.56.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^n)(a+cx^{2n})^3}\,dx = \int \frac{1}{(a+c\,x^{2\,n})^3\,(d+e\,x^n)}\,dx$$

input
```
int(1/((a + c*x^(2*n))^3*(d + e*x^n)),x)
```

output
```
int(1/((a + c*x^(2*n))^3*(d + e*x^n)), x)
```

3.57 $\int \frac{1}{(d+ex^n)^2 (a+cx^{2n})^3} \, dx$

3.57.1 Optimal result

Integrand size = 21, antiderivative size = 701

$$\begin{aligned}
&\int \frac{1}{(d+ex^n)^2\,(a+cx^{2n})^3}\,dx \\
&= \frac{cx(cd^2-ae^2-2cdex^n)}{4a\,(cd^2+ae^2)^2\,n\,(a+cx^{2n})^2}+\frac{ce^2x(3cd^2-ae^2-4cdex^n)}{2a\,(cd^2+ae^2)^3\,n\,(a+cx^{2n})} \\
&\quad-\frac{cx((cd^2-ae^2)\,(1-4n)-2cde(1-3n)x^n)}{8a^2\,(cd^2+ae^2)^2\,n^2\,(a+cx^{2n})} \\
&\quad+\frac{ce^4(5cd^2-ae^2)\,x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a\,(cd^2+ae^2)^4} \\
&\quad+\frac{c(cd^2-ae^2)\,(1-4n)(1-2n)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{8a^3\,(cd^2+ae^2)^2\,n^2} \\
&\quad-\frac{ce^2(3cd^2-ae^2)\,(1-2n)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{2a^2\,(cd^2+ae^2)^3\,n} \\
&\quad+\frac{6ce^6x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)}{(cd^2+ae^2)^4} \\
&\quad-\frac{6c^2de^5x^{1+n}\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a\,(cd^2+ae^2)^4\,(1+n)} \\
&\quad-\frac{c^2de(1-3n)(1-n)x^{1+n}\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{4a^3\,(cd^2+ae^2)^2\,n^2(1+n)} \\
&\quad+\frac{2c^2de^3(1-n)x^{1+n}\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a^2\,(cd^2+ae^2)^3\,n(1+n)} \\
&\quad+\frac{e^6x\,\text{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)}{d^2\,(cd^2+ae^2)^3}
\end{aligned}$$

output
```
1/4*c*x*(c*d^2-a*e^2-2*c*d*e*x^n)/a/(a*e^2+c*d^2)^2/n/(a+c*x^(2*n))^2+1/2*
c*e^2*x*(3*c*d^2-a*e^2-4*c*d*e*x^n)/a/(a*e^2+c*d^2)^3/n/(a+c*x^(2*n))-1/8*
c*x*((-a*e^2+c*d^2)*(1-4*n)-2*c*d*e*(1-3*n)*x^n)/a^2/(a*e^2+c*d^2)^2/n^2/(
a+c*x^(2*n))+c*e^4*(-a*e^2+5*c*d^2)*x*hypergeom([1, 1/2/n],[1+1/2/n],-c*x^
(2*n)/a)/a/(a*e^2+c*d^2)^4+1/8*c*(-a*e^2+c*d^2)*(1-4*n)*(1-2*n)*x*hypergeo
m([1, 1/2/n],[1+1/2/n],-c*x^(2*n)/a)/a^3/(a*e^2+c*d^2)^2/n^2-1/2*c*e^2*(-a
*e^2+3*c*d^2)*(1-2*n)*x*hypergeom([1, 1/2/n],[1+1/2/n],-c*x^(2*n)/a)/a^2/(
a*e^2+c*d^2)^3/n+6*c*e^6*x*hypergeom([1, 1/n],[1+1/n],-e*x^n/d)/(a*e^2+c*d
^2)^4-6*c^2*d*e^5*x^(1+n)*hypergeom([1, 1/2*(1+n)/n],[3/2+1/2/n],-c*x^(2*n
)/a)/a/(a*e^2+c*d^2)^4/(1+n)-1/4*c^2*d*e*(1-3*n)*(1-n)*x^(1+n)*hypergeom([
1, 1/2*(1+n)/n],[3/2+1/2/n],-c*x^(2*n)/a)/a^3/(a*e^2+c*d^2)^2/n^2/(1+n)+2*
c^2*d*e^3*(1-n)*x^(1+n)*hypergeom([1, 1/2*(1+n)/n],[3/2+1/2/n],-c*x^(2*n)/
a)/a^2/(a*e^2+c*d^2)^3/n/(1+n)+e^6*x*hypergeom([2, 1/n],[1+1/n],-e*x^n/d)/
d^2/(a*e^2+c*d^2)^3
```

3.57.2 Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 426, normalized size of antiderivative = 0.61

$$\int \frac{1}{(d+ex^n)^2 (a+cx^{2n})^3} \, dx = \frac{x\left(\frac{ce^4(5cd^2-ae^2)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a} + 6ce^6\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right) - \frac{6c^2de^5x^n}{\cdots}\right.}{\cdots}$$

input
```
Integrate[1/((d + e*x^n)^2*(a + c*x^(2*n))^3),x]
```

output
```
(x*((c*e^4*(5*c*d^2 - a*e^2)*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2,
 -((c*x^(2*n))/a)])/a + 6*c*e^6*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -
((e*x^n)/d)] - (6*c^2*d*e^5*x^n*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n
^(-1))/2, -((c*x^(2*n))/a)])/(a*(1 + n)) + (c*e^2*(3*c*d^2 - a*e^2)*(c*d^2
 + a*e^2)*Hypergeometric2F1[2, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])
/a^2 + (e^6*(c*d^2 + a*e^2)*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((e*
x^n)/d)])/d^2 - (4*c^2*d*e^3*(c*d^2 + a*e^2)*x^n*Hypergeometric2F1[2, (1 +
 n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a^2*(1 + n)) + (c*(c*d^2 -
a*e^2)*(c*d^2 + a*e^2)^2*Hypergeometric2F1[3, 1/(2*n), (2 + n^(-1))/2, -((
c*x^(2*n))/a)])/a^3 - (2*c^2*d*e*(c*d^2 + a*e^2)^2*x^n*Hypergeometric2F1[3
, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a^3*(1 + n))))/(c*d^2
 + a*e^2)^4
```

3.57.3 Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 701, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1767, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^{2n})^3 (d+ex^n)^2}dx$$

$$\Big\downarrow \text{1767}$$

$$\int \left(-\frac{ce^2 (ae^2-3cd^2+4cdex^n)}{(ae^2+cd^2)^3 (a+cx^{2n})^2}-\frac{c(ae^2-cd^2+2cdex^n)}{(ae^2+cd^2)^2 (a+cx^{2n})^3}+\frac{6cde^6}{(ae^2+cd^2)^4 (d+ex^n)}+\frac{e^6}{(ae^2+cd^2)^3 (d+ex^n)^2}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c^2de(1-3n)(1-n)x^{n+1}\text{Hypergeometric2F1}\left(1,\frac{n+1}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{4a^3n^2(n+1)(ae^2+cd^2)^2}+\frac{c(1-4n)(1-2n)x(cd^2-ae^2)\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{8a^3n^2(ae^2+cd^2)^2}+\frac{2c^2de^3(1-n)x^{n+1}\text{Hypergeometric2F1}\left(1,\frac{n+1}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a^2n(n+1)(ae^2+cd^2)^3}-\frac{ce^2(1-2n)x(3cd^2-ae^2)\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{2a^2n(ae^2+cd^2)^3}-\frac{cx((1-4n)(cd^2-ae^2)-2cde(1-3n)x^n)}{8a^2n^2(ae^2+cd^2)^2(a+cx^{2n})}-\frac{6c^2de^5x^{n+1}\text{Hypergeometric2F1}\left(1,\frac{n+1}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a(n+1)(ae^2+cd^2)^4}+\frac{ce^2x(-ae^2+3cd^2-4cdex^n)}{2an(ae^2+cd^2)^3(a+cx^{2n})}+\frac{cx(-ae^2+cd^2-2cdex^n)}{4an(ae^2+cd^2)^2(a+cx^{2n})^2}+\frac{6ce^6x\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)}{(ae^2+cd^2)^4}+\frac{e^6x\text{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)}{d^2(ae^2+cd^2)^3}+\frac{ce^4x(5cd^2-ae^2)\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{a(ae^2+cd^2)^4}$$

input
```
Int[1/((d + e*x^n)^2*(a + c*x^(2*n))^3),x]
```

output
```
(c*x*(c*d^2 - a*e^2 - 2*c*d*e*x^n))/(4*a*(c*d^2 + a*e^2)^2*n*(a + c*x^(2*n
))^2) + (c*e^2*x*(3*c*d^2 - a*e^2 - 4*c*d*e*x^n))/(2*a*(c*d^2 + a*e^2)^3*n
*(a + c*x^(2*n))) - (c*x*((c*d^2 - a*e^2)*(1 - 4*n) - 2*c*d*e*(1 - 3*n)*x^
n))/(8*a^2*(c*d^2 + a*e^2)^2*n^2*(a + c*x^(2*n))) + (c*e^4*(5*c*d^2 - a*e^
2)*x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/(a*(
c*d^2 + a*e^2)^4) + (c*(c*d^2 - a*e^2)*(1 - 4*n)*(1 - 2*n)*x*Hypergeometri
c2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/(8*a^3*(c*d^2 + a*e^2)
^2*n^2) - (c*e^2*(3*c*d^2 - a*e^2)*(1 - 2*n)*x*Hypergeometric2F1[1, 1/(2*n
), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/(2*a^2*(c*d^2 + a*e^2)^3*n) + (6*c*e
^6*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((e*x^n)/d)])/(c*d^2 + a*e^
2)^4 - (6*c^2*d*e^5*x^(1 + n)*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(
-1))/2, -((c*x^(2*n))/a)])/(a*(c*d^2 + a*e^2)^4*(1 + n)) - (c^2*d*e*(1 - 3
*n)*(1 - n)*x^(1 + n)*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2,
-((c*x^(2*n))/a)])/(4*a^3*(c*d^2 + a*e^2)^2*n^2*(1 + n)) + (2*c^2*d*e^3*(1
 - n)*x^(1 + n)*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x
^(2*n))/a)])/(a^2*(c*d^2 + a*e^2)^3*n*(1 + n)) + (e^6*x*Hypergeometric2F1[
2, n^(-1), 1 + n^(-1), -((e*x^n)/d)])/(d^2*(c*d^2 + a*e^2)^3)
```

3.57.3.1 Defintions of rubi rules used

rule 1767
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + c*x^(2*n))^p, x], x] /; FreeQ[{a
, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && ((Integ
ersQ[p, q] &&  !IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] &&  !IntegerQ[n])
)
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.57.4 Maple [F]

$$\int \frac{1}{(d+e\,x^n)^2\,(a+c\,x^{2n})^3}dx$$

input
```
int(1/(d+e*x^n)^2/(a+c*x^(2*n))^3,x)
```

output
```
int(1/(d+e*x^n)^2/(a+c*x^(2*n))^3,x)
```

3.57.5 Fricas [F]

$$\int \frac{1}{\left(d+ex^n\right)^2\left(a+cx^{2n}\right)^3}\,dx=\int \frac{1}{\left(cx^{2n}+a\right)^3\left(ex^n+d\right)^2}\,dx$$

input
```
integrate(1/(d+e*x^n)^2/(a+c*x^(2*n))^3,x, algorithm="fricas")
```

output
```
integral(1/(a^3*e^2*x^(2*n) + 2*a^3*d*e*x^n + a^3*d^2 + (c^3*e^2*x^(2*n) +
 2*c^3*d*e*x^n + c^3*d^2)*x^(6*n) + 3*(a*c^2*e^2*x^(2*n) + 2*a*c^2*d*e*x^n
 + a*c^2*d^2)*x^(4*n) + 3*(a^2*c*e^2*x^(2*n) + 2*a^2*c*d*e*x^n + a^2*c*d^2
)*x^(2*n)), x)
```

3.57.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+ex^n\right)^2\left(a+cx^{2n}\right)^3}\,dx=\text{Timed out}$$

input
```
integrate(1/(d+e*x**n)**2/(a+c*x**(2*n))**3,x)
```

output
```
Timed out
```

3.57.7 Maxima [F]

$$\int \frac{1}{\left(d+ex^n\right)^2\left(a+cx^{2n}\right)^3}\,dx=\int \frac{1}{\left(cx^{2n}+a\right)^3\left(ex^n+d\right)^2}\,dx$$

input
```
integrate(1/(d+e*x^n)^2/(a+c*x^(2*n))^3,x, algorithm="maxima")
```

output
```
(c*d^2*e^6*(7*n - 1) + a*e^8*(n - 1))*integrate(1/(c^4*d^10*n + 4*a*c^3*d^
8*e^2*n + 6*a^2*c^2*d^6*e^4*n + 4*a^3*c*d^4*e^6*n + a^4*d^2*e^8*n + (c^4*d
^9*e*n + 4*a*c^3*d^7*e^3*n + 6*a^2*c^2*d^5*e^5*n + 4*a^3*c*d^3*e^7*n + a^4
*d*e^9*n)*x^n), x) - 1/8*(2*(a*c^3*d^2*e^4*(11*n - 1) + c^4*d^4*e^2*(3*n -
 1) - 4*a^2*c^2*e^6*n)*x*x^(4*n) + (a^2*c^2*d*e^5*(8*n - 1) + 2*a*c^3*d^3*
e^3*(5*n - 1) + c^4*d^5*e*(2*n - 1))*x*x^(3*n) + (a^2*c^2*d^2*e^4*(34*n -
3) - c^4*d^6*(4*n - 1) - 2*a*c^3*d^4*e^2*(n + 1) - 16*a^3*c*e^6*n)*x*x^(2*
n) + (a^3*c*d*e^5*(10*n - 1) + 2*a^2*c^2*d^3*e^3*(7*n - 1) + a*c^3*d^5*e*(
4*n - 1))*x*x^n + (a^3*c*d^2*e^4*(10*n - 1) - a*c^3*d^6*(6*n - 1) - 12*a^2
*c^2*d^4*e^2*n - 8*a^4*e^6*n)*x)/(a^4*c^3*d^8*n^2 + 3*a^5*c^2*d^6*e^2*n^2
+ 3*a^6*c*d^4*e^4*n^2 + a^7*d^2*e^6*n^2 + (a^2*c^5*d^7*e*n^2 + 3*a^3*c^4*d
^5*e^3*n^2 + 3*a^4*c^3*d^3*e^5*n^2 + a^5*c^2*d*e^7*n^2)*x^(5*n) + (a^2*c^5
*d^8*n^2 + 3*a^3*c^4*d^6*e^2*n^2 + 3*a^4*c^3*d^4*e^4*n^2 + a^5*c^2*d^2*e^6
*n^2)*x^(4*n) + 2*(a^3*c^4*d^7*e*n^2 + 3*a^4*c^3*d^5*e^3*n^2 + 3*a^5*c^2*d
^3*e^5*n^2 + a^6*c*d*e^7*n^2)*x^(3*n) + 2*(a^3*c^4*d^8*n^2 + 3*a^4*c^3*d^6
*e^2*n^2 + 3*a^5*c^2*d^4*e^4*n^2 + a^6*c*d^2*e^6*n^2)*x^(2*n) + (a^4*c^3*d
^7*e*n^2 + 3*a^5*c^2*d^5*e^3*n^2 + 3*a^6*c*d^3*e^5*n^2 + a^7*d*e^7*n^2)*x^
n) - integrate(-1/8*((8*n^2 - 6*n + 1)*c^4*d^6 + (32*n^2 - 18*n + 1)*a*c^3
*d^4*e^2 + (48*n^2 - 2*n - 1)*a^2*c^2*d^2*e^4 - (24*n^2 - 10*n + 1)*a^3*c*
e^6 - 2*((3*n^2 - 4*n + 1)*c^4*d^5*e + 2*(7*n^2 - 8*n + 1)*a*c^3*d^3*e^...
```

3.57.8 Giac [F]

$$\int \frac{1}{(d+ex^n)^2\,(a+cx^{2n})^3}\,dx=\int \frac{1}{(cx^{2\,n}+a)^3(ex^n+d)^2}\,dx$$

input
```
integrate(1/(d+e*x^n)^2/(a+c*x^(2*n))^3,x, algorithm="giac")
```

output
```
integrate(1/((c*x^(2*n) + a)^3*(e*x^n + d)^2), x)
```

3.57.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^n)^2 (a+cx^{2n})^3}\,dx = \int \frac{1}{(a+c\,x^{2\,n})^3\,(d+e\,x^n)^2}\,dx$$

input
```
int(1/((a + c*x^(2*n))^3*(d + e*x^n)^2),x)
```

output
```
int(1/((a + c*x^(2*n))^3*(d + e*x^n)^2), x)
```

3.58 $\int \frac{1}{(d+ex^n)\sqrt{a+cx^{2n}}}\,dx$

3.58.1 Optimal result

Integrand size = 23, antiderivative size = 171

$$\int \frac{1}{(d+ex^n)\sqrt{a+cx^{2n}}}\,dx = \frac{x\sqrt{1+\frac{cx^{2n}}{a}}\,\text{AppellF1}\left(\frac{1}{2n},\frac{1}{2},1,\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d\sqrt{a+cx^{2n}}} - \frac{ex^{1+n}\sqrt{1+\frac{cx^{2n}}{a}}\,\text{AppellF1}\left(\frac{1+n}{2n},\frac{1}{2},1,\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^2(1+n)\sqrt{a+cx^{2n}}}$$

output
```
x*AppellF1(1/2/n,1,1/2,1+1/2/n,e^2*x^(2*n)/d^2,-c*x^(2*n)/a)*(1+c*x^(2*n)/
a)^(1/2)/d/(a+c*x^(2*n))^(1/2)-e*x^(1+n)*AppellF1(1/2*(1+n)/n,1,1/2,3/2+1/
2/n,e^2*x^(2*n)/d^2,-c*x^(2*n)/a)*(1+c*x^(2*n)/a)^(1/2)/d^2/(1+n)/(a+c*x^(
2*n))^(1/2)
```

3.58.2 Mathematica [F]

$$\int \frac{1}{(d+ex^n)\sqrt{a+cx^{2n}}}\,dx = \int \frac{1}{(d+ex^n)\sqrt{a+cx^{2n}}}\,dx$$

input
```
Integrate[1/((d + e*x^n)*Sqrt[a + c*x^(2*n)]),x]
```

output
```
Integrate[1/((d + e*x^n)*Sqrt[a + c*x^(2*n)]), x]
```

3.58.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1768, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+cx^{2n}}\,(d+ex^n)}\,dx$$

$$\Big\downarrow \text{1768}$$

$$\int \left(\frac{d}{\sqrt{a+cx^{2n}}\,(d^2-e^2x^{2n})}+\frac{ex^n}{\sqrt{a+cx^{2n}}\,(e^2x^{2n}-d^2)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x\sqrt{\frac{cx^{2n}}{a}+1}\,\text{AppellF1}\left(\frac{1}{2n},\frac{1}{2},1,\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d\sqrt{a+cx^{2n}}}-\frac{ex^{n+1}\sqrt{\frac{cx^{2n}}{a}+1}\,\text{AppellF1}\left(\frac{n+1}{2n},\frac{1}{2},1,\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^2(n+1)\sqrt{a+cx^{2n}}}$$

input
```
Int[1/((d + e*x^n)*Sqrt[a + c*x^(2*n)]),x]
```

output
```
(x*Sqrt[1 + (c*x^(2*n))/a]*AppellF1[1/(2*n), 1/2, 1, (2 + n^(-1))/2, -((c*
x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d*Sqrt[a + c*x^(2*n)]) - (e*x^(1 + n)*Sq
rt[1 + (c*x^(2*n))/a]*AppellF1[(1 + n)/(2*n), 1/2, 1, (3 + n^(-1))/2, -((c
*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d^2*(1 + n)*Sqrt[a + c*x^(2*n)])
```

3.58.3.1 Defintions of rubi rules used

rule 1768
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
 :> Int[ExpandIntegrand[(a + c*x^(2*n))^p, (d/(d^2 - e^2*x^(2*n)) - e*(x^n/
(d^2 - e^2*x^(2*n))))^(-q), x], x] /; FreeQ[{a, c, d, e, n, p}, x] && EqQ[n
2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && ILtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.58.4 Maple [F]

$$\int \frac{1}{(d+e\,x^{n})\,\sqrt{a+c\,x^{2n}}}dx$$

input
```
int(1/(d+e*x^n)/(a+c*x^(2*n))^(1/2),x)
```

output
```
int(1/(d+e*x^n)/(a+c*x^(2*n))^(1/2),x)
```

3.58.5 Fricas [F]

$$\int \frac{1}{(d+ex^{n})\,\sqrt{a+cx^{2n}}}\,dx = \int \frac{1}{\sqrt{cx^{2\,n}+a}(ex^{n}+d)}\,dx$$

input
```
integrate(1/(d+e*x^n)/(a+c*x^(2*n))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^(2*n) + a)/(a*e*x^n + a*d + (c*e*x^n + c*d)*x^(2*n)), x)
```

3.58.6 Sympy [F]

$$\int \frac{1}{(d+ex^{n})\,\sqrt{a+cx^{2n}}}\,dx = \int \frac{1}{\sqrt{a+cx^{2n}}\,(d+ex^{n})}\,dx$$

input
```
integrate(1/(d+e*x**n)/(a+c*x**(2*n))**(1/2),x)
```

output
```
Integral(1/(sqrt(a + c*x**(2*n))*(d + e*x**n)), x)
```

3.58.7 Maxima [F]

$$\int \frac{1}{(d+ex^n)\sqrt{a+cx^{2n}}}\,dx = \int \frac{1}{\sqrt{cx^{2n}+a}(ex^n+d)}\,dx$$

input
```
integrate(1/(d+e*x^n)/(a+c*x^(2*n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^(2*n) + a)*(e*x^n + d)), x)
```

3.58.8 Giac [F]

$$\int \frac{1}{(d+ex^n)\sqrt{a+cx^{2n}}}\,dx = \int \frac{1}{\sqrt{cx^{2n}+a}(ex^n+d)}\,dx$$

input
```
integrate(1/(d+e*x^n)/(a+c*x^(2*n))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^(2*n) + a)*(e*x^n + d)), x)
```

3.58.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^n)\sqrt{a+cx^{2n}}}\,dx = \int \frac{1}{\sqrt{a+c\,x^{2n}}\,(d+e\,x^n)}\,dx$$

input
```
int(1/((a + c*x^(2*n))^(1/2)*(d + e*x^n)),x)
```

output
```
int(1/((a + c*x^(2*n))^(1/2)*(d + e*x^n)), x)
```

3.59 $\int (d+ex^n)^q \left(a+cx^{2n}\right)^p \, dx$

3.59.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int (d+ex^n)^q \left(a+cx^{2n}\right)^p \, dx = \text{Int}\left((d+ex^n)^q \left(a+cx^{2n}\right)^p, x\right)$$

output
```
Unintegrable((d+e*x^n)^q*(a+c*x^(2*n))^p,x)
```

3.59.2 Mathematica [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int (d+ex^n)^q \left(a+cx^{2n}\right)^p \, dx = \int (d+ex^n)^q \left(a+cx^{2n}\right)^p \, dx$$

input
```
Integrate[(d + e*x^n)^q*(a + c*x^(2*n))^p,x]
```

output
```
Integrate[(d + e*x^n)^q*(a + c*x^(2*n))^p, x]
```

3.59.3 Rubi [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {1770}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+cx^{2n})^p (d+ex^n)^q \, dx$$

$$\Big\downarrow \ 1770$$

$$\int (a+cx^{2n})^p (d+ex^n)^q dx$$

input
```
Int[(d + e*x^n)^q*(a + c*x^(2*n))^p,x]
```

output
```
$Aborted
```

3.59.3.1 Defintions of rubi rules used

rule 1770
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
 :> Unintegrable[(d + e*x^n)^q*(a + c*x^(2*n))^p, x] /; FreeQ[{a, c, d, e,
n, p, q}, x] && EqQ[n2, 2*n]
```

3.59.4 Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int (d+e\,x^n)^q \left(a+c\,x^{2n}\right)^p dx$$

input
```
int((d+e*x^n)^q*(a+c*x^(2*n))^p,x)
```

output
```
int((d+e*x^n)^q*(a+c*x^(2*n))^p,x)
```

3.59.5 Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int (d+ex^n)^q \left(a+cx^{2n}\right)^p \, dx = \int (cx^{2n}+a)^p(ex^n+d)^q \, dx$$

input
```
integrate((d+e*x^n)^q*(a+c*x^(2*n))^p,x, algorithm="fricas")
```

output
```
integral((c*x^(2*n) + a)^p*(e*x^n + d)^q, x)
```

3.59.6 Sympy [F(-1)]

Timed out.

$$\int (d+ex^n)^q \left(a+cx^{2n}\right)^p \, dx = \text{Timed out}$$

input
```
integrate((d+e*x**n)**q*(a+c*x**(2*n))**p,x)
```

output
```
Timed out
```

3.59.7 Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int (d+ex^n)^q \left(a+cx^{2n}\right)^p \, dx = \int (cx^{2n}+a)^p(ex^n+d)^q \, dx$$

input
```
integrate((d+e*x^n)^q*(a+c*x^(2*n))^p,x, algorithm="maxima")
```

output
```
integrate((c*x^(2*n) + a)^p*(e*x^n + d)^q, x)
```

3.59.8 Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int (d+ex^n)^q \left(a+cx^{2n}\right)^p \, dx = \int (cx^{2n}+a)^p (ex^n+d)^q \, dx$$

input
```
integrate((d+e*x^n)^q*(a+c*x^(2*n))^p,x, algorithm="giac")
```

output
```
integrate((c*x^(2*n) + a)^p*(e*x^n + d)^q, x)
```

3.59.9 Mupad [N/A]

Not integrable

Time = 8.93 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int (d+ex^n)^q \left(a+cx^{2n}\right)^p \, dx = \int \left(a+c\,x^{2\,n}\right)^p \left(d+e\,x^n\right)^q \, dx$$

input
```
int((a + c*x^(2*n))^p*(d + e*x^n)^q,x)
```

output
```
int((a + c*x^(2*n))^p*(d + e*x^n)^q, x)
```

3.60 $\int (d+ex^n)^3 (a+cx^{2n})^p \, dx$

3.60.1 Optimal result

Integrand size = 21, antiderivative size = 299

$$\int (d+ex^n)^3 (a+cx^{2n})^p \, dx = \frac{3de^2x^{1+2n}(a+cx^{2n})^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2}\left(2+\frac{1}{n}\right),-p,\frac{1}{2}\left(4+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{1+2n} + \frac{e^3x^{1+3n}(a+cx^{2n})^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2}\left(3+\frac{1}{n}\right),-p,\frac{1}{2}\left(5+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{1+3n} + d^3x(a+cx^{2n})^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2n},-p,\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right) + \frac{3d^2ex^{1+n}(a+cx^{2n})^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1+n}{2n},-p,\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{1+n}$$

output
```
3*d*e^2*x^(1+2*n)*(a+c*x^(2*n))^p*hypergeom([-p, 1+1/2/n],[2+1/2/n],-c*x^(
2*n)/a)/(1+2*n)/((1+c*x^(2*n)/a)^p)+e^3*x^(1+3*n)*(a+c*x^(2*n))^p*hypergeo
m([-p, 3/2+1/2/n],[5/2+1/2/n],-c*x^(2*n)/a)/(1+3*n)/((1+c*x^(2*n)/a)^p)+d^
3*x*(a+c*x^(2*n))^p*hypergeom([-p, 1/2/n],[1+1/2/n],-c*x^(2*n)/a)/((1+c*x^
(2*n)/a)^p)+3*d^2*e*x^(1+n)*(a+c*x^(2*n))^p*hypergeom([-p, 1/2*(1+n)/n],[3
/2+1/2/n],-c*x^(2*n)/a)/(1+n)/((1+c*x^(2*n)/a)^p)
```

3.60.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.71

$$\int (d+ex^n)^3 \left(a+cx^{2n}\right)^p \, dx = x\left(a+cx^{2n}\right)^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} \left(\frac{3de^2x^{2n}\,\text{Hypergeometric2F1}\left(\frac{1}{2}\left(2+\frac{1}{n}\right),-p,\frac{1}{2}\left(4+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{1+2n} + \frac{e^3x^{3n}\,\text{Hypergeometric2F1}\left(\frac{1}{2}\left(3+\frac{1}{n}\right),-p,\frac{1}{2}\left(5+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{1+3n} + d^2\left(d\,\text{Hypergeometric2F1}\left(\frac{1}{2n},-p,\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right) + \frac{3ex^n\,\text{Hypergeometric2F1}\left(\frac{1+n}{2n},-p,\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{1+n}\right)\right)$$

input
```
Integrate[(d + e*x^n)^3*(a + c*x^(2*n))^p,x]
```

output
```
(x*(a + c*x^(2*n))^p*((3*d*e^2*x^(2*n)*Hypergeometric2F1[(2 + n^(-1))/2, -
p, (4 + n^(-1))/2, -((c*x^(2*n))/a)])/(1 + 2*n) + (e^3*x^(3*n)*Hypergeomet
ric2F1[(3 + n^(-1))/2, -p, (5 + n^(-1))/2, -((c*x^(2*n))/a)])/(1 + 3*n) +
d^2*(d*Hypergeometric2F1[1/(2*n), -p, (2 + n^(-1))/2, -((c*x^(2*n))/a)] +
(3*e*x^n*Hypergeometric2F1[(1 + n)/(2*n), -p, (3 + n^(-1))/2, -((c*x^(2*n)
)/a)])/(1 + n))))/(1 + (c*x^(2*n))/a)^p
```

3.60.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1767, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex^n)^3 \left(a+cx^{2n}\right)^p \, dx$$

$$\downarrow 1767$$

$$\int \left(d^3\left(a+cx^{2n}\right)^p+3d^2ex^n\left(a+cx^{2n}\right)^p+3de^2x^{2n}\left(a+cx^{2n}\right)^p+e^3x^{3n}\left(a+cx^{2n}\right)^p\right)dx$$

$$\downarrow 2009$$

$$d^3x\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2n},-p,\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)+\frac{3d^2ex^{n+1}\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{n+1}{2n},-p,\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{n+1}+\frac{3de^2x^{2n+1}\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2}\left(2+\frac{1}{n}\right),-p,\frac{1}{2}\left(4+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{2n+1}+\frac{e^3x^{3n+1}\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2}\left(3+\frac{1}{n}\right),-p,\frac{1}{2}\left(5+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{3n+1}$$

input
```
Int[(d + e*x^n)^3*(a + c*x^(2*n))^p,x]
```

output
```
(3*d*e^2*x^(1 + 2*n)*(a + c*x^(2*n))^p*Hypergeometric2F1[(2 + n^(-1))/2, -
p, (4 + n^(-1))/2, -((c*x^(2*n))/a)])/((1 + 2*n)*(1 + (c*x^(2*n))/a)^p) +
(e^3*x^(1 + 3*n)*(a + c*x^(2*n))^p*Hypergeometric2F1[(3 + n^(-1))/2, -p, (
5 + n^(-1))/2, -((c*x^(2*n))/a)])/((1 + 3*n)*(1 + (c*x^(2*n))/a)^p) + (d^3
*x*(a + c*x^(2*n))^p*Hypergeometric2F1[1/(2*n), -p, (2 + n^(-1))/2, -((c*x
^(2*n))/a)])/(1 + (c*x^(2*n))/a)^p + (3*d^2*e*x^(1 + n)*(a + c*x^(2*n))^p*
Hypergeometric2F1[(1 + n)/(2*n), -p, (3 + n^(-1))/2, -((c*x^(2*n))/a)])/((
1 + n)*(1 + (c*x^(2*n))/a)^p)
```

3.60.3.1 Defintions of rubi rules used

rule 1767
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + c*x^(2*n))^p, x], x] /; FreeQ[{a
, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && ((Integ
ersQ[p, q] &&  !IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] &&  !IntegerQ[n])
)
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.60.4 Maple [F]

$$\int (d+e\,x^n)^3\,(a+c\,x^{2n})^p\,dx$$

input
```
int((d+e*x^n)^3*(a+c*x^(2*n))^p,x)
```

output
```
int((d+e*x^n)^3*(a+c*x^(2*n))^p,x)
```

3.60.5 Fricas [F]

$$\int (d+ex^n)^3\,(a+cx^{2n})^p\;dx=\int (ex^n+d)^3(cx^{2\,n}+a)^p\,dx$$

input
```
integrate((d+e*x^n)^3*(a+c*x^(2*n))^p,x, algorithm="fricas")
```

output
```
integral((e^3*x^(3*n) + 3*d*e^2*x^(2*n) + 3*d^2*e*x^n + d^3)*(c*x^(2*n) +
a)^p, x)
```

3.60.6 Sympy [F(-1)]

Timed out.

$$\int (d+ex^n)^3\,(a+cx^{2n})^p\;dx=\text{Timed out}$$

input
```
integrate((d+e*x**n)**3*(a+c*x**(2*n))**p,x)
```

output
```
Timed out
```

3.60.7 Maxima [F]

$$\int (d+ex^n)^3 \left(a+cx^{2n}\right)^p \, dx = \int (ex^n+d)^3 (cx^{2n}+a)^p \, dx$$

input
```
integrate((d+e*x^n)^3*(a+c*x^(2*n))^p,x, algorithm="maxima")
```

output
```
integrate((e*x^n + d)^3*(c*x^(2*n) + a)^p, x)
```

3.60.8 Giac [F(-2)]

Exception generated.

$$\int (d+ex^n)^3 \left(a+cx^{2n}\right)^p \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((d+e*x^n)^3*(a+c*x^(2*n))^p,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{96,[1,0,6,4,3,5,4,1,2]%%%}+%%%{480,[1,0,6,4,3,4,4,1,2]%%%}
+%%%{960,
```

3.60.9 Mupad [F(-1)]

Timed out.

$$\int (d+ex^n)^3 \left(a+cx^{2n}\right)^p \, dx = \int (a+c\,x^{2\,n})^p\,(d+e\,x^n)^3 \, dx$$

input
```
int((a + c*x^(2*n))^p*(d + e*x^n)^3,x)
```

output
```
int((a + c*x^(2*n))^p*(d + e*x^n)^3, x)
```

3.61 $\int (d+ex^n)^2 (a+cx^{2n})^p \, dx$

3.61.1 Optimal result

Integrand size = 21, antiderivative size = 217

$$\int (d+ex^n)^2 (a+cx^{2n})^p \, dx = \frac{e^2x^{1+2n}(a+cx^{2n})^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2}\left(2+\frac{1}{n}\right),-p,\frac{1}{2}\left(4+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{1+2n} + d^2x(a+cx^{2n})^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2n},-p,\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right) + \frac{2dex^{1+n}(a+cx^{2n})^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1+n}{2n},-p,\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{1+n}$$

output
```
e^2*x^(1+2*n)*(a+c*x^(2*n))^p*hypergeom([-p, 1+1/2/n],[2+1/2/n],-c*x^(2*n)
/a)/(1+2*n)/((1+c*x^(2*n)/a)^p)+d^2*x*(a+c*x^(2*n))^p*hypergeom([-p, 1/2/n
],[1+1/2/n],-c*x^(2*n)/a)/((1+c*x^(2*n)/a)^p)+2*d*e*x^(1+n)*(a+c*x^(2*n))^
p*hypergeom([-p, 1/2*(1+n)/n],[3/2+1/2/n],-c*x^(2*n)/a)/(1+n)/((1+c*x^(2*n
)/a)^p)
```

3.61.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.79

$$\int (d+ex^n)^2 \left(a+cx^{2n}\right)^p \, dx = \frac{x(a+cx^{2n})^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} \left(e^2(1+n)x^{2n}\,\text{Hypergeometric2F1}\left(\frac{1}{2}\left(2+\frac{1}{n}\right),-p,\frac{1}{2}\left(4+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)+d(1+\right.}{}$$

input
```
Integrate[(d + e*x^n)^2*(a + c*x^(2*n))^p,x]
```

output
```
(x*(a + c*x^(2*n))^p*(e^2*(1 + n)*x^(2*n)*Hypergeometric2F1[(2 + n^(-1))/2
, -p, (4 + n^(-1))/2, -((c*x^(2*n))/a)] + d*(1 + 2*n)*(d*(1 + n)*Hypergeom
etric2F1[1/(2*n), -p, (2 + n^(-1))/2, -((c*x^(2*n))/a)] + 2*e*x^n*Hypergeo
metric2F1[(1 + n)/(2*n), -p, (3 + n^(-1))/2, -((c*x^(2*n))/a)])))/((1 + n)
*(1 + 2*n)*(1 + (c*x^(2*n))/a)^p)
```

3.61.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1767, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex^n)^2 \left(a+cx^{2n}\right)^p \, dx$$

$$\Big\downarrow \text{1767}$$

$$\int \left(d^2\left(a+cx^{2n}\right)^p+2dex^n\left(a+cx^{2n}\right)^p+e^2x^{2n}\left(a+cx^{2n}\right)^p\right)dx$$

$$\Big\downarrow \text{2009}$$

$$d^2x\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2n},-p,\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)+\frac{2dex^{n+1}\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{n+1}{2n},-p,\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{n+1}+\frac{e^2x^{2n+1}\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2}\left(2+\frac{1}{n}\right),-p,\frac{1}{2}\left(4+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{2n+1}$$

input
```
Int[(d + e*x^n)^2*(a + c*x^(2*n))^p,x]
```

output
```
(e^2*x^(1 + 2*n)*(a + c*x^(2*n))^p*Hypergeometric2F1[(2 + n^(-1))/2, -p, (
4 + n^(-1))/2, -((c*x^(2*n))/a)])/((1 + 2*n)*(1 + (c*x^(2*n))/a)^p) + (d^2
*x*(a + c*x^(2*n))^p*Hypergeometric2F1[1/(2*n), -p, (2 + n^(-1))/2, -((c*x
^(2*n))/a)])/(1 + (c*x^(2*n))/a)^p + (2*d*e*x^(1 + n)*(a + c*x^(2*n))^p*Hy
pergeometric2F1[(1 + n)/(2*n), -p, (3 + n^(-1))/2, -((c*x^(2*n))/a)])/((1
+ n)*(1 + (c*x^(2*n))/a)^p)
```

3.61.3.1 Defintions of rubi rules used

rule 1767
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + c*x^(2*n))^p, x], x] /; FreeQ[{a
, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && ((Integ
ersQ[p, q] &&  !IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] &&  !IntegerQ[n])
)
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.61.4 Maple [F]

$$\int (d+e\,x^{n})^{2}\left(a+c\,x^{2n}\right)^{p}dx$$

input
```
int((d+e*x^n)^2*(a+c*x^(2*n))^p,x)
```

output
```
int((d+e*x^n)^2*(a+c*x^(2*n))^p,x)
```

3.61.5 Fricas [F]

$$\int (d+ex^n)^2 \left(a+cx^{2n}\right)^p \,dx = \int (ex^n+d)^2 (cx^{2n}+a)^p \,dx$$

input
```
integrate((d+e*x^n)^2*(a+c*x^(2*n))^p,x, algorithm="fricas")
```

output
```
integral((e^2*x^(2*n) + 2*d*e*x^n + d^2)*(c*x^(2*n) + a)^p, x)
```

3.61.6 Sympy [F(-1)]

Timed out.

$$\int (d+ex^n)^2 \left(a+cx^{2n}\right)^p \,dx = \text{Timed out}$$

input
```
integrate((d+e*x**n)**2*(a+c*x**(2*n))**p,x)
```

output
```
Timed out
```

3.61.7 Maxima [F]

$$\int (d+ex^n)^2 \left(a+cx^{2n}\right)^p \,dx = \int (ex^n+d)^2 (cx^{2n}+a)^p \,dx$$

input
```
integrate((d+e*x^n)^2*(a+c*x^(2*n))^p,x, algorithm="maxima")
```

output
```
integrate((e*x^n + d)^2*(c*x^(2*n) + a)^p, x)
```

3.61.8 Giac [F(-2)]

Exception generated.

$$\int (d+ex^n)^2 (a+cx^{2n})^p \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((d+e*x^n)^2*(a+c*x^(2*n))^p,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{64,[1,0,4,3,1,4,3,1,1]%%%}+%%%{256,[1,0,4,3,1,3,3,1,1]%%%}
+%%%{384,
```

3.61.9 Mupad [F(-1)]

Timed out.

$$\int (d+ex^n)^2 (a+cx^{2n})^p \, dx = \int (a+c\,x^{2\,n})^p\,(d+e\,x^n)^2 \, dx$$

input
```
int((a + c*x^(2*n))^p*(d + e*x^n)^2,x)
```

output
```
int((a + c*x^(2*n))^p*(d + e*x^n)^2, x)
```

3.62 $\int (d+ex^n)(a+cx^{2n})^p \, dx$

3.62.1 Optimal result

Integrand size = 19, antiderivative size = 135

$$\int (d+ex^n)(a+cx^{2n})^p \, dx = dx(a+cx^{2n})^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2n},-p,\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right) + \frac{ex^{1+n}(a+cx^{2n})^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1+n}{2n},-p,\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{1+n}$$

output
```
d*x*(a+c*x^(2*n))^p*hypergeom([-p, 1/2/n],[1+1/2/n],-c*x^(2*n)/a)/((1+c*x^
(2*n)/a)^p)+e*x^(1+n)*(a+c*x^(2*n))^p*hypergeom([-p, 1/2*(1+n)/n],[3/2+1/2
/n],-c*x^(2*n)/a)/(1+n)/((1+c*x^(2*n)/a)^p)
```

3.62.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.81

$$\int (d+ex^n)(a+cx^{2n})^p \, dx = \frac{x(a+cx^{2n})^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\left(d(1+n)\text{Hypergeometric2F1}\left(\frac{1}{2n},-p,\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)+ex^n\text{Hypergeomet}\right.}{1+n}$$

input
```
Integrate[(d + e*x^n)*(a + c*x^(2*n))^p,x]
```

output
```
(x*(a + c*x^(2*n))^p*(d*(1 + n)*Hypergeometric2F1[1/(2*n), -p, (2 + n^(-1)
)/2, -((c*x^(2*n))/a)] + e*x^n*Hypergeometric2F1[(1 + n)/(2*n), -p, (3 + n
^(-1))/2, -((c*x^(2*n))/a)]))/((1 + n)*(1 + (c*x^(2*n))/a)^p)
```

3.62.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1763, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex^n)\left(a+cx^{2n}\right)^p\,dx$$

$$\Big\downarrow \text{1763}$$

$$\int \left(d\left(a+cx^{2n}\right)^p+ex^n\left(a+cx^{2n}\right)^p\right)dx$$

$$\Big\downarrow \text{2009}$$

$$dx\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2n},-p,\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)+\frac{ex^{n+1}\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{n+1}{2n},-p,\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a}\right)}{n+1}$$

input
```
Int[(d + e*x^n)*(a + c*x^(2*n))^p,x]
```

output
```
(d*x*(a + c*x^(2*n))^p*Hypergeometric2F1[1/(2*n), -p, (2 + n^(-1))/2, -((c
*x^(2*n))/a)])/(1 + (c*x^(2*n))/a)^p + (e*x^(1 + n)*(a + c*x^(2*n))^p*Hype
rgeometric2F1[(1 + n)/(2*n), -p, (3 + n^(-1))/2, -((c*x^(2*n))/a)])/((1 +
n)*(1 + (c*x^(2*n))/a)^p)
```

3.62.3.1 Defintions of rubi rules used

rule 1763
```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^n)*(a + c*x^(2*n))^p, x], x] /; FreeQ[{a, c, d,
 e, n}, x] && EqQ[n2, 2*n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.62.4 Maple [F]

$$\int (d+e\,x^{n})\left(a+c\,x^{2n}\right)^{p}dx$$

input
```
int((d+e*x^n)*(a+c*x^(2*n))^p,x)
```

output
```
int((d+e*x^n)*(a+c*x^(2*n))^p,x)
```

3.62.5 Fricas [F]

$$\int (d+ex^{n})\left(a+cx^{2n}\right)^{p}\,dx=\int (ex^{n}+d)\left(cx^{2\,n}+a\right)^{p}\,dx$$

input
```
integrate((d+e*x^n)*(a+c*x^(2*n))^p,x, algorithm="fricas")
```

output
```
integral((e*x^n + d)*(c*x^(2*n) + a)^p, x)
```

3.62.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 133.83 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.07

$$\int \left(d+ex^n\right)\left(a+cx^{2n}\right)^p\,dx=\frac{a^{\frac{1}{2n}}a^{p-\frac{1}{2n}}dx\Gamma\left(\frac{1}{2n}\right){}_2F_1\left(\begin{matrix}\frac{1}{2n},-p\\1+\frac{1}{2n}\end{matrix}\middle|\frac{cx^{2n}e^{i\pi}}{a}\right)}{2n\Gamma\left(1+\frac{1}{2n}\right)}+\frac{a^{\frac{1}{2}+\frac{1}{2n}}a^{p-\frac{1}{2}-\frac{1}{2n}}ex^{n+1}\Gamma\left(\frac{1}{2}+\frac{1}{2n}\right){}_2F_1\left(\begin{matrix}-p,\frac{1}{2}+\frac{1}{2n}\\\frac{3}{2}+\frac{1}{2n}\end{matrix}\middle|\frac{cx^{2n}e^{i\pi}}{a}\right)}{2n\Gamma\left(\frac{3}{2}+\frac{1}{2n}\right)}$$

input
```
integrate((d+e*x**n)*(a+c*x**(2*n))**p,x)
```

output
```
a**(1/(2*n))*a**(p - 1/(2*n))*d*x*gamma(1/(2*n))*hyper((1/(2*n), -p), (1 +
 1/(2*n),), c*x**(2*n)*exp_polar(I*pi)/a)/(2*n*gamma(1 + 1/(2*n))) + a**(1
/2 + 1/(2*n))*a**(p - 1/2 - 1/(2*n))*e*x**(n + 1)*gamma(1/2 + 1/(2*n))*hyp
er((-p, 1/2 + 1/(2*n)), (3/2 + 1/(2*n),), c*x**(2*n)*exp_polar(I*pi)/a)/(2
*n*gamma(3/2 + 1/(2*n)))
```

3.62.7 Maxima [F]

$$\int \left(d+ex^n\right)\left(a+cx^{2n}\right)^p\,dx=\int \left(ex^n+d\right)\left(cx^{2n}+a\right)^p\,dx$$

input
```
integrate((d+e*x^n)*(a+c*x^(2*n))^p,x, algorithm="maxima")
```

output
```
integrate((e*x^n + d)*(c*x^(2*n) + a)^p, x)
```

3.62.8 Giac [F]

$$\int \left(d+ex^n\right)\left(a+cx^{2n}\right)^p\,dx=\int \left(ex^n+d\right)\left(cx^{2n}+a\right)^p\,dx$$

input
```
integrate((d+e*x^n)*(a+c*x^(2*n))^p,x, algorithm="giac")
```

output
```
integrate((e*x^n + d)*(c*x^(2*n) + a)^p, x)
```

3.62.9 Mupad [F(-1)]

Timed out.

$$\int (d+ex^n) \left(a+cx^{2n}\right)^p \,dx = \int \left(a+c\,x^{2\,n}\right)^p \left(d+e\,x^n\right) \,dx$$

input
```
int((a + c*x^(2*n))^p*(d + e*x^n),x)
```

output
```
int((a + c*x^(2*n))^p*(d + e*x^n), x)
```

3.63 $\int \frac{(a+cx^{2n})^p}{d+ex^n} dx$

3.63.1 Optimal result

Integrand size = 21, antiderivative size = 167

$$\int \frac{(a+cx^{2n})^p}{d+ex^n} dx = \frac{x(a+cx^{2n})^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} \text{AppellF1}\left(\frac{1}{2n},-p,1,\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d} - \frac{ex^{1+n}(a+cx^{2n})^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} \text{AppellF1}\left(\frac{1+n}{2n},-p,1,\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^2(1+n)}$$

output
```
x*(a+c*x^(2*n))^p*AppellF1(1/2/n,1,-p,1+1/2/n,e^2*x^(2*n)/d^2,-c*x^(2*n)/a
)/d/((1+c*x^(2*n)/a)^p)-e*x^(1+n)*(a+c*x^(2*n))^p*AppellF1(1/2*(1+n)/n,1,-
p,3/2+1/2/n,e^2*x^(2*n)/d^2,-c*x^(2*n)/a)/d^2/(1+n)/((1+c*x^(2*n)/a)^p)
```

3.63.2 Mathematica [F]

$$\int \frac{(a+cx^{2n})^p}{d+ex^n} dx = \int \frac{(a+cx^{2n})^p}{d+ex^n} dx$$

input
```
Integrate[(a + c*x^(2*n))^p/(d + e*x^n),x]
```

output
```
Integrate[(a + c*x^(2*n))^p/(d + e*x^n), x]
```

3.63.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1768, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^{2n})^p}{d+ex^n}\,dx$$

$$\Big\downarrow \text{ 1768}$$

$$\int \left(\frac{d(a+cx^{2n})^p}{d^2-e^2x^{2n}}+\frac{ex^n(a+cx^{2n})^p}{e^2x^{2n}-d^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x(a+cx^{2n})^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2n},-p,1,\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d}-\frac{ex^{n+1}(a+cx^{2n})^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{n+1}{2n},-p,1,\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^2(n+1)}$$

input
```
Int[(a + c*x^(2*n))^p/(d + e*x^n),x]
```

output
```
(x*(a + c*x^(2*n))^p*AppellF1[1/(2*n), -p, 1, (2 + n^(-1))/2, -((c*x^(2*n)
)/a), (e^2*x^(2*n))/d^2])/(d*(1 + (c*x^(2*n))/a)^p) - (e*x^(1 + n)*(a + c*
x^(2*n))^p*AppellF1[(1 + n)/(2*n), -p, 1, (3 + n^(-1))/2, -((c*x^(2*n))/a)
, (e^2*x^(2*n))/d^2])/(d^2*(1 + n)*(1 + (c*x^(2*n))/a)^p)
```

3.63.3.1 Defintions of rubi rules used

rule 1768
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
 :> Int[ExpandIntegrand[(a + c*x^(2*n))^p, (d/(d^2 - e^2*x^(2*n)) - e*(x^n/
(d^2 - e^2*x^(2*n))))^(-q), x], x] /; FreeQ[{a, c, d, e, n, p}, x] && EqQ[n
2, 2*n] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.63.4 Maple [F]

$$\int \frac{(a+c\,x^{2n})^p}{d+e\,x^n}dx$$

input
```
int((a+c*x^(2*n))^p/(d+e*x^n),x)
```

output
```
int((a+c*x^(2*n))^p/(d+e*x^n),x)
```

3.63.5 Fricas [F]

$$\int \frac{(a+cx^{2n})^p}{d+ex^n}\,dx = \int \frac{(cx^{2\,n}+a)^p}{ex^n+d}\,dx$$

input
```
integrate((a+c*x^(2*n))^p/(d+e*x^n),x, algorithm="fricas")
```

output
```
integral((c*x^(2*n) + a)^p/(e*x^n + d), x)
```

3.63.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{(a+cx^{2n})^p}{d+ex^n}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((a+c*x**(2*n))**p/(d+e*x**n),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.63.7 Maxima [F]

$$\int \frac{(a+cx^{2n})^p}{d+ex^n}\,dx=\int \frac{(cx^{2n}+a)^p}{ex^n+d}\,dx$$

input
```
integrate((a+c*x^(2*n))^p/(d+e*x^n),x, algorithm="maxima")
```

output
```
integrate((c*x^(2*n) + a)^p/(e*x^n + d), x)
```

3.63.8 Giac [F]

$$\int \frac{(a+cx^{2n})^p}{d+ex^n}\,dx=\int \frac{(cx^{2n}+a)^p}{ex^n+d}\,dx$$

input
```
integrate((a+c*x^(2*n))^p/(d+e*x^n),x, algorithm="giac")
```

output
```
integrate((c*x^(2*n) + a)^p/(e*x^n + d), x)
```

3.63.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+cx^{2n})^p}{d+ex^n}\,dx=\int \frac{(a+c\,x^{2\,n})^p}{d+e\,x^n}\,dx$$

input
```
int((a + c*x^(2*n))^p/(d + e*x^n),x)
```

output
```
int((a + c*x^(2*n))^p/(d + e*x^n), x)
```

3.64 $\int \frac{(a+cx^{2n})^p}{(d+ex^n)^2} dx$

3.64.1 Optimal result

Integrand size = 21, antiderivative size = 261

$$\int \frac{(a+cx^{2n})^p}{(d+ex^n)^2} dx = \frac{e^2x^{1+2n}(a+cx^{2n})^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\text{AppellF1}\left(\frac{1}{2}\left(2+\frac{1}{n}\right),-p,2,\frac{1}{2}\left(4+\frac{1}{n}\right),-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^4(1+2n)} + \frac{x(a+cx^{2n})^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\text{AppellF1}\left(\frac{1}{2n},-p,2,\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^2} - \frac{2ex^{1+n}(a+cx^{2n})^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\text{AppellF1}\left(\frac{1+n}{2n},-p,2,\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^3(1+n)}$$

output
```
e^2*x^(1+2*n)*(a+c*x^(2*n))^p*AppellF1(1+1/2/n,2,-p,2+1/2/n,e^2*x^(2*n)/d^
2,-c*x^(2*n)/a)/d^4/(1+2*n)/((1+c*x^(2*n)/a)^p)+x*(a+c*x^(2*n))^p*AppellF1
(1/2/n,2,-p,1+1/2/n,e^2*x^(2*n)/d^2,-c*x^(2*n)/a)/d^2/((1+c*x^(2*n)/a)^p)-
2*e*x^(1+n)*(a+c*x^(2*n))^p*AppellF1(1/2*(1+n)/n,2,-p,3/2+1/2/n,e^2*x^(2*n
)/d^2,-c*x^(2*n)/a)/d^3/(1+n)/((1+c*x^(2*n)/a)^p)
```

3.64.2 Mathematica [F]

$$\int \frac{(a+cx^{2n})^p}{(d+ex^n)^2}\,dx=\int \frac{(a+cx^{2n})^p}{(d+ex^n)^2}\,dx$$

input
```
Integrate[(a + c*x^(2*n))^p/(d + e*x^n)^2,x]
```

output
```
Integrate[(a + c*x^(2*n))^p/(d + e*x^n)^2, x]
```

3.64.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1768, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^{2n})^p}{(d+ex^n)^2}\,dx$$

$$\Big\downarrow \text{1768}$$

$$\int \left(\frac{e^2x^{2n}(a+cx^{2n})^p}{(e^2x^{2n}-d^2)^2}+\frac{d^2(a+cx^{2n})^p}{(d^2-e^2x^{2n})^2}-\frac{2dex^n(a+cx^{2n})^p}{(e^2x^{2n}-d^2)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x(a+cx^{2n})^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2n},-p,2,\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^2}+\frac{e^2x^{2n+1}(a+cx^{2n})^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2}\left(2+\frac{1}{n}\right),-p,2,\frac{1}{2}\left(4+\frac{1}{n}\right),-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^4(2n+1)}-\frac{2ex^{n+1}(a+cx^{2n})^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{n+1}{2n},-p,2,\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^3(n+1)}$$

input
```
Int[(a + c*x^(2*n))^p/(d + e*x^n)^2,x]
```

output
```
(e^2*x^(1 + 2*n)*(a + c*x^(2*n))^p*AppellF1[(2 + n^(-1))/2, -p, 2, (4 + n^
(-1))/2, -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d^4*(1 + 2*n)*(1 + (c*x^(2
*n))/a)^p) + (x*(a + c*x^(2*n))^p*AppellF1[1/(2*n), -p, 2, (2 + n^(-1))/2,
 -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d^2*(1 + (c*x^(2*n))/a)^p) - (2*e*
x^(1 + n)*(a + c*x^(2*n))^p*AppellF1[(1 + n)/(2*n), -p, 2, (3 + n^(-1))/2,
 -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d^3*(1 + n)*(1 + (c*x^(2*n))/a)^p)
```

3.64.3.1 Defintions of rubi rules used

rule 1768
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
 :> Int[ExpandIntegrand[(a + c*x^(2*n))^p, (d/(d^2 - e^2*x^(2*n)) - e*(x^n/
(d^2 - e^2*x^(2*n))))^(-q), x], x] /; FreeQ[{a, c, d, e, n, p}, x] && EqQ[n
2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && ILtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.64.4 Maple [F]

$$\int \frac{(a+c\,x^{2n})^p}{(d+e\,x^n)^2}dx$$

input
```
int((a+c*x^(2*n))^p/(d+e*x^n)^2,x)
```

output
```
int((a+c*x^(2*n))^p/(d+e*x^n)^2,x)
```

3.64.5 Fricas [F]

$$\int \frac{(a+cx^{2n})^p}{(d+ex^n)^2}\,dx = \int \frac{(cx^{2\,n}+a)^p}{(ex^n+d)^2}\,dx$$

input
```
integrate((a+c*x^(2*n))^p/(d+e*x^n)^2,x, algorithm="fricas")
```

output
```
integral((c*x^(2*n) + a)^p/(e^2*x^(2*n) + 2*d*e*x^n + d^2), x)
```

3.64.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+cx^{2n})^p}{(d+ex^n)^2}\,dx = \text{Timed out}$$

input
```
integrate((a+c*x**(2*n))**p/(d+e*x**n)**2,x)
```

output
```
Timed out
```

3.64.7 Maxima [F]

$$\int \frac{(a+cx^{2n})^p}{(d+ex^n)^2}\,dx = \int \frac{(cx^{2n}+a)^p}{(ex^n+d)^2}\,dx$$

input
```
integrate((a+c*x^(2*n))^p/(d+e*x^n)^2,x, algorithm="maxima")
```

output
```
integrate((c*x^(2*n) + a)^p/(e*x^n + d)^2, x)
```

3.64.8 Giac [F]

$$\int \frac{(a+cx^{2n})^p}{(d+ex^n)^2}\,dx = \int \frac{(cx^{2n}+a)^p}{(ex^n+d)^2}\,dx$$

input
```
integrate((a+c*x^(2*n))^p/(d+e*x^n)^2,x, algorithm="giac")
```

output
```
integrate((c*x^(2*n) + a)^p/(e*x^n + d)^2, x)
```

3.64.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+cx^{2n})^p}{(d+ex^n)^2} \, dx = \int \frac{(a+c\,x^{2\,n})^p}{(d+e\,x^n)^2} \, dx$$

input
```
int((a + c*x^(2*n))^p/(d + e*x^n)^2,x)
```

output
```
int((a + c*x^(2*n))^p/(d + e*x^n)^2, x)
```

3.65 $\int \frac{(a+cx^{2n})^p}{(d+ex^n)^3} dx$

3.65.1 Optimal result

Integrand size = 21, antiderivative size = 357

$$\begin{aligned}\int \frac{(a+cx^{2n})^p}{(d+ex^n)^3}\,dx &= \frac{3e^2x^{1+2n}(a+cx^{2n})^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{2}\left(2+\frac{1}{n}\right),-p,3,\frac{1}{2}\left(4+\frac{1}{n}\right),-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^5(1+2n)}\\ &\quad-\frac{e^3x^{1+3n}(a+cx^{2n})^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{2}\left(3+\frac{1}{n}\right),-p,3,\frac{1}{2}\left(5+\frac{1}{n}\right),-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^6(1+3n)}\\ &\quad+\frac{x(a+cx^{2n})^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{2n},-p,3,\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^3}\\ &\quad-\frac{3ex^{1+n}(a+cx^{2n})^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{1+n}{2n},-p,3,\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^4(1+n)}\end{aligned}$$

output
```
3*e^2*x^(1+2*n)*(a+c*x^(2*n))^p*AppellF1(1+1/2/n,3,-p,2+1/2/n,e^2*x^(2*n)/
d^2,-c*x^(2*n)/a)/d^5/(1+2*n)/((1+c*x^(2*n)/a)^p)-e^3*x^(1+3*n)*(a+c*x^(2*
n))^p*AppellF1(3/2+1/2/n,3,-p,5/2+1/2/n,e^2*x^(2*n)/d^2,-c*x^(2*n)/a)/d^6/
(1+3*n)/((1+c*x^(2*n)/a)^p)+x*(a+c*x^(2*n))^p*AppellF1(1/2/n,3,-p,1+1/2/n,
e^2*x^(2*n)/d^2,-c*x^(2*n)/a)/d^3/((1+c*x^(2*n)/a)^p)-3*e*x^(1+n)*(a+c*x^(
2*n))^p*AppellF1(1/2*(1+n)/n,3,-p,3/2+1/2/n,e^2*x^(2*n)/d^2,-c*x^(2*n)/a)/
d^4/(1+n)/((1+c*x^(2*n)/a)^p)
```

3.65.2 Mathematica [F]

$$\int \frac{(a+cx^{2n})^p}{(d+ex^n)^3}\,dx=\int \frac{(a+cx^{2n})^p}{(d+ex^n)^3}\,dx$$

input
```
Integrate[(a + c*x^(2*n))^p/(d + e*x^n)^3,x]
```

output
```
Integrate[(a + c*x^(2*n))^p/(d + e*x^n)^3, x]
```

3.65.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1768, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^{2n})^p}{(d+ex^n)^3}\,dx$$

$$\Big\downarrow \text{1768}$$

$$\int \left(-\frac{3de^2x^{2n}(a+cx^{2n})^p}{(e^2x^{2n}-d^2)^3}+\frac{3d^2ex^n(a+cx^{2n})^p}{(e^2x^{2n}-d^2)^3}+\frac{e^3x^{3n}(a+cx^{2n})^p}{(e^2x^{2n}-d^2)^3}+\frac{d^3(a+cx^{2n})^p}{(d^2-e^2x^{2n})^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{e^3x^{3n+1}(a+cx^{2n})^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2}\left(3+\frac{1}{n}\right),-p,3,\frac{1}{2}\left(5+\frac{1}{n}\right),-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^6(3n+1)}+\frac{3e^2x^{2n+1}(a+cx^{2n})^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2}\left(2+\frac{1}{n}\right),-p,3,\frac{1}{2}\left(4+\frac{1}{n}\right),-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^5(2n+1)}-\frac{3ex^{n+1}(a+cx^{2n})^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{n+1}{2n},-p,3,\frac{1}{2}\left(3+\frac{1}{n}\right),-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^4(n+1)}+\frac{x(a+cx^{2n})^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2n},-p,3,\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^3}$$

input
```
Int[(a + c*x^(2*n))^p/(d + e*x^n)^3,x]
```

output
```
(3*e^2*x^(1 + 2*n)*(a + c*x^(2*n))^p*AppellF1[(2 + n^(-1))/2, -p, 3, (4 +
n^(-1))/2, -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d^5*(1 + 2*n)*(1 + (c*x^
(2*n))/a)^p) - (e^3*x^(1 + 3*n)*(a + c*x^(2*n))^p*AppellF1[(3 + n^(-1))/2,
 -p, 3, (5 + n^(-1))/2, -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d^6*(1 + 3*
n)*(1 + (c*x^(2*n))/a)^p) + (x*(a + c*x^(2*n))^p*AppellF1[1/(2*n), -p, 3,
(2 + n^(-1))/2, -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d^3*(1 + (c*x^(2*n)
)/a)^p) - (3*e*x^(1 + n)*(a + c*x^(2*n))^p*AppellF1[(1 + n)/(2*n), -p, 3,
(3 + n^(-1))/2, -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d^4*(1 + n)*(1 + (c
*x^(2*n))/a)^p)
```

3.65.3.1 Defintions of rubi rules used

rule 1768
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
 :> Int[ExpandIntegrand[(a + c*x^(2*n))^p, (d/(d^2 - e^2*x^(2*n)) - e*(x^n/
(d^2 - e^2*x^(2*n))))^(-q), x], x] /; FreeQ[{a, c, d, e, n, p}, x] && EqQ[n
2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && ILtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.65.4 Maple [F]

$$\int \frac{(a + c\,x^{2n})^p}{(d + e\,x^n)^3}\,dx$$

input
```
int((a+c*x^(2*n))^p/(d+e*x^n)^3,x)
```

output
```
int((a+c*x^(2*n))^p/(d+e*x^n)^3,x)
```

3.65.5 Fricas [F]

$$\int \frac{(a + cx^{2n})^p}{(d + ex^n)^3}\,dx = \int \frac{(cx^{2\,n} + a)^p}{(ex^n + d)^3}\,dx$$

input
```
integrate((a+c*x^(2*n))^p/(d+e*x^n)^3,x, algorithm="fricas")
```

output
```
integral((c*x^(2*n) + a)^p/(e^3*x^(3*n) + 3*d*e^2*x^(2*n) + 3*d^2*e*x^n +
d^3), x)
```

3.65.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+cx^{2n})^p}{(d+ex^n)^3}\,dx = \text{Timed out}$$

input
```
integrate((a+c*x**(2*n))**p/(d+e*x**n)**3,x)
```

output
```
Timed out
```

3.65.7 Maxima [F]

$$\int \frac{(a+cx^{2n})^p}{(d+ex^n)^3}\,dx = \int \frac{(cx^{2n}+a)^p}{(ex^n+d)^3}\,dx$$

input
```
integrate((a+c*x^(2*n))^p/(d+e*x^n)^3,x, algorithm="maxima")
```

output
```
integrate((c*x^(2*n) + a)^p/(e*x^n + d)^3, x)
```

3.65.8 Giac [F]

$$\int \frac{(a+cx^{2n})^p}{(d+ex^n)^3}\,dx = \int \frac{(cx^{2n}+a)^p}{(ex^n+d)^3}\,dx$$

input
```
integrate((a+c*x^(2*n))^p/(d+e*x^n)^3,x, algorithm="giac")
```

output
```
integrate((c*x^(2*n) + a)^p/(e*x^n + d)^3, x)
```

3.65.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+cx^{2n})^p}{(d+ex^n)^3}\,dx = \int \frac{(a+c\,x^{2\,n})^p}{(d+e\,x^n)^3}\,dx$$

input
```
int((a + c*x^(2*n))^p/(d + e*x^n)^3,x)
```

output
```
int((a + c*x^(2*n))^p/(d + e*x^n)^3, x)
```

3.66 $\int (d+ex^n)(a+bx^n+cx^{2n})\,dx$

3.66.1 Optimal result

Integrand size = 22, antiderivative size = 62

$$\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)\,dx = adx+\frac{(bd+ae)x^{1+n}}{1+n}+\frac{(cd+be)x^{1+2n}}{1+2n}+\frac{cex^{1+3n}}{1+3n}$$

output
```
a*d*x+(a*e+b*d)*x^(1+n)/(1+n)+(b*e+c*d)*x^(1+2*n)/(1+2*n)+c*e*x^(1+3*n)/(1
+3*n)
```

3.66.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.92

$$\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)\,dx = x\left(ad+\frac{(bd+ae)x^n}{1+n}+\frac{(cd+be)x^{2n}}{1+2n}+\frac{cex^{3n}}{1+3n}\right)$$

input
```
Integrate[(d + e*x^n)*(a + b*x^n + c*x^(2*n)),x]
```

output
```
x*(a*d + ((b*d + a*e)*x^n)/(1 + n) + ((c*d + b*e)*x^(2*n))/(1 + 2*n) + (c*
e*x^(3*n))/(1 + 3*n))
```

3.66.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1737, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)dx$$

$$\Big\downarrow \text{1737}$$

$$\int \left(x^n(ae+bd)+ad+x^{2n}(be+cd)+cex^{3n}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^{n+1}(ae+bd)}{n+1}+adx+\frac{x^{2n+1}(be+cd)}{2n+1}+\frac{cex^{3n+1}}{3n+1}$$

input
```
Int[(d + e*x^n)*(a + b*x^n + c*x^(2*n)),x]
```

output
```
a*d*x + ((b*d + a*e)*x^(1 + n))/(1 + n) + ((c*d + b*e)*x^(1 + 2*n))/(1 + 2
*n) + (c*e*x^(1 + 3*n))/(1 + 3*n)
```

3.66.3.1 Defintions of rubi rules used

rule 1737
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2
_)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))
, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.66.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.97

method	result
risch	$adx+\frac{(ae+bd)x\,x^{n}}{1+n}+\frac{(be+cd)x\,x^{2n}}{1+2n}+\frac{ecx\,x^{3n}}{1+3n}$
norman	$adx+\frac{(ae+bd)x\,e^{n\ln(x)}}{1+n}+\frac{(be+cd)x\,e^{2n\ln(x)}}{1+2n}+\frac{ecx\,e^{3n\ln(x)}}{1+3n}$
parallelrisch	$\frac{3x\,x^{2n}be\,n^{2}+2x\,x^{n}x^{2n}ce\,n^{2}+4x\,x^{2n}ben+3x\,x^{n}x^{2n}cen+6x\,x^{n}ae\,n^{2}+6x\,x^{n}bd\,n^{2}+3x\,x^{2n}cd\,n^{2}+6xad\,n^{3}+x\,x^{2n}be+x\,x^{n}x^{2n}ce}{(1+n)(1+2n)(1+3n)}$

input
```
int((d+e*x^n)*(a+b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

output
```
a*d*x+(a*e+b*d)/(1+n)*x*x^n+(b*e+c*d)/(1+2*n)*x*(x^n)^2+e*c/(1+3*n)*x*(x^n
)^3
```

3.66.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. 2(62) = 124.

Time = 0.31 (sec) , antiderivative size = 137, normalized size of antiderivative = 2.21

$$\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)\,dx$$
$$=\frac{(2\,cen^2+3\,cen+ce)xx^{3\,n}+(3\,(cd+be)n^2+cd+be+4\,(cd+be)n)xx^{2\,n}+(6\,(bd+ae)n^2+bd+ae+5}{6\,n^3+11\,n^2+6\,n+1}$$

input
```
integrate((d+e*x^n)*(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
((2*c*e*n^2 + 3*c*e*n + c*e)*x*x^(3*n) + (3*(c*d + b*e)*n^2 + c*d + b*e +
4*(c*d + b*e)*n)*x*x^(2*n) + (6*(b*d + a*e)*n^2 + b*d + a*e + 5*(b*d + a*e
)*n)*x*x^n + (6*a*d*n^3 + 11*a*d*n^2 + 6*a*d*n + a*d)*x)/(6*n^3 + 11*n^2 +
 6*n + 1)
```

3.66.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 656 vs. $2(53) = 106$.

Time = 0.39 (sec) , antiderivative size = 656, normalized size of antiderivative = 10.58

$$\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)\,dx$$
$$=\begin{cases} adx + ae\log(x) + bd\log(x) - \frac{be}{x} - \frac{cd}{x} - \frac{ce}{2x^2} \\ adx + 2ae\sqrt{x} + 2bd\sqrt{x} + be\log(x) + cd\log(x) - \frac{2ce}{\sqrt{x}} \\ adx + \frac{3aex^{\frac{2}{3}}}{2} + \frac{3bdx^{\frac{2}{3}}}{2} + 3be\sqrt[3]{x} + 3cd\sqrt[3]{x} + ce\log(x) \\ \frac{6adn^3x}{6n^3+11n^2+6n+1} + \frac{11adn^2x}{6n^3+11n^2+6n+1} + \frac{6adnx}{6n^3+11n^2+6n+1} + \frac{adx}{6n^3+11n^2+6n+1} + \frac{6aen^2xx^n}{6n^3+11n^2+6n+1} + \frac{5aenxx^n}{6n^3+11n^2+6n+1} + \frac{}{6n^3+} \end{cases}$$

input
```
integrate((d+e*x**n)*(a+b*x**n+c*x**(2*n)),x)
```

output
```
Piecewise((a*d*x + a*e*log(x) + b*d*log(x) - b*e/x - c*d/x - c*e/(2*x**2),
 Eq(n, -1)), (a*d*x + 2*a*e*sqrt(x) + 2*b*d*sqrt(x) + b*e*log(x) + c*d*log
(x) - 2*c*e/sqrt(x), Eq(n, -1/2)), (a*d*x + 3*a*e*x**(2/3)/2 + 3*b*d*x**(2
/3)/2 + 3*b*e*x**(1/3) + 3*c*d*x**(1/3) + c*e*log(x), Eq(n, -1/3)), (6*a*d
*n**3*x/(6*n**3 + 11*n**2 + 6*n + 1) + 11*a*d*n**2*x/(6*n**3 + 11*n**2 + 6
*n + 1) + 6*a*d*n*x/(6*n**3 + 11*n**2 + 6*n + 1) + a*d*x/(6*n**3 + 11*n**2
 + 6*n + 1) + 6*a*e*n**2*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 5*a*e*n*x*x
**n/(6*n**3 + 11*n**2 + 6*n + 1) + a*e*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1)
 + 6*b*d*n**2*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 5*b*d*n*x*x**n/(6*n**3
 + 11*n**2 + 6*n + 1) + b*d*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 3*b*e*n*
*2*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 4*b*e*n*x*x**(2*n)/(6*n**3 +
11*n**2 + 6*n + 1) + b*e*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 3*c*d*n
**2*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 4*c*d*n*x*x**(2*n)/(6*n**3 +
 11*n**2 + 6*n + 1) + c*d*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 2*c*e*
n**2*x*x**(3*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 3*c*e*n*x*x**(3*n)/(6*n**3
+ 11*n**2 + 6*n + 1) + c*e*x*x**(3*n)/(6*n**3 + 11*n**2 + 6*n + 1), True))
```

3.66.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.32

$$\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)\,dx = adx + \frac{cex^{3\,n+1}}{3\,n+1} + \frac{cdx^{2\,n+1}}{2\,n+1} + \frac{bex^{2\,n+1}}{2\,n+1} + \frac{bdx^{n+1}}{n+1} + \frac{aex^{n+1}}{n+1}$$

input
```
integrate((d+e*x^n)*(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
a*d*x + c*e*x^(3*n + 1)/(3*n + 1) + c*d*x^(2*n + 1)/(2*n + 1) + b*e*x^(2*n
 + 1)/(2*n + 1) + b*d*x^(n + 1)/(n + 1) + a*e*x^(n + 1)/(n + 1)
```

3.66.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 198 vs. 2(62) = 124.

Time = 0.30 (sec) , antiderivative size = 198, normalized size of antiderivative = 3.19

$$\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)\,dx$$
$$= \frac{6\,adn^3x + 2\,cen^2xx^{3\,n} + 3\,cdn^2xx^{2\,n} + 3\,ben^2xx^{2\,n} + 6\,bdn^2xx^n + 6\,aen^2xx^n + 11\,adn^2x + 3\,cenxx^{3\,n} + 4\,\cdots}{6\,n^3 + 11\cdots}$$

input
```
integrate((d+e*x^n)*(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
(6*a*d*n^3*x + 2*c*e*n^2*x*x^(3*n) + 3*c*d*n^2*x*x^(2*n) + 3*b*e*n^2*x*x^(
2*n) + 6*b*d*n^2*x*x^n + 6*a*e*n^2*x*x^n + 11*a*d*n^2*x + 3*c*e*n*x*x^(3*n
) + 4*c*d*n*x*x^(2*n) + 4*b*e*n*x*x^(2*n) + 5*b*d*n*x*x^n + 5*a*e*n*x*x^n
+ 6*a*d*n*x + c*e*x*x^(3*n) + c*d*x*x^(2*n) + b*e*x*x^(2*n) + b*d*x*x^n +
a*e*x*x^n + a*d*x)/(6*n^3 + 11*n^2 + 6*n + 1)
```

3.66.9 Mupad [B] (verification not implemented)

Time = 8.39 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.95

$$\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)\,dx = a\,d\,x + \frac{x\,x^{2\,n}\,(b\,e+c\,d)}{2\,n+1} + \frac{x\,x^n\,(a\,e+b\,d)}{n+1} + \frac{c\,e\,x\,x^{3\,n}}{3\,n+1}$$

input
```
int((d + e*x^n)*(a + b*x^n + c*x^(2*n)),x)
```

output
```
a*d*x + (x*x^(2*n)*(b*e + c*d))/(2*n + 1) + (x*x^n*(a*e + b*d))/(n + 1) +
(c*e*x*x^(3*n))/(3*n + 1)
```

3.66. $\int (d + ex^n) (a + bx^n + cx^{2n}) \, dx$

3.67 $\int (d+ex^n)(a+bx^n+cx^{2n})^2\,dx$

3.67.1 Optimal result

Integrand size = 24, antiderivative size = 132

$$\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)^2\,dx = a^2dx+\frac{a(2bd+ae)x^{1+n}}{1+n}+\frac{\left(b^2d+2acd+2abe\right)x^{1+2n}}{1+2n}+\frac{\left(2bcd+b^2e+2ace\right)x^{1+3n}}{1+3n}+\frac{c(cd+2be)x^{1+4n}}{1+4n}+\frac{c^2ex^{1+5n}}{1+5n}$$

output
```
a^2*d*x+a*(a*e+2*b*d)*x^(1+n)/(1+n)+(2*a*b*e+2*a*c*d+b^2*d)*x^(1+2*n)/(1+2
*n)+(2*a*c*e+b^2*e+2*b*c*d)*x^(1+3*n)/(1+3*n)+c*(2*b*e+c*d)*x^(1+4*n)/(1+4
*n)+c^2*e*x^(1+5*n)/(1+5*n)
```

3.67.2 Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.93

$$\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)^2\,dx = x\left(a^2d+\frac{a(2bd+ae)x^n}{1+n}+\frac{\left(b^2d+2acd+2abe\right)x^{2n}}{1+2n}+\frac{\left(2bcd+b^2e+2ace\right)x^{3n}}{1+3n}+\frac{c(cd+2be)x^{4n}}{1+4n}+\frac{c^2ex^{5n}}{1+5n}\right)$$

input
```
Integrate[(d + e*x^n)*(a + b*x^n + c*x^(2*n))^2,x]
```

output
```
x*(a^2*d + (a*(2*b*d + a*e)*x^n)/(1 + n) + ((b^2*d + 2*a*c*d + 2*a*b*e)*x^
(2*n))/(1 + 2*n) + ((2*b*c*d + b^2*e + 2*a*c*e)*x^(3*n))/(1 + 3*n) + (c*(c
*d + 2*b*e)*x^(4*n))/(1 + 4*n) + (c^2*e*x^(5*n))/(1 + 5*n))
```

3.67.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1762, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)^2\,dx$$

$$\Big\downarrow \text{1762}$$

$$\int \left(a^2d+x^{2n}\left(2abe+2acd+b^2d\right)+x^{3n}\left(2ace+b^2e+2bcd\right)+ax^n(ae+2bd)+cx^{4n}(2be+cd)+c^2ex^{5n}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$a^2dx+\frac{x^{2n+1}\left(2abe+2acd+b^2d\right)}{2n+1}+\frac{x^{3n+1}\left(2ace+b^2e+2bcd\right)}{3n+1}+\frac{ax^{n+1}(ae+2bd)}{n+1}+\frac{cx^{4n+1}(2be+cd)}{4n+1}+\frac{c^2ex^{5n+1}}{5n+1}$$

input
```
Int[(d + e*x^n)*(a + b*x^n + c*x^(2*n))^2,x]
```

output
```
a^2*d*x + (a*(2*b*d + a*e)*x^(1 + n))/(1 + n) + ((b^2*d + 2*a*c*d + 2*a*b*
e)*x^(1 + 2*n))/(1 + 2*n) + ((2*b*c*d + b^2*e + 2*a*c*e)*x^(1 + 3*n))/(1 +
 3*n) + (c*(c*d + 2*b*e)*x^(1 + 4*n))/(1 + 4*n) + (c^2*e*x^(1 + 5*n))/(1 +
 5*n)
```

3.67.3.1 Defintions of rubi rules used

rule 1762
```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p
_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)*(a + b*x^n + c*x^(2*n))^p,
 x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.67.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.97

method	result
risch	$a^2dx+\frac{(2ace+b^2e+2bcd)x\,x^{3n}}{1+3n}+\frac{(2abe+2acd+b^2d)x\,x^{2n}}{1+2n}+\frac{a(ae+2bd)x\,x^n}{1+n}+\frac{c(2be+cd)x\,x^{4n}}{1+4n}+\frac{e\,c^2x\,x^{5n}}{1+5n}$
norman	$a^2dx+\frac{(2ace+b^2e+2bcd)x\,\mathrm{e}^{3n\ln(x)}}{1+3n}+\frac{(2abe+2acd+b^2d)x\,\mathrm{e}^{2n\ln(x)}}{1+2n}+\frac{a(ae+2bd)x\,\mathrm{e}^{n\ln(x)}}{1+n}+\frac{c(2be+cd)x\,\mathrm{e}^{4n\ln(x)}}{1+4n}+$
parallelrisch	$\frac{142x\,x^n abd\,n^2+118x\,x^{2n}acd\,n^2+2x\,x^n x^{2n}ace+2x\,x^n x^{2n}bcd+28x\,x^n abdn+26x\,x^{2n}acdn+120x\,x^n a^2e\,n^4+154x\,x^n a^2e\,n^3+22}{}$

input
```
int((d+e*x^n)*(a+b*x^n+c*x^(2*n))^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*d*x+(2*a*c*e+b^2*e+2*b*c*d)/(1+3*n)*x*(x^n)^3+(2*a*b*e+2*a*c*d+b^2*d)/
(1+2*n)*x*(x^n)^2+a*(a*e+2*b*d)/(1+n)*x*x^n+c*(2*b*e+c*d)/(1+4*n)*x*(x^n)^
4+e*c^2/(1+5*n)*x*(x^n)^5
```

3.67.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 495 vs. $2(132) = 264$.

Time = 0.37 (sec) , antiderivative size = 495, normalized size of antiderivative = 3.75

$$\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)^2\,dx$$
$$=\frac{(24\,c^2en^4+50\,c^2en^3+35\,c^2en^2+10\,c^2en+c^2e)xx^{5\,n}+(30\,(c^2d+2\,bce)n^4+61\,(c^2d+2\,bce)n^3+c^2d+2}{}$$

input
```
integrate((d+e*x^n)*(a+b*x^n+c*x^(2*n))^2,x, algorithm="fricas")
```

output
```
((24*c^2*e*n^4 + 50*c^2*e*n^3 + 35*c^2*e*n^2 + 10*c^2*e*n + c^2*e)*x*x^(5*
n) + (30*(c^2*d + 2*b*c*e)*n^4 + 61*(c^2*d + 2*b*c*e)*n^3 + c^2*d + 2*b*c*
e + 41*(c^2*d + 2*b*c*e)*n^2 + 11*(c^2*d + 2*b*c*e)*n)*x*x^(4*n) + (40*(2*
b*c*d + (b^2 + 2*a*c)*e)*n^4 + 78*(2*b*c*d + (b^2 + 2*a*c)*e)*n^3 + 2*b*c*
d + 49*(2*b*c*d + (b^2 + 2*a*c)*e)*n^2 + (b^2 + 2*a*c)*e + 12*(2*b*c*d + (
b^2 + 2*a*c)*e)*n)*x*x^(3*n) + (60*(2*a*b*e + (b^2 + 2*a*c)*d)*n^4 + 107*(
2*a*b*e + (b^2 + 2*a*c)*d)*n^3 + 2*a*b*e + 59*(2*a*b*e + (b^2 + 2*a*c)*d)*
n^2 + (b^2 + 2*a*c)*d + 13*(2*a*b*e + (b^2 + 2*a*c)*d)*n)*x*x^(2*n) + (120
*(2*a*b*d + a^2*e)*n^4 + 154*(2*a*b*d + a^2*e)*n^3 + 2*a*b*d + a^2*e + 71*
(2*a*b*d + a^2*e)*n^2 + 14*(2*a*b*d + a^2*e)*n)*x*x^n + (120*a^2*d*n^5 + 2
74*a^2*d*n^4 + 225*a^2*d*n^3 + 85*a^2*d*n^2 + 15*a^2*d*n + a^2*d)*x)/(120*
n^5 + 274*n^4 + 225*n^3 + 85*n^2 + 15*n + 1)
```

3.67.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3128 vs. 2(124) = 248.

Time = 1.34 (sec) , antiderivative size = 3128, normalized size of antiderivative = 23.70

$$\int (d + ex^n) \left(a + bx^n + cx^{2n}\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate((d+e*x**n)*(a+b*x**n+c*x**(2*n))**2,x)
```

output
```
Piecewise((a**2*d*x + a**2*e*log(x) + 2*a*b*d*log(x) - 2*a*b*e/x - 2*a*c*d
/x - a*c*e/x**2 - b**2*d/x - b**2*e/(2*x**2) - b*c*d/x**2 - 2*b*c*e/(3*x**
3) - c**2*d/(3*x**3) - c**2*e/(4*x**4), Eq(n, -1)), (a**2*d*x + 2*a**2*e*s
qrt(x) + 4*a*b*d*sqrt(x) + 2*a*b*e*log(x) + 2*a*c*d*log(x) - 4*a*c*e/sqrt(
x) + b**2*d*log(x) - 2*b**2*e/sqrt(x) - 4*b*c*d/sqrt(x) - 2*b*c*e/x - c**2
*d/x - 2*c**2*e/(3*x**(3/2)), Eq(n, -1/2)), (a**2*d*x + 3*a**2*e*x**(2/3)/
2 + 3*a*b*d*x**(2/3) + 6*a*b*e*x**(1/3) + 6*a*c*d*x**(1/3) + 2*a*c*e*log(x
) + 3*b**2*d*x**(1/3) + b**2*e*log(x) + 2*b*c*d*log(x) - 6*b*c*e/x**(1/3)
- 3*c**2*d/x**(1/3) - 3*c**2*e/(2*x**(2/3)), Eq(n, -1/3)), (a**2*d*x + 4*a
**2*e*x**(3/4)/3 + 8*a*b*d*x**(3/4)/3 + 4*a*b*e*sqrt(x) + 4*a*c*d*sqrt(x)
+ 8*a*c*e*x**(1/4) + 2*b**2*d*sqrt(x) + 4*b**2*e*x**(1/4) + 8*b*c*d*x**(1/
4) + 2*b*c*e*log(x) + c**2*d*log(x) - 4*c**2*e/x**(1/4), Eq(n, -1/4)), (a*
*2*d*x + 5*a**2*e*x**(4/5)/4 + 5*a*b*d*x**(4/5)/2 + 10*a*b*e*x**(3/5)/3 +
10*a*c*d*x**(3/5)/3 + 5*a*c*e*x**(2/5) + 5*b**2*d*x**(3/5)/3 + 5*b**2*e*x*
*(2/5)/2 + 5*b*c*d*x**(2/5) + 10*b*c*e*x**(1/5) + 5*c**2*d*x**(1/5) + c**2
*e*log(x), Eq(n, -1/5)), (120*a**2*d*n**5*x/(120*n**5 + 274*n**4 + 225*n**
3 + 85*n**2 + 15*n + 1) + 274*a**2*d*n**4*x/(120*n**5 + 274*n**4 + 225*n**
3 + 85*n**2 + 15*n + 1) + 225*a**2*d*n**3*x/(120*n**5 + 274*n**4 + 225*n**
3 + 85*n**2 + 15*n + 1) + 85*a**2*d*n**2*x/(120*n**5 + 274*n**4 + 225*n**3
 + 85*n**2 + 15*n + 1) + 15*a**2*d*n*x/(120*n**5 + 274*n**4 + 225*n**3 ...
```

3.67.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.58

$$\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)^2\,dx = a^2dx+\frac{c^2ex^{5n+1}}{5n+1}+\frac{c^2dx^{4n+1}}{4n+1}+\frac{2\,bcex^{4n+1}}{4n+1}+\frac{2\,bcdx^{3n+1}}{3n+1}+\frac{b^2ex^{3n+1}}{3n+1}+\frac{2\,acex^{3n+1}}{3n+1}+\frac{b^2dx^{2n+1}}{2n+1}+\frac{2\,acdx^{2n+1}}{2n+1}+\frac{2\,abex^{2n+1}}{2n+1}+\frac{2\,abdx^{n+1}}{n+1}+\frac{a^2ex^{n+1}}{n+1}$$

input
```
integrate((d+e*x^n)*(a+b*x^n+c*x^(2*n))^2,x, algorithm="maxima")
```

output
```
a^2*d*x + c^2*e*x^(5*n + 1)/(5*n + 1) + c^2*d*x^(4*n + 1)/(4*n + 1) + 2*b*
c*e*x^(4*n + 1)/(4*n + 1) + 2*b*c*d*x^(3*n + 1)/(3*n + 1) + b^2*e*x^(3*n +
 1)/(3*n + 1) + 2*a*c*e*x^(3*n + 1)/(3*n + 1) + b^2*d*x^(2*n + 1)/(2*n + 1
) + 2*a*c*d*x^(2*n + 1)/(2*n + 1) + 2*a*b*e*x^(2*n + 1)/(2*n + 1) + 2*a*b*
d*x^(n + 1)/(n + 1) + a^2*e*x^(n + 1)/(n + 1)
```

3.67.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 798 vs. $2(132) = 264$.

Time = 0.38 (sec) , antiderivative size = 798, normalized size of antiderivative = 6.05

$$\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)^2\,dx = \frac{120\,a^2dn^5x+24\,c^2en^4xx^{5\,n}+30\,c^2dn^4xx^{4\,n}+60\,bcen^4xx^{4\,n}+80\,bcdn^4xx^{3\,n}+40\,b^2en^4xx^{3\,n}+80\,acen^4xx}{}$$

input
```
integrate((d+e*x^n)*(a+b*x^n+c*x^(2*n))^2,x, algorithm="giac")
```

output
```
(120*a^2*d*n^5*x + 24*c^2*e*n^4*x*x^(5*n) + 30*c^2*d*n^4*x*x^(4*n) + 60*b*
c*e*n^4*x*x^(4*n) + 80*b*c*d*n^4*x*x^(3*n) + 40*b^2*e*n^4*x*x^(3*n) + 80*a
*c*e*n^4*x*x^(3*n) + 60*b^2*d*n^4*x*x^(2*n) + 120*a*c*d*n^4*x*x^(2*n) + 12
0*a*b*e*n^4*x*x^(2*n) + 240*a*b*d*n^4*x*x^n + 120*a^2*e*n^4*x*x^n + 274*a^
2*d*n^4*x + 50*c^2*e*n^3*x*x^(5*n) + 61*c^2*d*n^3*x*x^(4*n) + 122*b*c*e*n^
3*x*x^(4*n) + 156*b*c*d*n^3*x*x^(3*n) + 78*b^2*e*n^3*x*x^(3*n) + 156*a*c*e
*n^3*x*x^(3*n) + 107*b^2*d*n^3*x*x^(2*n) + 214*a*c*d*n^3*x*x^(2*n) + 214*a
*b*e*n^3*x*x^(2*n) + 308*a*b*d*n^3*x*x^n + 154*a^2*e*n^3*x*x^n + 225*a^2*d
*n^3*x + 35*c^2*e*n^2*x*x^(5*n) + 41*c^2*d*n^2*x*x^(4*n) + 82*b*c*e*n^2*x*
x^(4*n) + 98*b*c*d*n^2*x*x^(3*n) + 49*b^2*e*n^2*x*x^(3*n) + 98*a*c*e*n^2*x
*x^(3*n) + 59*b^2*d*n^2*x*x^(2*n) + 118*a*c*d*n^2*x*x^(2*n) + 118*a*b*e*n^
2*x*x^(2*n) + 142*a*b*d*n^2*x*x^n + 71*a^2*e*n^2*x*x^n + 85*a^2*d*n^2*x +
10*c^2*e*n*x*x^(5*n) + 11*c^2*d*n*x*x^(4*n) + 22*b*c*e*n*x*x^(4*n) + 24*b*
c*d*n*x*x^(3*n) + 12*b^2*e*n*x*x^(3*n) + 24*a*c*e*n*x*x^(3*n) + 13*b^2*d*n
*x*x^(2*n) + 26*a*c*d*n*x*x^(2*n) + 26*a*b*e*n*x*x^(2*n) + 28*a*b*d*n*x*x^
n + 14*a^2*e*n*x*x^n + 15*a^2*d*n*x + c^2*e*x*x^(5*n) + c^2*d*x*x^(4*n) +
2*b*c*e*x*x^(4*n) + 2*b*c*d*x*x^(3*n) + b^2*e*x*x^(3*n) + 2*a*c*e*x*x^(3*n
) + b^2*d*x*x^(2*n) + 2*a*c*d*x*x^(2*n) + 2*a*b*e*x*x^(2*n) + 2*a*b*d*x*x^
n + a^2*e*x*x^n + a^2*d*x)/(120*n^5 + 274*n^4 + 225*n^3 + 85*n^2 + 15*n +
1)
```

3.67.9 Mupad [B] (verification not implemented)

Time = 8.49 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.99

$$\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)^2\,dx = a^2\,d\,x+\frac{x\,x^{4\,n}\,\left(d\,c^2+2\,b\,e\,c\right)}{4\,n+1}+\frac{x\,x^n\,\left(e\,a^2+2\,b\,d\,a\right)}{n+1}+\frac{x\,x^{2\,n}\,\left(d\,b^2+2\,a\,e\,b+2\,a\,c\,d\right)}{2\,n+1}+\frac{x\,x^{3\,n}\,\left(e\,b^2+2\,c\,d\,b+2\,a\,c\,e\right)}{3\,n+1}+\frac{c^2\,e\,x\,x^{5\,n}}{5\,n+1}$$

input
```
int((d + e*x^n)*(a + b*x^n + c*x^(2*n))^2,x)
```

output
```
a^2*d*x + (x*x^(4*n)*(c^2*d + 2*b*c*e))/(4*n + 1) + (x*x^n*(a^2*e + 2*a*b*
d))/(n + 1) + (x*x^(2*n)*(b^2*d + 2*a*b*e + 2*a*c*d))/(2*n + 1) + (x*x^(3*
n)*(b^2*e + 2*a*c*e + 2*b*c*d))/(3*n + 1) + (c^2*e*x*x^(5*n))/(5*n + 1)
```

3.68 $\int (d+ex^n) (a+bx^n+cx^{2n})^3 \, dx$

3.68.1 Optimal result

Integrand size = 24, antiderivative size = 218

$$\int (d+ex^n) (a+bx^n+cx^{2n})^3 \, dx = a^3dx + \frac{a^2(3bd+ae)x^{1+n}}{1+n} + \frac{3a(b^2d+acd+abe)\, x^{1+2n}}{1+2n} + \frac{(b^3d+6abcd+3ab^2e+3a^2ce)\, x^{1+3n}}{1+3n} + \frac{(3b^2cd+3ac^2d+b^3e+6abce)\, x^{1+4n}}{1+4n} + \frac{3c(bcd+b^2e+ace)\, x^{1+5n}}{1+5n} + \frac{c^2(cd+3be)x^{1+6n}}{1+6n} + \frac{c^3ex^{1+7n}}{1+7n}$$

output
```
a^3*d*x+a^2*(a*e+3*b*d)*x^(1+n)/(1+n)+3*a*(a*b*e+a*c*d+b^2*d)*x^(1+2*n)/(1
+2*n)+(3*a^2*c*e+3*a*b^2*e+6*a*b*c*d+b^3*d)*x^(1+3*n)/(1+3*n)+(6*a*b*c*e+3
*a*c^2*d+b^3*e+3*b^2*c*d)*x^(1+4*n)/(1+4*n)+3*c*(a*c*e+b^2*e+b*c*d)*x^(1+5
*n)/(1+5*n)+c^2*(3*b*e+c*d)*x^(1+6*n)/(1+6*n)+c^3*e*x^(1+7*n)/(1+7*n)
```

3.68.2 Mathematica [A] (verified)

Time = 3.84 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.94

$$\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)^3\,dx=x\left(a^3d+\frac{a^2(3bd+ae)x^n}{1+n}+\frac{3a(b^2d+acd+abe)\,x^{2n}}{1+2n}+\frac{(b^3d+6abcd+3ab^2e+3a^2ce)\,x^{3n}}{1+3n}+\frac{(3b^2cd+3ac^2d+b^3e+6abce)\,x^{4n}}{1+4n}+\frac{3c(bcd+b^2e+ace)\,x^{5n}}{1+5n}+\frac{c^2(cd+3be)x^{6n}}{1+6n}+\frac{c^3ex^{7n}}{1+7n}\right)$$

input
```
Integrate[(d + e*x^n)*(a + b*x^n + c*x^(2*n))^3,x]
```

output
```
x*(a^3*d + (a^2*(3*b*d + a*e)*x^n)/(1 + n) + (3*a*(b^2*d + a*c*d + a*b*e)*
x^(2*n))/(1 + 2*n) + ((b^3*d + 6*a*b*c*d + 3*a*b^2*e + 3*a^2*c*e)*x^(3*n))
/(1 + 3*n) + ((3*b^2*c*d + 3*a*c^2*d + b^3*e + 6*a*b*c*e)*x^(4*n))/(1 + 4*
n) + (3*c*(b*c*d + b^2*e + a*c*e)*x^(5*n))/(1 + 5*n) + (c^2*(c*d + 3*b*e)*
x^(6*n))/(1 + 6*n) + (c^3*e*x^(7*n))/(1 + 7*n))
```

3.68.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1762, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)^3\,dx$$

$$\Big\downarrow \text{ 1762}$$

$$\int \left(a^3d+x^{3n}\left(3a^2ce+3ab^2e+6abcd+b^3d\right)+a^2x^n(ae+3bd)+3ax^{2n}\left(abe+acd+b^2d\right)+3cx^{5n}\left(ace+b^2e+bc\right.\right.$$

$$\Big\downarrow \text{ 2009}$$

$$a^3dx+\frac{x^{3n+1}(3a^2ce+3ab^2e+6abcd+b^3d)}{3n+1}+\frac{a^2x^{n+1}(ae+3bd)}{n+1}+\frac{3ax^{2n+1}(abe+acd+b^2d)}{2n+1}+\frac{3cx^{5n+1}(ace+b^2e+bcd)}{5n+1}+\frac{x^{4n+1}(6abce+3ac^2d+b^3e+3b^2cd)}{4n+1}+\frac{c^2x^{6n+1}(3be+cd)}{6n+1}+\frac{c^3ex^{7n+1}}{7n+1}$$

input
```
Int[(d + e*x^n)*(a + b*x^n + c*x^(2*n))^3,x]
```

output
```
a^3*d*x + (a^2*(3*b*d + a*e)*x^(1 + n))/(1 + n) + (3*a*(b^2*d + a*c*d + a*
b*e)*x^(1 + 2*n))/(1 + 2*n) + ((b^3*d + 6*a*b*c*d + 3*a*b^2*e + 3*a^2*c*e)
*x^(1 + 3*n))/(1 + 3*n) + ((3*b^2*c*d + 3*a*c^2*d + b^3*e + 6*a*b*c*e)*x^(
1 + 4*n))/(1 + 4*n) + (3*c*(b*c*d + b^2*e + a*c*e)*x^(1 + 5*n))/(1 + 5*n)
+ (c^2*(c*d + 3*b*e)*x^(1 + 6*n))/(1 + 6*n) + (c^3*e*x^(1 + 7*n))/(1 + 7*n
)
```

3.68.3.1 Defintions of rubi rules used

rule 1762
```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p
_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)*(a + b*x^n + c*x^(2*n))^p,
 x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.68.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.97

method	result
risch	$a^3dx+\frac{(6abce+3a\,c^2d+b^3e+3b^2cd)x\,x^{4n}}{1+4n}+\frac{(3a^2ce+3a\,b^2e+6abcd+b^3d)x\,x^{3n}}{1+3n}+\frac{a^2(ae+3bd)x\,x^n}{1+n}+\frac{c^2(3be+cd)x\,x^{6n}}{1+6n}$
norman	$a^3dx+\frac{(6abce+3a\,c^2d+b^3e+3b^2cd)x\,e^{4n\ln(x)}}{1+4n}+\frac{(3a^2ce+3a\,b^2e+6abcd+b^3d)x\,e^{3n\ln(x)}}{1+3n}+\frac{a^2(ae+3bd)x\,e^{n\ln(x)}}{1+n}+\frac{c^2(}{}$
parallelrisch	Expression too large to display

input
```
int((d+e*x^n)*(a+b*x^n+c*x^(2*n))^3,x,method=_RETURNVERBOSE)
```

output
```
a^3*d*x+(6*a*b*c*e+3*a*c^2*d+b^3*e+3*b^2*c*d)/(1+4*n)*x*(x^n)^4+(3*a^2*c*e
+3*a*b^2*e+6*a*b*c*d+b^3*d)/(1+3*n)*x*(x^n)^3+a^2*(a*e+3*b*d)/(1+n)*x*x^n+
c^2*(3*b*e+c*d)/(1+6*n)*x*(x^n)^6+c^3*e/(1+7*n)*x*(x^n)^7+3*a*(a*b*e+a*c*d
+b^2*d)/(1+2*n)*x*(x^n)^2+3*c*(a*c*e+b^2*e+b*c*d)/(1+5*n)*x*(x^n)^5
```

3.68.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1209 vs. $2(218) = 436$.

Time = 0.37 (sec) , antiderivative size = 1209, normalized size of antiderivative = 5.55

$$\int (d + ex^n) \left(a + bx^n + cx^{2n}\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate((d+e*x^n)*(a+b*x^n+c*x^(2*n))^3,x, algorithm="fricas")
```

output
```
((720*c^3*e*n^6 + 1764*c^3*e*n^5 + 1624*c^3*e*n^4 + 735*c^3*e*n^3 + 175*c^
3*e*n^2 + 21*c^3*e*n + c^3*e)*x*x^(7*n) + (840*(c^3*d + 3*b*c^2*e)*n^6 + 2
038*(c^3*d + 3*b*c^2*e)*n^5 + 1849*(c^3*d + 3*b*c^2*e)*n^4 + c^3*d + 3*b*c
^2*e + 820*(c^3*d + 3*b*c^2*e)*n^3 + 190*(c^3*d + 3*b*c^2*e)*n^2 + 22*(c^3
*d + 3*b*c^2*e)*n)*x*x^(6*n) + 3*(1008*(b*c^2*d + (b^2*c + a*c^2)*e)*n^6 +
 2412*(b*c^2*d + (b^2*c + a*c^2)*e)*n^5 + 2144*(b*c^2*d + (b^2*c + a*c^2)*
e)*n^4 + b*c^2*d + 925*(b*c^2*d + (b^2*c + a*c^2)*e)*n^3 + 207*(b*c^2*d +
(b^2*c + a*c^2)*e)*n^2 + (b^2*c + a*c^2)*e + 23*(b*c^2*d + (b^2*c + a*c^2)
*e)*n)*x*x^(5*n) + (1260*(3*(b^2*c + a*c^2)*d + (b^3 + 6*a*b*c)*e)*n^6 + 2
952*(3*(b^2*c + a*c^2)*d + (b^3 + 6*a*b*c)*e)*n^5 + 2545*(3*(b^2*c + a*c^2
)*d + (b^3 + 6*a*b*c)*e)*n^4 + 1056*(3*(b^2*c + a*c^2)*d + (b^3 + 6*a*b*c)
*e)*n^3 + 226*(3*(b^2*c + a*c^2)*d + (b^3 + 6*a*b*c)*e)*n^2 + 3*(b^2*c + a
*c^2)*d + (b^3 + 6*a*b*c)*e + 24*(3*(b^2*c + a*c^2)*d + (b^3 + 6*a*b*c)*e)
*n)*x*x^(4*n) + (1680*((b^3 + 6*a*b*c)*d + 3*(a*b^2 + a^2*c)*e)*n^6 + 3796
*((b^3 + 6*a*b*c)*d + 3*(a*b^2 + a^2*c)*e)*n^5 + 3112*((b^3 + 6*a*b*c)*d +
 3*(a*b^2 + a^2*c)*e)*n^4 + 1219*((b^3 + 6*a*b*c)*d + 3*(a*b^2 + a^2*c)*e)
*n^3 + 247*((b^3 + 6*a*b*c)*d + 3*(a*b^2 + a^2*c)*e)*n^2 + (b^3 + 6*a*b*c)
*d + 3*(a*b^2 + a^2*c)*e + 25*((b^3 + 6*a*b*c)*d + 3*(a*b^2 + a^2*c)*e)*n)
*x*x^(3*n) + 3*(2520*(a^2*b*e + (a*b^2 + a^2*c)*d)*n^6 + 5274*(a^2*b*e + (
a*b^2 + a^2*c)*d)*n^5 + 3929*(a^2*b*e + (a*b^2 + a^2*c)*d)*n^4 + a^2*b*...
```

3.68.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9190 vs. $2(212) = 424$.

Time = 5.81 (sec) , antiderivative size = 9190, normalized size of antiderivative = 42.16

$$\int (d + ex^n) \left(a + bx^n + cx^{2n}\right)^3 dx = \text{Too large to display}$$

input
```
integrate((d+e*x**n)*(a+b*x**n+c*x**(2*n))**3,x)
```

output
```
Piecewise((a**3*d*x + a**3*e*log(x) + 3*a**2*b*d*log(x) - 3*a**2*b*e/x - 3
*a**2*c*d/x - 3*a**2*c*e/(2*x**2) - 3*a*b**2*d/x - 3*a*b**2*e/(2*x**2) - 3
*a*b*c*d/x**2 - 2*a*b*c*e/x**3 - a*c**2*d/x**3 - 3*a*c**2*e/(4*x**4) - b**
3*d/(2*x**2) - b**3*e/(3*x**3) - b**2*c*d/x**3 - 3*b**2*c*e/(4*x**4) - 3*b
*c**2*d/(4*x**4) - 3*b*c**2*e/(5*x**5) - c**3*d/(5*x**5) - c**3*e/(6*x**6)
, Eq(n, -1)), (a**3*d*x + 2*a**3*e*sqrt(x) + 6*a**2*b*d*sqrt(x) + 3*a**2*b
*e*log(x) + 3*a**2*c*d*log(x) - 6*a**2*c*e/sqrt(x) + 3*a*b**2*d*log(x) - 6
*a*b**2*e/sqrt(x) - 12*a*b*c*d/sqrt(x) - 6*a*b*c*e/x - 3*a*c**2*d/x - 2*a*
c**2*e/x**(3/2) - 2*b**3*d/sqrt(x) - b**3*e/x - 3*b**2*c*d/x - 2*b**2*c*e/
x**(3/2) - 2*b*c**2*d/x**(3/2) - 3*b*c**2*e/(2*x**2) - c**3*d/(2*x**2) - 2
*c**3*e/(5*x**(5/2)), Eq(n, -1/2)), (a**3*d*x + 3*a**3*e*x**(2/3)/2 + 9*a*
*2*b*d*x**(2/3)/2 + 9*a**2*b*e*x**(1/3) + 9*a**2*c*d*x**(1/3) + 3*a**2*c*e
*log(x) + 9*a*b**2*d*x**(1/3) + 3*a*b**2*e*log(x) + 6*a*b*c*d*log(x) - 18*
a*b*c*e/x**(1/3) - 9*a*c**2*d/x**(1/3) - 9*a*c**2*e/(2*x**(2/3)) + b**3*d*
log(x) - 3*b**3*e/x**(1/3) - 9*b**2*c*d/x**(1/3) - 9*b**2*c*e/(2*x**(2/3))
 - 9*b*c**2*d/(2*x**(2/3)) - 3*b*c**2*e/x - c**3*d/x - 3*c**3*e/(4*x**(4/3
)), Eq(n, -1/3)), (a**3*d*x + 4*a**3*e*x**(3/4)/3 + 4*a**2*b*d*x**(3/4) +
6*a**2*b*e*sqrt(x) + 6*a**2*c*d*sqrt(x) + 12*a**2*c*e*x**(1/4) + 6*a*b**2*
d*sqrt(x) + 12*a*b**2*e*x**(1/4) + 24*a*b*c*d*x**(1/4) + 6*a*b*c*e*log(x)
+ 3*a*c**2*d*log(x) - 12*a*c**2*e/x**(1/4) + 4*b**3*d*x**(1/4) + b**3*e...
```

3.68.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.77

$$\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)^3\,dx = a^3dx + \frac{c^3ex^{7n+1}}{7n+1} + \frac{c^3dx^{6n+1}}{6n+1} + \frac{3bc^2ex^{6n+1}}{6n+1} + \frac{3bc^2dx^{5n+1}}{5n+1} + \frac{3b^2cex^{5n+1}}{5n+1} + \frac{3ac^2ex^{5n+1}}{5n+1} + \frac{3b^2cdx^{4n+1}}{4n+1} + \frac{3ac^2dx^{4n+1}}{4n+1} + \frac{b^3ex^{4n+1}}{4n+1} + \frac{6abcex^{4n+1}}{4n+1} + \frac{b^3dx^{3n+1}}{3n+1} + \frac{6abcdx^{3n+1}}{3n+1} + \frac{3ab^2ex^{3n+1}}{3n+1} + \frac{3a^2cex^{3n+1}}{3n+1} + \frac{3ab^2dx^{2n+1}}{2n+1} + \frac{3a^2cdx^{2n+1}}{2n+1} + \frac{3a^2bex^{2n+1}}{2n+1} + \frac{3a^2bdx^{n+1}}{n+1} + \frac{a^3ex^{n+1}}{n+1}$$

input
```
integrate((d+e*x^n)*(a+b*x^n+c*x^(2*n))^3,x, algorithm="maxima")
```

output
```
a^3*d*x + c^3*e*x^(7*n + 1)/(7*n + 1) + c^3*d*x^(6*n + 1)/(6*n + 1) + 3*b*
c^2*e*x^(6*n + 1)/(6*n + 1) + 3*b*c^2*d*x^(5*n + 1)/(5*n + 1) + 3*b^2*c*e*
x^(5*n + 1)/(5*n + 1) + 3*a*c^2*e*x^(5*n + 1)/(5*n + 1) + 3*b^2*c*d*x^(4*n
 + 1)/(4*n + 1) + 3*a*c^2*d*x^(4*n + 1)/(4*n + 1) + b^3*e*x^(4*n + 1)/(4*n
 + 1) + 6*a*b*c*e*x^(4*n + 1)/(4*n + 1) + b^3*d*x^(3*n + 1)/(3*n + 1) + 6*
a*b*c*d*x^(3*n + 1)/(3*n + 1) + 3*a*b^2*e*x^(3*n + 1)/(3*n + 1) + 3*a^2*c*
e*x^(3*n + 1)/(3*n + 1) + 3*a*b^2*d*x^(2*n + 1)/(2*n + 1) + 3*a^2*c*d*x^(2
*n + 1)/(2*n + 1) + 3*a^2*b*e*x^(2*n + 1)/(2*n + 1) + 3*a^2*b*d*x^(n + 1)/
(n + 1) + a^3*e*x^(n + 1)/(n + 1)
```

3.68.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2064 vs. 2(218) = 436.

Time = 0.38 (sec) , antiderivative size = 2064, normalized size of antiderivative = 9.47

$$\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)^3\,dx = \text{Too large to display}$$

input
```
integrate((d+e*x^n)*(a+b*x^n+c*x^(2*n))^3,x, algorithm="giac")
```

output
```
(5040*a^3*d*n^7*x + 720*c^3*e*n^6*x*x^(7*n) + 840*c^3*d*n^6*x*x^(6*n) + 25
20*b*c^2*e*n^6*x*x^(6*n) + 3024*b*c^2*d*n^6*x*x^(5*n) + 3024*b^2*c*e*n^6*x
*x^(5*n) + 3024*a*c^2*e*n^6*x*x^(5*n) + 3780*b^2*c*d*n^6*x*x^(4*n) + 3780*
a*c^2*d*n^6*x*x^(4*n) + 1260*b^3*e*n^6*x*x^(4*n) + 7560*a*b*c*e*n^6*x*x^(4
*n) + 1680*b^3*d*n^6*x*x^(3*n) + 10080*a*b*c*d*n^6*x*x^(3*n) + 5040*a*b^2*
e*n^6*x*x^(3*n) + 5040*a^2*c*e*n^6*x*x^(3*n) + 7560*a*b^2*d*n^6*x*x^(2*n)
+ 7560*a^2*c*d*n^6*x*x^(2*n) + 7560*a^2*b*e*n^6*x*x^(2*n) + 15120*a^2*b*d*
n^6*x*x^n + 5040*a^3*e*n^6*x*x^n + 13068*a^3*d*n^6*x + 1764*c^3*e*n^5*x*x^
(7*n) + 2038*c^3*d*n^5*x*x^(6*n) + 6114*b*c^2*e*n^5*x*x^(6*n) + 7236*b*c^2
*d*n^5*x*x^(5*n) + 7236*b^2*c*e*n^5*x*x^(5*n) + 7236*a*c^2*e*n^5*x*x^(5*n)
 + 8856*b^2*c*d*n^5*x*x^(4*n) + 8856*a*c^2*d*n^5*x*x^(4*n) + 2952*b^3*e*n^
5*x*x^(4*n) + 17712*a*b*c*e*n^5*x*x^(4*n) + 3796*b^3*d*n^5*x*x^(3*n) + 227
76*a*b*c*d*n^5*x*x^(3*n) + 11388*a*b^2*e*n^5*x*x^(3*n) + 11388*a^2*c*e*n^5
*x*x^(3*n) + 15822*a*b^2*d*n^5*x*x^(2*n) + 15822*a^2*c*d*n^5*x*x^(2*n) + 1
5822*a^2*b*e*n^5*x*x^(2*n) + 24084*a^2*b*d*n^5*x*x^n + 8028*a^3*e*n^5*x*x^
n + 13132*a^3*d*n^5*x + 1624*c^3*e*n^4*x*x^(7*n) + 1849*c^3*d*n^4*x*x^(6*n
) + 5547*b*c^2*e*n^4*x*x^(6*n) + 6432*b*c^2*d*n^4*x*x^(5*n) + 6432*b^2*c*e
*n^4*x*x^(5*n) + 6432*a*c^2*e*n^4*x*x^(5*n) + 7635*b^2*c*d*n^4*x*x^(4*n) +
 7635*a*c^2*d*n^4*x*x^(4*n) + 2545*b^3*e*n^4*x*x^(4*n) + 15270*a*b*c*e*n^4
*x*x^(4*n) + 3112*b^3*d*n^4*x*x^(3*n) + 18672*a*b*c*d*n^4*x*x^(3*n) + 9...
```

3.68.9 Mupad [B] (verification not implemented)

Time = 8.55 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.04

$$\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)^3\,dx = a^3\,d\,x+\frac{x\,x^n\,\left(e\,a^3+3\,b\,d\,a^2\right)}{n+1}+\frac{x\,x^{2\,n}\,\left(3\,e\,a^2\,b+3\,c\,d\,a^2+3\,d\,a\,b^2\right)}{2\,n+1}+\frac{x\,x^{5\,n}\,\left(3\,e\,b^2\,c+3\,d\,b\,c^2+3\,a\,e\,c^2\right)}{5\,n+1}+\frac{x\,x^{3\,n}\,\left(3\,c\,e\,a^2+3\,e\,a\,b^2+6\,c\,d\,a\,b+d\,b^3\right)}{3\,n+1}+\frac{x\,x^{4\,n}\,\left(e\,b^3+3\,d\,b^2\,c+6\,a\,e\,b\,c+3\,a\,d\,c^2\right)}{4\,n+1}+\frac{x\,x^{6\,n}\,\left(d\,c^3+3\,b\,e\,c^2\right)}{6\,n+1}+\frac{c^3\,e\,x\,x^{7\,n}}{7\,n+1}$$

input
```
int((d + e*x^n)*(a + b*x^n + c*x^(2*n))^3,x)
```

output
```
a^3*d*x + (x*x^n*(a^3*e + 3*a^2*b*d))/(n + 1) + (x*x^(2*n)*(3*a*b^2*d + 3*
a^2*b*e + 3*a^2*c*d))/(2*n + 1) + (x*x^(5*n)*(3*a*c^2*e + 3*b*c^2*d + 3*b^
2*c*e))/(5*n + 1) + (x*x^(3*n)*(b^3*d + 3*a*b^2*e + 3*a^2*c*e + 6*a*b*c*d)
)/(3*n + 1) + (x*x^(4*n)*(b^3*e + 3*a*c^2*d + 3*b^2*c*d + 6*a*b*c*e))/(4*n
 + 1) + (x*x^(6*n)*(c^3*d + 3*b*c^2*e))/(6*n + 1) + (c^3*e*x*x^(7*n))/(7*n
 + 1)
```

3.68. $\int (d + ex^n) (a + bx^n + cx^{2n})^3 \, dx$

3.69 $\int \frac{(d+ex^n)^3}{a+bx^n+cx^{2n}}\,dx$

3.69.1 Optimal result

Integrand size = 26, antiderivative size = 308

$$\int \frac{(d+ex^n)^3}{a+bx^n+cx^{2n}}\,dx = \frac{e^2(3cd-be)x}{c^2}+\frac{e^3x^{1+n}}{c(1+n)} + \frac{\left(3c^2d^2e-3bcde^2+b^2e^3-ace^3+\frac{(2cd-be)(c^2d^2+b^2e^2-ce(bd+3ae))}{\sqrt{b^2-4ac}}\right)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2}{b-\sqrt{}}\right.}{c^2\left(b-\sqrt{b^2-4ac}\right)} + \frac{\left(3c^2d^2e-3bcde^2+b^2e^3-ace^3-\frac{(2cd-be)(c^2d^2+b^2e^2-ce(bd+3ae))}{\sqrt{b^2-4ac}}\right)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2}{b+\sqrt{}}\right.}{c^2\left(b+\sqrt{b^2-4ac}\right)}$$

output
```
e^2*(-b*e+3*c*d)*x/c^2+e^3*x^(1+n)/c/(1+n)+x*hypergeom([1, 1/n],[1+1/n],-2
*c*x^n/(b-(-4*a*c+b^2)^(1/2)))*(3*c^2*d^2*e-3*b*c*d*e^2+b^2*e^3-a*c*e^3+(-
b*e+2*c*d)*(c^2*d^2+b^2*e^2-c*e*(3*a*e+b*d))/(-4*a*c+b^2)^(1/2))/c^2/(b-(-
4*a*c+b^2)^(1/2))+x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1
/2)))*(3*c^2*d^2*e-3*b*c*d*e^2+b^2*e^3-a*c*e^3-(-b*e+2*c*d)*(c^2*d^2+b^2*e
^2-c*e*(3*a*e+b*d))/(-4*a*c+b^2)^(1/2))/c^2/(b+(-4*a*c+b^2)^(1/2))
```

3.69.2 Mathematica [A] (verified)

Time = 1.88 (sec) , antiderivative size = 295, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex^n)^3}{a+bx^n+cx^{2n}}\,dx = \frac{x\left(e^2(3cd-be)+\frac{ce^3x^n}{1+n}+\frac{\left(3c^2d^2e-3bcde^2+b^2e^3-ace^3+\frac{(2cd-be)\left(c^2d^2+b^2e^2-ce(bd+3ae)\right)}{\sqrt{b^2-4ac}}\right)\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},\frac{2cx^n}{-b+\sqrt{b^2-}}\right.}{b-\sqrt{b^2-4ac}}\right.}{c^2}$$

input
```
Integrate[(d + e*x^n)^3/(a + b*x^n + c*x^(2*n)),x]
```

output
```
(x*(e^2*(3*c*d - b*e) + (c*e^3*x^n)/(1 + n) + ((3*c^2*d^2*e - 3*b*c*d*e^2
+ b^2*e^3 - a*c*e^3 + ((2*c*d - b*e)*(c^2*d^2 + b^2*e^2 - c*e*(b*d + 3*a*e
)))/Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (2*c*x^n)/
(-b + Sqrt[b^2 - 4*a*c])])/(b - Sqrt[b^2 - 4*a*c]) + ((3*c^2*d^2*e - 3*b*c
*d*e^2 + b^2*e^3 - a*c*e^3 + ((-2*c*d + b*e)*(c^2*d^2 + b^2*e^2 - c*e*(b*d
 + 3*a*e)))/Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-
2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(b + Sqrt[b^2 - 4*a*c])))/c^2
```

3.69.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1754, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^n)^3}{a+bx^n+cx^{2n}}\,dx$$

$$\Big\downarrow \; 1754$$

$$\int \left(\frac{x^n\left(-ace^3+b^2e^3-3bcde^2+3c^2d^2e\right)+abe^3-3acde^2+c^2d^3}{c^2\left(a+bx^n+cx^{2n}\right)}+\frac{e^2(3cd-be)}{c^2}+\frac{e^3x^n}{c}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{x\left(\frac{(2cd-be)(-ce(3ae+bd)+b^2e^2+c^2d^2)}{\sqrt{b^2-4ac}}-ace^3+b^2e^3-3bcde^2+3c^2d^2e\right)\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{c^2\left(b-\sqrt{b^2-4ac}\right)}$$
$$\frac{x\left(-\frac{(2cd-be)(-ce(3ae+bd)+b^2e^2+c^2d^2e)}{\sqrt{b^2-4ac}}-ace^3+b^2e^3-3bcde^2+3c^2d^2e\right)\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b+\sqrt{b^2-4a}}\right.}{c^2\left(\sqrt{b^2-4ac}+b\right)}$$
$$\frac{e^2x(3cd-be)}{c^2}+\frac{e^3x^{n+1}}{c(n+1)}$$

input
```
Int[(d + e*x^n)^3/(a + b*x^n + c*x^(2*n)),x]
```

output
```
(e^2*(3*c*d - b*e)*x)/c^2 + (e^3*x^(1 + n))/(c*(1 + n)) + ((3*c^2*d^2*e -
3*b*c*d*e^2 + b^2*e^3 - a*c*e^3 + ((2*c*d - b*e)*(c^2*d^2 + b^2*e^2 - c*e*
(b*d + 3*a*e)))/Sqrt[b^2 - 4*a*c])*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-
1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(c^2*(b - Sqrt[b^2 - 4*a*c])) + (
(3*c^2*d^2*e - 3*b*c*d*e^2 + b^2*e^3 - a*c*e^3 - ((2*c*d - b*e)*(c^2*d^2 +
 b^2*e^2 - c*e*(b*d + 3*a*e)))/Sqrt[b^2 - 4*a*c])*x*Hypergeometric2F1[1, n
^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(c^2*(b + Sqrt[b^2
 - 4*a*c]))
```

3.69.3.1 Defintions of rubi rules used

rule 1754
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q/(a + b*x^n + c*x^(2*n)),
 x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.69.4 Maple [F]

$$\int \frac{(d+e\,x^n)^3}{a+b\,x^n+c\,x^{2n}}dx$$

input
```
int((d+e*x^n)^3/(a+b*x^n+c*x^(2*n)),x)
```

output
```
int((d+e*x^n)^3/(a+b*x^n+c*x^(2*n)),x)
```

3.69.5 Fricas [F]

$$\int \frac{(d+ex^n)^3}{a+bx^n+cx^{2n}}\,dx = \int \frac{(ex^n+d)^3}{cx^{2n}+bx^n+a}\,dx$$

input
```
integrate((d+e*x^n)^3/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
integral((e^3*x^(3*n) + 3*d*e^2*x^(2*n) + 3*d^2*e*x^n + d^3)/(c*x^(2*n) +
b*x^n + a), x)
```

3.69.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^n)^3}{a+bx^n+cx^{2n}}\,dx = \text{Timed out}$$

input
```
integrate((d+e*x**n)**3/(a+b*x**n+c*x**(2*n)),x)
```

output
```
Timed out
```

3.69.7 Maxima [F]

$$\int \frac{(d+ex^n)^3}{a+bx^n+cx^{2n}}\,dx = \int \frac{(ex^n+d)^3}{cx^{2n}+bx^n+a}\,dx$$

input
```
integrate((d+e*x^n)^3/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
(c*e^3*x*x^n + (3*c*d*e^2*(n + 1) - b*e^3*(n + 1))*x)/(c^2*(n + 1)) - inte
grate(-(c^2*d^3 - (3*c*d*e^2 - b*e^3)*a + (3*c^2*d^2*e - 3*b*c*d*e^2 + b^2
*e^3 - a*c*e^3)*x^n)/(c^3*x^(2*n) + b*c^2*x^n + a*c^2), x)
```

3.69.8 Giac [F]

$$\int \frac{(d+ex^n)^3}{a+bx^n+cx^{2n}}\,dx = \int \frac{(ex^n+d)^3}{cx^{2n}+bx^n+a}\,dx$$

input
```
integrate((d+e*x^n)^3/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate((e*x^n + d)^3/(c*x^(2*n) + b*x^n + a), x)
```

3.69.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^n)^3}{a+bx^n+cx^{2n}}\,dx = \int \frac{(d+e\,x^n)^3}{a+b\,x^n+c\,x^{2n}}\,dx$$

input
```
int((d + e*x^n)^3/(a + b*x^n + c*x^(2*n)),x)
```

output
```
int((d + e*x^n)^3/(a + b*x^n + c*x^(2*n)), x)
```

3.70 $\int \frac{(d+ex^n)^2}{a+bx^n+cx^{2n}}\,dx$

3.70.1 Optimal result

Integrand size = 26, antiderivative size = 224

$$\int \frac{(d+ex^n)^2}{a+bx^n+cx^{2n}}\,dx = \frac{e^2x}{c} + \frac{\left(2cde-be^2+\frac{2c^2d^2+b^2e^2-2ce(bd+ae)}{\sqrt{b^2-4ac}}\right)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{c\left(b-\sqrt{b^2-4ac}\right)} + \frac{\left(2cde-be^2-\frac{2c^2d^2+b^2e^2-2ce(bd+ae)}{\sqrt{b^2-4ac}}\right)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{c\left(b+\sqrt{b^2-4ac}\right)}$$

output
```
e^2*x/c+x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))*(2*c
*d*e-b*e^2+(2*c^2*d^2+b^2*e^2-2*c*e*(a*e+b*d))/(-4*a*c+b^2)^(1/2))/c/(b-(-
4*a*c+b^2)^(1/2))+x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1
/2)))*(2*c*d*e-b*e^2+(-2*c^2*d^2-b^2*e^2+2*c*e*(a*e+b*d))/(-4*a*c+b^2)^(1/
2))/c/(b+(-4*a*c+b^2)^(1/2))
```

3.70.2 Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex^n)^2}{a+bx^n+cx^{2n}}\,dx = \frac{x\left(e^2+\frac{\left(2cde-be^2+\frac{2c^2d^2+b^2e^2-2ce(bd+ae)}{\sqrt{b^2-4ac}}\right)\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)}{b-\sqrt{b^2-4ac}}+\frac{\left(2cde-be^2-\frac{2c^2d^2+b^2e^2-2ce(bd+ae)}{\sqrt{b^2-4ac}}\right)\text{Hyp}}{b+\sqrt{b}}\right.}{c}$$

input
```
Integrate[(d + e*x^n)^2/(a + b*x^n + c*x^(2*n)),x]
```

output
```
(x*(e^2 + ((2*c*d*e - b*e^2 + (2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))/Sq
rt[b^2 - 4*a*c])*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (2*c*x^n)/(-b +
Sqrt[b^2 - 4*a*c])])/(b - Sqrt[b^2 - 4*a*c]) + ((2*c*d*e - b*e^2 - (2*c^2*
d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))/Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1,
 n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(b + Sqrt[b^2 -
4*a*c])))/c
```

3.70.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1754, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^n)^2}{a+bx^n+cx^{2n}}\,dx$$

$$\Big\downarrow \text{1754}$$

$$\int \left(\frac{-ae^2+x^n\left(2cde-be^2\right)+cd^2}{c\left(a+bx^n+cx^{2n}\right)}+\frac{e^2}{c}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x\left(\frac{-2ce(ae+bd)+b^2e^2+2c^2d^2}{\sqrt{b^2-4ac}}-be^2+2cde\right)\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{c\left(b-\sqrt{b^2-4ac}\right)}+\frac{x\left(-\frac{-2ce(ae+bd)+b^2e^2+2c^2d^2}{\sqrt{b^2-4ac}}-be^2+2cde\right)\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{c\left(\sqrt{b^2-4ac}+b\right)}+\frac{e^2x}{c}$$

input
```
Int[(d + e*x^n)^2/(a + b*x^n + c*x^(2*n)),x]
```

output
```
(e^2*x)/c + ((2*c*d*e - b*e^2 + (2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))/
Sqrt[b^2 - 4*a*c])*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(
b - Sqrt[b^2 - 4*a*c])])/(c*(b - Sqrt[b^2 - 4*a*c])) + ((2*c*d*e - b*e^2 -
 (2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))/Sqrt[b^2 - 4*a*c])*x*Hypergeome
tric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(c*(b
+ Sqrt[b^2 - 4*a*c]))
```

3.70.3.1 Defintions of rubi rules used

rule 1754
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q/(a + b*x^n + c*x^(2*n)),
 x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.70.4 Maple [F]

$$\int \frac{(d+e\,x^n)^2}{a+b\,x^n+c\,x^{2n}}dx$$

input
```
int((d+e*x^n)^2/(a+b*x^n+c*x^(2*n)),x)
```

output
```
int((d+e*x^n)^2/(a+b*x^n+c*x^(2*n)),x)
```

3.70.5 Fricas [F]

$$\int \frac{(d+ex^n)^2}{a+bx^n+cx^{2n}}\,dx = \int \frac{(ex^n+d)^2}{cx^{2n}+bx^n+a}\,dx$$

input
```
integrate((d+e*x^n)^2/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
integral((e^2*x^(2*n) + 2*d*e*x^n + d^2)/(c*x^(2*n) + b*x^n + a), x)
```

3.70.6 Sympy [F]

$$\int \frac{(d+ex^n)^2}{a+bx^n+cx^{2n}}\,dx = \int \frac{(d+ex^n)^2}{a+bx^n+cx^{2n}}\,dx$$

input
```
integrate((d+e*x**n)**2/(a+b*x**n+c*x**(2*n)),x)
```

output
```
Integral((d + e*x**n)**2/(a + b*x**n + c*x**(2*n)), x)
```

3.70.7 Maxima [F]

$$\int \frac{(d+ex^n)^2}{a+bx^n+cx^{2n}}\,dx = \int \frac{(ex^n+d)^2}{cx^{2n}+bx^n+a}\,dx$$

input
```
integrate((d+e*x^n)^2/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
e^2*x/c - integrate(-(c*d^2 - a*e^2 + (2*c*d*e - b*e^2)*x^n)/(c^2*x^(2*n)
+ b*c*x^n + a*c), x)
```

3.70.8 Giac [F]

$$\int \frac{(d+ex^n)^2}{a+bx^n+cx^{2n}}\,dx=\int \frac{(ex^n+d)^2}{cx^{2n}+bx^n+a}\,dx$$

input
```
integrate((d+e*x^n)^2/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate((e*x^n + d)^2/(c*x^(2*n) + b*x^n + a), x)
```

3.70.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^n)^2}{a+bx^n+cx^{2n}}\,dx=\int \frac{(d+e\,x^n)^2}{a+b\,x^n+c\,x^{2\,n}}\,dx$$

input
```
int((d + e*x^n)^2/(a + b*x^n + c*x^(2*n)),x)
```

output
```
int((d + e*x^n)^2/(a + b*x^n + c*x^(2*n)), x)
```

3.71 $\int \frac{d+ex^n}{a+bx^n+cx^{2n}}\,dx$

3.71.1 Optimal result

Integrand size = 24, antiderivative size = 154

$$\int \frac{d+ex^n}{a+bx^n+cx^{2n}}\,dx = \frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{b-\sqrt{b^2-4ac}} + \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{b+\sqrt{b^2-4ac}}$$

output
```
x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))*(e+(-b*e+2*c
*d)/(-4*a*c+b^2)^(1/2))/(b-(-4*a*c+b^2)^(1/2))+x*hypergeom([1, 1/n],[1+1/n
],-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(e+(b*e-2*c*d)/(-4*a*c+b^2)^(1/2))/(b+(
-4*a*c+b^2)^(1/2))
```

3.71.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.87

$$\int \frac{d+ex^n}{a+bx^n+cx^{2n}}\,dx = \frac{x\left(\left(bd+\sqrt{b^2-4ac}\,d-2ae\right)\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)+\left(-bd+\sqrt{b^2-4ac}\,d+2ae\right)\right.}{2a\sqrt{b^2-4ac}}$$

input
```
Integrate[(d + e*x^n)/(a + b*x^n + c*x^(2*n)),x]
```

output
```
(x*((b*d + Sqrt[b^2 - 4*a*c]*d - 2*a*e)*Hypergeometric2F1[1, n^(-1), 1 + n
^(-1), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + (-(b*d) + Sqrt[b^2 - 4*a*c]*d
 + 2*a*e)*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^
2 - 4*a*c])]))/(2*a*Sqrt[b^2 - 4*a*c])
```

3.71.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1752, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^n}{a+bx^n+cx^{2n}}dx$$
$$\Big\downarrow \text{1752}$$
$$\frac{1}{2}\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\int \frac{1}{cx^n+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^n+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx$$
$$\Big\downarrow \text{778}$$
$$\frac{x\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{b-\sqrt{b^2-4ac}}+\frac{x\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}+b}$$

input
```
Int[(d + e*x^n)/(a + b*x^n + c*x^(2*n)),x]
```

output
```
((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*x*Hypergeometric2F1[1, n^(-1), 1 +
n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(b - Sqrt[b^2 - 4*a*c]) + ((e
 - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*x*Hypergeometric2F1[1, n^(-1), 1 + n^(
-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(b + Sqrt[b^2 - 4*a*c])
```

3.71.3.1 Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 1752
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2
 - 4*a*c] ||  !IGtQ[n/2, 0])
```

3.71.4 Maple [F]

$$\int \frac{d + e\,x^{n}}{a + b\,x^{n} + c\,x^{2n}}dx$$

input
```
int((d+e*x^n)/(a+b*x^n+c*x^(2*n)),x)
```

output
```
int((d+e*x^n)/(a+b*x^n+c*x^(2*n)),x)
```

3.71.5 Fricas [F]

$$\int \frac{d + ex^n}{a + bx^n + cx^{2n}}\,dx = \int \frac{ex^n + d}{cx^{2n} + bx^n + a}\,dx$$

input
```
integrate((d+e*x^n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
integral((e*x^n + d)/(c*x^(2*n) + b*x^n + a), x)
```

3.71.6 Sympy [F]

$$\int \frac{d+ex^n}{a+bx^n+cx^{2n}}\,dx=\int \frac{d+ex^n}{a+bx^n+cx^{2n}}\,dx$$

input
```
integrate((d+e*x**n)/(a+b*x**n+c*x**(2*n)),x)
```

output
```
Integral((d + e*x**n)/(a + b*x**n + c*x**(2*n)), x)
```

3.71.7 Maxima [F]

$$\int \frac{d+ex^n}{a+bx^n+cx^{2n}}\,dx=\int \frac{ex^n+d}{cx^{2n}+bx^n+a}\,dx$$

input
```
integrate((d+e*x^n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
integrate((e*x^n + d)/(c*x^(2*n) + b*x^n + a), x)
```

3.71.8 Giac [F]

$$\int \frac{d+ex^n}{a+bx^n+cx^{2n}}\,dx=\int \frac{ex^n+d}{cx^{2n}+bx^n+a}\,dx$$

input
```
integrate((d+e*x^n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate((e*x^n + d)/(c*x^(2*n) + b*x^n + a), x)
```

3.71.9 Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^n}{a+bx^n+cx^{2n}}\,dx = \int \frac{d+e\,x^n}{a+b\,x^n+c\,x^{2\,n}}\,dx$$

input
```
int((d + e*x^n)/(a + b*x^n + c*x^(2*n)),x)
```

output
```
int((d + e*x^n)/(a + b*x^n + c*x^(2*n)), x)
```

3.72 $\int \frac{1}{(d+ex^n)(a+bx^n+cx^{2n})}\,dx$

3.72.1 Optimal result

Integrand size = 26, antiderivative size = 243

$$\begin{aligned}\int \frac{1}{(d+ex^n)(a+bx^n+cx^{2n})}\,dx &= -\frac{c\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac-b\sqrt{b^2-4ac}\right)(cd^2-bde+ae^2)}\\
&\quad-\frac{c\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\left(b+\sqrt{b^2-4ac}\right)(cd^2-bde+ae^2)}\\
&\quad+\frac{e^2x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)}{d\,(cd^2-bde+ae^2)}\end{aligned}$$

output
```
e^2*x*hypergeom([1, 1/n],[1+1/n],-e*x^n/d)/d/(a*e^2-b*d*e+c*d^2)-c*x*hyper
geom([1, 1/n],[1+1/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(e+(-b*e+2*c*d)/(-4
*a*c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)/(b+(-4*a*c+b^2)^(1/2))-c*x*hypergeom(
[1, 1/n],[1+1/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))*(2*c*d-e*(b+(-4*a*c+b^2)
^(1/2)))/(a*e^2-b*d*e+c*d^2)/(b^2-4*a*c-b*(-4*a*c+b^2)^(1/2))
```

3.72.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.82

$$\int \frac{1}{(d+ex^n)(a+bx^n+cx^{2n})}\,dx$$
$$= \frac{x\left(-\frac{c\left(e+\frac{-2cd+be}{\sqrt{b^2-4ac}}\right)\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)}{b-\sqrt{b^2-4ac}}-\frac{c\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{b+\sqrt{b^2-4ac}}+\right.}{cd^2+e(-bd+ae)}$$

input
```
Integrate[1/((d + e*x^n)*(a + b*x^n + c*x^(2*n))),x]
```

output
```
(x*(-((c*(e + (-2*c*d + b*e)/Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, n^(-1
), 1 + n^(-1), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/(b - Sqrt[b^2 - 4*a*c]
)) - (c*(e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, n^(-1),
 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(b + Sqrt[b^2 - 4*a*c])
+ (e^2*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((e*x^n)/d)])/d))/(c*d^2
+ e*(-(b*d) + a*e))
```

3.72.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1754, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^n)(a+bx^n+cx^{2n})}\,dx$$
$$\Big\downarrow \text{ 1754}$$
$$\int \left(\frac{e^2}{(d+ex^n)(ae^2-bde+cd^2)}+\frac{-be+cd-cex^n}{(ae^2-bde+cd^2)(a+bx^n+cx^{2n})}\right)dx$$
$$\Big\downarrow \text{ 2009}$$

$$-\frac{cx\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{\left(-b\sqrt{b^2-4ac}-4ac+b^2\right)(ae^2-bde+cd^2)}-\frac{cx\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\left(\sqrt{b^2-4ac}+b\right)(ae^2-bde+cd^2)}+\frac{e^2x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)}{d\,(ae^2-bde+cd^2)}$$

input
```
Int[1/((d + e*x^n)*(a + b*x^n + c*x^(2*n))),x]
```

output
```
-((c*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x*Hypergeometric2F1[1, n^(-1), 1
+ n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/((b^2 - 4*a*c - b*Sqrt[b^2
- 4*a*c])*(c*d^2 - b*d*e + a*e^2))) - (c*(e + (2*c*d - b*e)/Sqrt[b^2 - 4*a
*c])*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 -
 4*a*c])])/((b + Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)) + (e^2*x*Hype
rgeometric2F1[1, n^(-1), 1 + n^(-1), -((e*x^n)/d)])/(d*(c*d^2 - b*d*e + a*
e^2))
```

3.72.3.1 Defintions of rubi rules used

rule 1754
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q/(a + b*x^n + c*x^(2*n)),
 x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.72.4 Maple [F]

$$\int \frac{1}{(d+e\,x^n)\,(a+b\,x^n+c\,x^{2n})}dx$$

input
```
int(1/(d+e*x^n)/(a+b*x^n+c*x^(2*n)),x)
```

output
```
int(1/(d+e*x^n)/(a+b*x^n+c*x^(2*n)),x)
```

3.72.5 Fricas [F]

$$\int \frac{1}{(d+ex^n)(a+bx^n+cx^{2n})}\,dx = \int \frac{1}{(cx^{2n}+bx^n+a)(ex^n+d)}\,dx$$

input
```
integrate(1/(d+e*x^n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
integral(1/(b*e*x^(2*n) + a*d + (c*e*x^n + c*d)*x^(2*n) + (b*d + a*e)*x^n)
, x)
```

3.72.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex^n)(a+bx^n+cx^{2n})}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(1/(d+e*x**n)/(a+b*x**n+c*x**(2*n)),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.72.7 Maxima [F]

$$\int \frac{1}{(d+ex^n)(a+bx^n+cx^{2n})}\,dx = \int \frac{1}{(cx^{2n}+bx^n+a)(ex^n+d)}\,dx$$

input
```
integrate(1/(d+e*x^n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^(2*n) + b*x^n + a)*(e*x^n + d)), x)
```

3.72.8 Giac [F]

$$\int \frac{1}{(d + ex^n)(a + bx^n + cx^{2n})}\, dx = \int \frac{1}{(cx^{2n} + bx^n + a)(ex^n + d)}\, dx$$

input
```
integrate(1/(d+e*x^n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate(1/((c*x^(2*n) + b*x^n + a)*(e*x^n + d)), x)
```

3.72.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d + ex^n)(a + bx^n + cx^{2n})}\, dx = \int \frac{1}{(d + e\,x^n)\,(a + b\,x^n + c\,x^{2\,n})}\, dx$$

input
```
int(1/((d + e*x^n)*(a + b*x^n + c*x^(2*n))),x)
```

output
```
int(1/((d + e*x^n)*(a + b*x^n + c*x^(2*n))), x)
```

3.73 $\int \frac{1}{(d+ex^n)^2(a+bx^n+cx^{2n})}\,dx$

3.73.1 Optimal result

Integrand size = 26, antiderivative size = 368

$$\int \frac{1}{(d+ex^n)^2(a+bx^n+cx^{2n})}\,dx = -\frac{c\left(2c^2d^2+b\left(b+\sqrt{b^2-4ac}\right)e^2-2ce\left(bd+\sqrt{b^2-4ac}\,d+ae\right)\right)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2}{b-\sqrt{}}\right.}{\left(b^2-4ac-b\sqrt{b^2-4ac}\right)(cd^2-bde+ae^2)^2} - \frac{c\left(2c^2d^2+b\left(b-\sqrt{b^2-4ac}\right)e^2-2ce\left(bd-\sqrt{b^2-4ac}\,d+ae\right)\right)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2}{b+\sqrt{}}\right.}{\left(b^2-4ac+b\sqrt{b^2-4ac}\right)(cd^2-bde+ae^2)^2} + \frac{e^2(2cd-be)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)}{d\,(cd^2-bde+ae^2)^2} + \frac{e^2x\,\text{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)}{d^2\,(cd^2-bde+ae^2)}$$

output
```
e^2*(-b*e+2*c*d)*x*hypergeom([1, 1/n],[1+1/n],-e*x^n/d)/d/(a*e^2-b*d*e+c*d
^2)^2+e^2*x*hypergeom([2, 1/n],[1+1/n],-e*x^n/d)/d^2/(a*e^2-b*d*e+c*d^2)-c
*x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(2*c^2*d^2+
b*e^2*(b-(-4*a*c+b^2)^(1/2))-2*c*e*(b*d+a*e-d*(-4*a*c+b^2)^(1/2)))/(a*e^2-
b*d*e+c*d^2)^2/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))-c*x*hypergeom([1, 1/n],[1+
1/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))*(2*c^2*d^2+b*e^2*(b+(-4*a*c+b^2)^(1/
2))-2*c*e*(b*d+a*e+d*(-4*a*c+b^2)^(1/2)))/(a*e^2-b*d*e+c*d^2)^2/(b^2-4*a*c
-b*(-4*a*c+b^2)^(1/2))
```

3.73.2 Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 327, normalized size of antiderivative = 0.89

$$\int \frac{1}{(d+ex^n)^2 (a+bx^n+cx^{2n})} \, dx$$
$$= \frac{x\left(\frac{c\left(2c^2d^2+b\left(b+\sqrt{b^2-4ac}\right)e^2-2ce\left(bd+\sqrt{b^2-4ac}d+ae\right)\right)\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)}{-b^2+4ac+b\sqrt{b^2-4ac}}+\frac{c\left(-2c^2d^2+b\left(-b+\sqrt{b^2-4ac}\right)e^2\right.}{}\right.}{}$$

input
```
Integrate[1/((d + e*x^n)^2*(a + b*x^n + c*x^(2*n))),x]
```

output
```
(x*((c*(2*c^2*d^2 + b*(b + Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d + Sqrt[b^2
- 4*a*c]*d + a*e))*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (2*c*x^n)/(-b
+ Sqrt[b^2 - 4*a*c])])/(-b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c]) + (c*(-2*c^2*d
^2 + b*(-b + Sqrt[b^2 - 4*a*c])*e^2 + 2*c*e*(b*d - Sqrt[b^2 - 4*a*c]*d + a
*e))*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4
*a*c])])/(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c]) + (e^2*(2*c*d - b*e)*Hypergeo
metric2F1[1, n^(-1), 1 + n^(-1), -((e*x^n)/d)])/d + (e^2*(c*d^2 + e*(-(b*d
) + a*e))*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((e*x^n)/d)])/d^2))/(c
*d^2 + e*(-(b*d) + a*e))^2
```

3.73.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1754, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^n)^2 (a+bx^n+cx^{2n})} \, dx$$
$$\Big\downarrow \; 1754$$
$$\int \left(\frac{-ace^2+b^2e^2-\left(x^n\left(2c^2de-bce^2\right)\right)-2bcde+c^2d^2}{(ae^2-bde+cd^2)^2 (a+bx^n+cx^{2n})}-\frac{e^2(be-2cd)}{(d+ex^n)(ae^2-bde+cd^2)^2}+\frac{e^2}{(d+ex^n)^2 (ae^2-bde}\right.$$
$$\Big\downarrow \; 2009$$

$$-\frac{cx\left(-2ce\left(d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(\sqrt{b^2-4ac}+b\right)+2c^2d^2\right)\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{\left(-b\sqrt{b^2-4ac}-4ac+b^2\right)\left(ae^2-bde+cd^2\right)^2}$$
$$\frac{cx\left(-2ce\left(-d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(b-\sqrt{b^2-4ac}\right)+2c^2d^2\right)\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\left(b\sqrt{b^2-4ac}-4ac+b^2\right)\left(ae^2-bde+cd^2\right)^2}$$
$$\frac{e^2x(2cd-be)\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)}{d\left(ae^2-bde+cd^2\right)^2}+$$
$$\frac{e^2x\text{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)}{d^2\left(ae^2-bde+cd^2\right)}$$

input
```
Int[1/((d + e*x^n)^2*(a + b*x^n + c*x^(2*n))),x]
```

output
```
-((c*(2*c^2*d^2 + b*(b + Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d + Sqrt[b^2 -
4*a*c]*d + a*e))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b
- Sqrt[b^2 - 4*a*c])])/((b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e
 + a*e^2)^2)) - (c*(2*c^2*d^2 + b*(b - Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d
 - Sqrt[b^2 - 4*a*c]*d + a*e))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1),
(-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/((b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*
(c*d^2 - b*d*e + a*e^2)^2) + (e^2*(2*c*d - b*e)*x*Hypergeometric2F1[1, n^(
-1), 1 + n^(-1), -((e*x^n)/d)])/(d*(c*d^2 - b*d*e + a*e^2)^2) + (e^2*x*Hyp
ergeometric2F1[2, n^(-1), 1 + n^(-1), -((e*x^n)/d)])/(d^2*(c*d^2 - b*d*e +
 a*e^2))
```

3.73.3.1 Defintions of rubi rules used

rule 1754
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q/(a + b*x^n + c*x^(2*n)),
 x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.73.4 Maple [F]

$$\int \frac{1}{\left(d+e\,x^{n}\right)^{2}\left(a+b\,x^{n}+c\,x^{2n}\right)}dx$$

input
```
int(1/(d+e*x^n)^2/(a+b*x^n+c*x^(2*n)),x)
```

output
```
int(1/(d+e*x^n)^2/(a+b*x^n+c*x^(2*n)),x)
```

3.73.5 Fricas [F]

$$\int \frac{1}{(d+ex^n)^2\,(a+bx^n+cx^{2n})}\,dx = \int \frac{1}{(cx^{2\,n}+bx^n+a)(ex^n+d)^2}\,dx$$

input
```
integrate(1/(d+e*x^n)^2/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
integral(1/(b*e^2*x^(3*n) + a*d^2 + (c*e^2*x^(2*n) + 2*c*d*e*x^n + c*d^2)*
x^(2*n) + (2*b*d*e + a*e^2)*x^(2*n) + (b*d^2 + 2*a*d*e)*x^n), x)
```

3.73.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex^n)^2\,(a+bx^n+cx^{2n})}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(1/(d+e*x**n)**2/(a+b*x**n+c*x**(2*n)),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.73.7 Maxima [F]

$$\int \frac{1}{(d+ex^n)^2(a+bx^n+cx^{2n})}\,dx = \int \frac{1}{(cx^{2n}+bx^n+a)(ex^n+d)^2}\,dx$$

input
```
integrate(1/(d+e*x^n)^2/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
e^2*x/(c*d^4*n - b*d^3*e*n + a*d^2*e^2*n + (c*d^3*e*n - b*d^2*e^2*n + a*d*
e^3*n)*x^n) + (c*d^2*e^2*(3*n - 1) - b*d*e^3*(2*n - 1) + a*e^4*(n - 1))*in
tegrate(1/(c^2*d^6*n - 2*b*c*d^5*e*n + b^2*d^4*e^2*n + a^2*d^2*e^4*n + 2*(
c*d^4*e^2*n - b*d^3*e^3*n)*a + (c^2*d^5*e*n - 2*b*c*d^4*e^2*n + b^2*d^3*e^
3*n + a^2*d*e^5*n + 2*(c*d^3*e^3*n - b*d^2*e^4*n)*a)*x^n), x) + integrate(
(c^2*d^2 - 2*b*c*d*e + b^2*e^2 - a*c*e^2 - (2*c^2*d*e - b*c*e^2)*x^n)/(a^3
*e^4 + 2*(c*d^2*e^2 - b*d*e^3)*a^2 + (c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2)
*a + (c^3*d^4 - 2*b*c^2*d^3*e + b^2*c*d^2*e^2 + a^2*c*e^4 + 2*(c^2*d^2*e^2
 - b*c*d*e^3)*a)*x^(2*n) + (b*c^2*d^4 - 2*b^2*c*d^3*e + b^3*d^2*e^2 + a^2*
b*e^4 + 2*(b*c*d^2*e^2 - b^2*d*e^3)*a)*x^n), x)
```

3.73.8 Giac [F]

$$\int \frac{1}{(d+ex^n)^2(a+bx^n+cx^{2n})}\,dx = \int \frac{1}{(cx^{2n}+bx^n+a)(ex^n+d)^2}\,dx$$

input
```
integrate(1/(d+e*x^n)^2/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate(1/((c*x^(2*n) + b*x^n + a)*(e*x^n + d)^2), x)
```

3.73.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^n)^2(a+bx^n+cx^{2n})}\,dx = \int \frac{1}{(d+e\,x^n)^2\,(a+b\,x^n+c\,x^{2\,n})}\,dx$$

input
```
int(1/((d + e*x^n)^2*(a + b*x^n + c*x^(2*n))),x)
```

output
```
int(1/((d + e*x^n)^2*(a + b*x^n + c*x^(2*n))), x)
```

3.74 $\int \frac{1}{(d+ex^n)^3(a+bx^n+cx^{2n})}\,dx$

3.74.1 Optimal result

Integrand size = 26, antiderivative size = 552

$$\int \frac{1}{(d+ex^n)^3(a+bx^n+cx^{2n})}\,dx =$$

$$-\frac{c\left(2c^3d^3-b^2\left(b+\sqrt{b^2-4ac}\right)e^3-3c^2de\left(bd+\sqrt{b^2-4ac}d+2ae\right)+ce^2\left(3b^2d+a\sqrt{b^2-4ac}e+3b\left(\sqrt{b^2\right.\right.}{\left(b^2-4ac-b\sqrt{b^2-4ac}\right)\left(cd^2-bde+ae\right.}$$

$$-\frac{c\left(2c^3d^3-b^2\left(b-\sqrt{b^2-4ac}\right)e^3-3c^2de\left(bd-\sqrt{b^2-4ac}d+2ae\right)+ce^2\left(3b^2d-3b\sqrt{b^2-4ac}d+3abe-\right.}{\left(b^2-4ac+b\sqrt{b^2-4ac}\right)\left(cd^2-bde+ae\right.}$$

$$+\frac{e^2(3c^2d^2+b^2e^2-ce(3bd+ae))\,x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)}{d\left(cd^2-bde+ae^2\right)^3}$$

$$+\frac{e^2(2cd-be)x\,\text{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)}{d^2\left(cd^2-bde+ae^2\right)^2}$$

$$+\frac{e^2x\,\text{Hypergeometric2F1}\left(3,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)}{d^3\left(cd^2-bde+ae^2\right)}$$

output
```
e^2*(3*c^2*d^2+b^2*e^2-c*e*(a*e+3*b*d))*x*hypergeom([1, 1/n],[1+1/n],-e*x^
n/d)/d/(a*e^2-b*d*e+c*d^2)^3+e^2*(-b*e+2*c*d)*x*hypergeom([2, 1/n],[1+1/n]
,-e*x^n/d)/d^2/(a*e^2-b*d*e+c*d^2)^2+e^2*x*hypergeom([3, 1/n],[1+1/n],-e*x
^n/d)/d^3/(a*e^2-b*d*e+c*d^2)-c*x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b+(
-4*a*c+b^2)^(1/2)))*(2*c^3*d^3-b^2*e^3*(b-(-4*a*c+b^2)^(1/2))-3*c^2*d*e*(b
*d+2*a*e-d*(-4*a*c+b^2)^(1/2))+c*e^2*(3*b^2*d+3*a*b*e-3*b*d*(-4*a*c+b^2)^(
1/2)-(-4*a*c+b^2)^(1/2)*a*e))/(a*e^2-b*d*e+c*d^2)^3/(b^2-4*a*c+b*(-4*a*c+b
^2)^(1/2))-c*x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))
*(2*c^3*d^3-b^2*e^3*(b+(-4*a*c+b^2)^(1/2))-3*c^2*d*e*(b*d+2*a*e+d*(-4*a*c+
b^2)^(1/2))+c*e^2*(3*b^2*d+(-4*a*c+b^2)^(1/2)*a*e+3*b*(a*e+d*(-4*a*c+b^2)^
(1/2))))/(a*e^2-b*d*e+c*d^2)^3/(b^2-4*a*c-b*(-4*a*c+b^2)^(1/2))
```

3.74.2 Mathematica [A] (verified)

Time = 1.76 (sec) , antiderivative size = 509, normalized size of antiderivative = 0.92

$$\int \frac{1}{(d+ex^n)^3\,(a+bx^n+cx^{2n})}\,dx$$

$$= \frac{x\left(\frac{c\left(-2c^3d^3+b^2\left(b+\sqrt{b^2-4ac}\right)e^3+3c^2de\left(bd+\sqrt{b^2-4ac}d+2ae\right)-ce^2\left(3b^2d+a\sqrt{b^2-4ac}e+3b\left(\sqrt{b^2-4ac}d+ae\right)\right)\right)\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\right.}{b^2-4ac-b\sqrt{b^2-4ac}}\right.}{}$$

input
```
Integrate[1/((d + e*x^n)^3*(a + b*x^n + c*x^(2*n))),x]
```

output
```
(x*((c*(-2*c^3*d^3 + b^2*(b + Sqrt[b^2 - 4*a*c])*e^3 + 3*c^2*d*e*(b*d + Sq
rt[b^2 - 4*a*c]*d + 2*a*e) - c*e^2*(3*b^2*d + a*Sqrt[b^2 - 4*a*c]*e + 3*b*
(Sqrt[b^2 - 4*a*c]*d + a*e)))*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (2*
c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c]) - (c
*(2*c^3*d^3 + b^2*(-b + Sqrt[b^2 - 4*a*c])*e^3 + 3*c^2*d*e*(-(b*d) + Sqrt[
b^2 - 4*a*c]*d - 2*a*e) + c*e^2*(3*b^2*d - 3*b*Sqrt[b^2 - 4*a*c]*d + 3*a*b
*e - a*Sqrt[b^2 - 4*a*c]*e))*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*
c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c]) + (e^
2*(3*c^2*d^2 + b^2*e^2 - c*e*(3*b*d + a*e))*Hypergeometric2F1[1, n^(-1), 1
 + n^(-1), -((e*x^n)/d)])/d + (e^2*(2*c*d - b*e)*(c*d^2 + e*(-(b*d) + a*e)
)*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((e*x^n)/d)])/d^2 + (e^2*(c*d^
2 + e*(-(b*d) + a*e))^2*Hypergeometric2F1[3, n^(-1), 1 + n^(-1), -((e*x^n)
/d)])/d^3))/(c*d^2 + e*(-(b*d) + a*e))^3
```

3.74.3 Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 552, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1754, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^n)^3\,(a+bx^n+cx^{2n})}\,dx$$

$$\Big\downarrow \; 1754$$

$$\int \left(\frac{e^2\left(-ce(ae+3bd)+b^2e^2+3c^2d^2\right)}{(d+ex^n)\left(ae^2-bde+cd^2\right)^3}+\frac{-\left(x^n\left(-ac^2e^3+b^2ce^3-3bc^2de^2+3c^3d^2e\right)\right)+2abce^3-3ac^2de^2-b^3e^3\cdots}{\left(ae^2-bde+cd^2\right)^3\left(a+bx^n+cx^{2n}\right)}\cdots\right)$$

$$\Big\downarrow \; 2009$$

$$\frac{e^2x\left(-ce(ae+3bd)+b^2e^2+3c^2d^2\right)\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)}{d\left(ae^2-bde+cd^2\right)^3}-$$
$$\frac{cx\left(-3c^2de\left(d\sqrt{b^2-4ac}+2ae+bd\right)+ce^2\left(3b\left(d\sqrt{b^2-4ac}+ae\right)+ae\sqrt{b^2-4ac}+3b^2d\right)-b^2e^3\left(\sqrt{b^2-4ac}+b\cdots\right)\cdots\right)}{\left(-b\sqrt{b^2-4ac}-4ac+b^2\right)\left(ae^2-bde+cd^2\right)^3}\cdots$$
$$\frac{cx\left(-3c^2de\left(-d\sqrt{b^2-4ac}+2ae+bd\right)+ce^2\left(-3bd\sqrt{b^2-4ac}-ae\sqrt{b^2-4ac}+3abe+3b^2d\right)-b^2e^3\left(b-\sqrt{b^2-4\cdots}\right)\cdots\right)}{\left(b\sqrt{b^2-4ac}-4ac+b^2\right)\left(ae^2-bde+cd^2\right)^3}\cdots$$
$$\frac{e^2x(2cd-be)\,\text{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)}{d^2\left(ae^2-bde+cd^2\right)^2}+$$
$$\frac{e^2x\,\text{Hypergeometric2F1}\left(3,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)}{d^3\left(ae^2-bde+cd^2\right)}$$

input
```
Int[1/((d + e*x^n)^3*(a + b*x^n + c*x^(2*n))),x]
```

output
```
-((c*(2*c^3*d^3 - b^2*(b + Sqrt[b^2 - 4*a*c])*e^3 - 3*c^2*d*e*(b*d + Sqrt[
b^2 - 4*a*c]*d + 2*a*e) + c*e^2*(3*b^2*d + a*Sqrt[b^2 - 4*a*c]*e + 3*b*(Sq
rt[b^2 - 4*a*c]*d + a*e)))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*
c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/((b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*(c*d
^2 - b*d*e + a*e^2)^3)) - (c*(2*c^3*d^3 - b^2*(b - Sqrt[b^2 - 4*a*c])*e^3
- 3*c^2*d*e*(b*d - Sqrt[b^2 - 4*a*c]*d + 2*a*e) + c*e^2*(3*b^2*d - 3*b*Sqr
t[b^2 - 4*a*c]*d + 3*a*b*e - a*Sqrt[b^2 - 4*a*c]*e))*x*Hypergeometric2F1[1
, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/((b^2 - 4*a*c +
 b*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)^3) + (e^2*(3*c^2*d^2 + b^2*e
^2 - c*e*(3*b*d + a*e))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((e*x^
n)/d)])/(d*(c*d^2 - b*d*e + a*e^2)^3) + (e^2*(2*c*d - b*e)*x*Hypergeometri
c2F1[2, n^(-1), 1 + n^(-1), -((e*x^n)/d)])/(d^2*(c*d^2 - b*d*e + a*e^2)^2)
 + (e^2*x*Hypergeometric2F1[3, n^(-1), 1 + n^(-1), -((e*x^n)/d)])/(d^3*(c*
d^2 - b*d*e + a*e^2))
```

3.74.3.1 Defintions of rubi rules used

rule 1754
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q/(a + b*x^n + c*x^(2*n)),
 x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.74.4 Maple [F]

$$\int \frac{1}{(d+e\,x^n)^3\,(a+b\,x^n+c\,x^{2n})}dx$$

input
```
int(1/(d+e*x^n)^3/(a+b*x^n+c*x^(2*n)),x)
```

output
```
int(1/(d+e*x^n)^3/(a+b*x^n+c*x^(2*n)),x)
```

3.74.5 Fricas [F]

$$\int \frac{1}{(d+ex^n)^3 (a+bx^n+cx^{2n})} dx = \int \frac{1}{(cx^{2n}+bx^n+a)(ex^n+d)^3} dx$$

input
```
integrate(1/(d+e*x^n)^3/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
integral(1/(b*e^3*x^(4*n) + a*d^3 + (3*b*d*e^2 + a*e^3)*x^(3*n) + (c*e^3*x
^(3*n) + 3*c*d*e^2*x^(2*n) + 3*c*d^2*e*x^n + c*d^3)*x^(2*n) + 3*(b*d^2*e +
 a*d*e^2)*x^(2*n) + (b*d^3 + 3*a*d^2*e)*x^n), x)
```

3.74.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex^n)^3 (a+bx^n+cx^{2n})} dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(1/(d+e*x**n)**3/(a+b*x**n+c*x**(2*n)),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.74.7 Maxima [F]

$$\int \frac{1}{(d+ex^n)^3 (a+bx^n+cx^{2n})} dx = \int \frac{1}{(cx^{2n}+bx^n+a)(ex^n+d)^3} dx$$

input
```
integrate(1/(d+e*x^n)^3/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
((12*n^2 - 7*n + 1)*c^2*d^4*e^2 - 2*(8*n^2 - 6*n + 1)*b*c*d^3*e^3 + (6*n^2
 - 5*n + 1)*b^2*d^2*e^4 + (2*n^2 - 3*n + 1)*a^2*e^6 + 2*((3*n^2 - 5*n + 1)
*c*d^2*e^4 - (3*n^2 - 4*n + 1)*b*d*e^5)*a)*integrate(1/2/(c^3*d^9*n^2 - 3*
b*c^2*d^8*e*n^2 + 3*b^2*c*d^7*e^2*n^2 - b^3*d^6*e^3*n^2 + a^3*d^3*e^6*n^2
+ 3*(c*d^5*e^4*n^2 - b*d^4*e^5*n^2)*a^2 + 3*(c^2*d^7*e^2*n^2 - 2*b*c*d^6*e
^3*n^2 + b^2*d^5*e^4*n^2)*a + (c^3*d^8*e*n^2 - 3*b*c^2*d^7*e^2*n^2 + 3*b^2
*c*d^6*e^3*n^2 - b^3*d^5*e^4*n^2 + a^3*d^2*e^7*n^2 + 3*(c*d^4*e^5*n^2 - b*
d^3*e^6*n^2)*a^2 + 3*(c^2*d^6*e^3*n^2 - 2*b*c*d^5*e^4*n^2 + b^2*d^4*e^5*n^
2)*a)*x^n), x) + 1/2*((c*d^2*e^3*(6*n - 1) - b*d*e^4*(4*n - 1) + a*e^5*(2*
n - 1))*x*x^n + (c*d^3*e^2*(7*n - 1) - b*d^2*e^3*(5*n - 1) + a*d*e^4*(3*n
- 1))*x)/(c^2*d^8*n^2 - 2*b*c*d^7*e*n^2 + b^2*d^6*e^2*n^2 + a^2*d^4*e^4*n^
2 + 2*(c*d^6*e^2*n^2 - b*d^5*e^3*n^2)*a + (c^2*d^6*e^2*n^2 - 2*b*c*d^5*e^3
*n^2 + b^2*d^4*e^4*n^2 + a^2*d^2*e^6*n^2 + 2*(c*d^4*e^4*n^2 - b*d^3*e^5*n^
2)*a)*x^(2*n) + 2*(c^2*d^7*e*n^2 - 2*b*c*d^6*e^2*n^2 + b^2*d^5*e^3*n^2 + a
^2*d^3*e^5*n^2 + 2*(c*d^5*e^3*n^2 - b*d^4*e^4*n^2)*a)*x^n) + integrate((c^
3*d^3 - 3*b*c^2*d^2*e + 3*b^2*c*d*e^2 - b^3*e^3 - (3*c^2*d*e^2 - 2*b*c*e^3
)*a - (3*c^3*d^2*e - 3*b*c^2*d*e^2 + b^2*c*e^3 - a*c^2*e^3)*x^n)/(a^4*e^6
+ 3*(c*d^2*e^4 - b*d*e^5)*a^3 + 3*(c^2*d^4*e^2 - 2*b*c*d^3*e^3 + b^2*d^2*e
^4)*a^2 + (c^3*d^6 - 3*b*c^2*d^5*e + 3*b^2*c*d^4*e^2 - b^3*d^3*e^3)*a + (c
^4*d^6 - 3*b*c^3*d^5*e + 3*b^2*c^2*d^4*e^2 - b^3*c*d^3*e^3 + a^3*c*e^6 ...
```

3.74.8 Giac [F]

$$\int \frac{1}{(d+ex^n)^3\,(a+bx^n+cx^{2n})}\,dx = \int \frac{1}{(cx^{2n}+bx^n+a)(ex^n+d)^3}\,dx$$

input
```
integrate(1/(d+e*x^n)^3/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate(1/((c*x^(2*n) + b*x^n + a)*(e*x^n + d)^3), x)
```

3.74.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^n)^3 (a+bx^n+cx^{2n})}\,dx = \int \frac{1}{(d+e\,x^n)^3\,(a+b\,x^n+c\,x^{2\,n})}\,dx$$

input
```
int(1/((d + e*x^n)^3*(a + b*x^n + c*x^(2*n))),x)
```

output
```
int(1/((d + e*x^n)^3*(a + b*x^n + c*x^(2*n))), x)
```

3.75 $\int \frac{(d+ex^n)^3}{(a+bx^n+cx^{2n})^2}\,dx$

3.75.1 Optimal result

Integrand size = 26, antiderivative size = 750

$$
\begin{aligned}
\int \frac{(d+ex^n)^3}{(a+bx^n+cx^{2n})^2}\,dx
&= \frac{x\left(b^2cd^3-2acd(cd^2-3ae^2)-abe(3cd^2+ae^2)-\left(ab^2e^3+2ace(3cd^2-ae^2)-bcd(cd^2+3ae^2)\right)x^n\right)}{ac\left(b^2-4ac\right)n\left(a+bx^n+cx^{2n}\right)} \\
&+ \frac{e^2\left(e+\frac{6cd-3be}{\sqrt{b^2-4ac}}\right)x\,\mathrm{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{c\left(b-\sqrt{b^2-4ac}\right)} \\
&+ \frac{\left(\left(ab^2e^3+2ace(3cd^2-ae^2)-bcd(cd^2+3ae^2)\right)(1-n)+\frac{b^2cd\left(3ae^2(1-3n)-cd^2(1-n)\right)-ab^3e^3(1-3n)+4ac^2d\left(cd^2-3a\cdots\right.}{\sqrt{b^2-4ac}}\cdots\right.}{ac\left(b^2-4ac\right)\left(b-\sqrt{b^2-4a\cdots\right.} \\
&+ \frac{e^2\left(e-\frac{3(2cd-be)}{\sqrt{b^2-4ac}}\right)x\,\mathrm{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{c\left(b+\sqrt{b^2-4ac}\right)} \\
&+ \frac{\left(\left(ab^2e^3+2ace(3cd^2-ae^2)-bcd(cd^2+3ae^2)\right)(1-n)-\frac{b^2cd\left(3ae^2(1-3n)-cd^2(1-n)\right)-ab^3e^3(1-3n)+4ac^2d\left(cd^2-3a\cdots\right.}{\sqrt{b^2-4ac}}\cdots\right.}{ac\left(b^2-4ac\right)\left(b+\sqrt{b^2-4a\cdots\right.}
\end{aligned}
$$

output
```
x*(b^2*c*d^3-2*a*c*d*(-3*a*e^2+c*d^2)-a*b*e*(a*e^2+3*c*d^2)-(a*b^2*e^3+2*a
*c*e*(-a*e^2+3*c*d^2)-b*c*d*(3*a*e^2+c*d^2))*x^n)/a/c/(-4*a*c+b^2)/n/(a+b*
x^n+c*x^(2*n))+e^2*x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b-(-4*a*c+b^2)^(
1/2)))*(e+(-3*b*e+6*c*d)/(-4*a*c+b^2)^(1/2))/c/(b-(-4*a*c+b^2)^(1/2))+x*hy
pergeom([1, 1/n],[1+1/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))*((a*b^2*e^3+2*a*
c*e*(-a*e^2+3*c*d^2)-b*c*d*(3*a*e^2+c*d^2))*(1-n)+(b^2*c*d*(3*a*e^2*(1-3*n
)-c*d^2*(1-n))-a*b^3*e^3*(1-3*n)+4*a*c^2*d*(-3*a*e^2+c*d^2)*(1-2*n)+2*a*b*
c*e*(a*e^2*(2-5*n)+3*c*d^2*n))/(-4*a*c+b^2)^(1/2))/a/c/(-4*a*c+b^2)/n/(b-(
-4*a*c+b^2)^(1/2))+e^2*x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b+(-4*a*c+b^
2)^(1/2)))*(e-3*(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))/c/(b+(-4*a*c+b^2)^(1/2))+
x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*((a*b^2*e^3+
2*a*c*e*(-a*e^2+3*c*d^2)-b*c*d*(3*a*e^2+c*d^2))*(1-n)+(-b^2*c*d*(3*a*e^2*(
1-3*n)-c*d^2*(1-n))+a*b^3*e^3*(1-3*n)-4*a*c^2*d*(-3*a*e^2+c*d^2)*(1-2*n)-2
*a*b*c*e*(a*e^2*(2-5*n)+3*c*d^2*n))/(-4*a*c+b^2)^(1/2))/a/c/(-4*a*c+b^2)/n
/(b+(-4*a*c+b^2)^(1/2))
```

3.75.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 5537 vs. $2(750) = 1500$.

Time = 7.56 (sec) , antiderivative size = 5537, normalized size of antiderivative = 7.38

$$\int \frac{(d+ex^n)^3}{(a+bx^n+cx^{2n})^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[(d + e*x^n)^3/(a + b*x^n + c*x^(2*n))^2,x]
```

output
```
Result too large to show
```

3.75.3 Rubi [A] (verified)

Time = 2.33 (sec) , antiderivative size = 750, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1766, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^n)^3}{(a+bx^n+cx^{2n})^2}\,dx$$

$$\Big\downarrow \text{1766}$$

$$\int \left(\frac{x^n\left(-ace^3+b^2e^3-3bcde^2+3c^2d^2e\right)+abe^3-3acde^2+c^2d^3}{c^2\left(a+bx^n+cx^{2n}\right)^2}+\frac{e^2\left(-be+3cd+cex^n\right)}{c^2\left(a+bx^n+cx^{2n}\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x\left(-\left(x^n\left(ab^2e^3-bcd\left(3ae^2+cd^2\right)+2ace\left(3cd^2-ae^2\right)\right)\right)-abe\left(ae^2+3cd^2\right)-2acd\left(cd^2-3ae^2\right)+b^2cd^3\right)}{acn\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)}+$$
$$\frac{e^2x\left(\frac{6cd-3be}{\sqrt{b^2-4ac}}+e\right)\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{c\left(b-\sqrt{b^2-4ac}\right)}+$$
$$\frac{e^2x\left(e-\frac{3(2cd-be)}{\sqrt{b^2-4ac}}\right)\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{c\left(\sqrt{b^2-4ac}+b\right)}+$$
$$\frac{x\left((1-n)\left(ab^2e^3-bcd\left(3ae^2+cd^2\right)+2ace\left(3cd^2-ae^2\right)\right)+\frac{-ab^3e^3(1-3n)+b^2cd\left(3ae^2(1-3n)-cd^2(1-n)\right)+2abce\left(ae^2(2-5n)+\cdots\right.}{\sqrt{b^2-4ac}}\cdots\right.}{acn\left(b^2-4ac\right)\left(b-\sqrt{b^2-4ac}\right)}$$
$$\frac{x\left((1-n)\left(ab^2e^3-bcd\left(3ae^2+cd^2\right)+2ace\left(3cd^2-ae^2\right)\right)-\frac{-ab^3e^3(1-3n)+b^2cd\left(3ae^2(1-3n)-cd^2(1-n)\right)+2abce\left(ae^2(2-5n)+\cdots\right.}{\sqrt{b^2-4ac}}\cdots\right.}{acn\left(b^2-4ac\right)\left(\sqrt{b^2-4ac}+b\right)}$$

input
```
Int[(d + e*x^n)^3/(a + b*x^n + c*x^(2*n))^2,x]
```

3.75. $\int \frac{(d+ex^n)^3}{(a+bx^n+cx^{2n})^2}\,dx$

output
```
(x*(b^2*c*d^3 - 2*a*c*d*(c*d^2 - 3*a*e^2) - a*b*e*(3*c*d^2 + a*e^2) - (a*b
^2*e^3 + 2*a*c*e*(3*c*d^2 - a*e^2) - b*c*d*(c*d^2 + 3*a*e^2))*x^n))/(a*c*(
b^2 - 4*a*c)*n*(a + b*x^n + c*x^(2*n))) + (e^2*(e + (6*c*d - 3*b*e)/Sqrt[b
^2 - 4*a*c])*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b - Sq
rt[b^2 - 4*a*c])])/(c*(b - Sqrt[b^2 - 4*a*c])) + (((a*b^2*e^3 + 2*a*c*e*(3
*c*d^2 - a*e^2) - b*c*d*(c*d^2 + 3*a*e^2))*(1 - n) + (b^2*c*d*(3*a*e^2*(1
- 3*n) - c*d^2*(1 - n)) - a*b^3*e^3*(1 - 3*n) + 4*a*c^2*d*(c*d^2 - 3*a*e^2
)*(1 - 2*n) + 2*a*b*c*e*(a*e^2*(2 - 5*n) + 3*c*d^2*n))/Sqrt[b^2 - 4*a*c])*
x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*
c])])/(a*c*(b^2 - 4*a*c)*(b - Sqrt[b^2 - 4*a*c])*n) + (e^2*(e - (3*(2*c*d
- b*e))/Sqrt[b^2 - 4*a*c])*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*
c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(c*(b + Sqrt[b^2 - 4*a*c])) + (((a*b^2*e^
3 + 2*a*c*e*(3*c*d^2 - a*e^2) - b*c*d*(c*d^2 + 3*a*e^2))*(1 - n) - (b^2*c*
d*(3*a*e^2*(1 - 3*n) - c*d^2*(1 - n)) - a*b^3*e^3*(1 - 3*n) + 4*a*c^2*d*(c
*d^2 - 3*a*e^2)*(1 - 2*n) + 2*a*b*c*e*(a*e^2*(2 - 5*n) + 3*c*d^2*n))/Sqrt[
b^2 - 4*a*c])*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + S
qrt[b^2 - 4*a*c])])/(a*c*(b^2 - 4*a*c)*(b + Sqrt[b^2 - 4*a*c])*n)
```

3.75.3.1 Defintions of rubi rules used

rule 1766
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
))^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2
*n))^p, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ((IntegersQ[p, q] &&
!IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] &&  !IntegerQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.75.4 Maple [F]

$$\int \frac{(d+e\,x^n)^3}{(a+b\,x^n+c\,x^{2n})^2}dx$$

input
```
int((d+e*x^n)^3/(a+b*x^n+c*x^(2*n))^2,x)
```

output
```
int((d+e*x^n)^3/(a+b*x^n+c*x^(2*n))^2,x)
```

3.75.5 Fricas [F]

$$\int \frac{(d+ex^n)^3}{(a+bx^n+cx^{2n})^2}\,dx = \int \frac{(ex^n+d)^3}{(cx^{2n}+bx^n+a)^2}\,dx$$

input
```
integrate((d+e*x^n)^3/(a+b*x^n+c*x^(2*n))^2,x, algorithm="fricas")
```

output
```
integral((e^3*x^(3*n) + 3*d*e^2*x^(2*n) + 3*d^2*e*x^n + d^3)/(c^2*x^(4*n)
+ b^2*x^(2*n) + 2*a*b*x^n + a^2 + 2*(b*c*x^n + a*c)*x^(2*n)), x)
```

3.75.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^n)^3}{(a+bx^n+cx^{2n})^2}\,dx = \text{Timed out}$$

input
```
integrate((d+e*x**n)**3/(a+b*x**n+c*x**(2*n))**2,x)
```

output
```
Timed out
```

3.75.7 Maxima [F]

$$\int \frac{(d+ex^n)^3}{(a+bx^n+cx^{2n})^2}\,dx = \int \frac{(ex^n+d)^3}{(cx^{2n}+bx^n+a)^2}\,dx$$

input
```
integrate((d+e*x^n)^3/(a+b*x^n+c*x^(2*n))^2,x, algorithm="maxima")
```

output
```
((b*c^2*d^3 + 2*a^2*c*e^3 - (6*c^2*d^2*e - 3*b*c*d*e^2 + b^2*e^3)*a)*x*x^n
 + (b^2*c*d^3 + (6*c*d*e^2 - b*e^3)*a^2 - (2*c^2*d^3 + 3*b*c*d^2*e)*a)*x)/
(a^2*b^2*c*n - 4*a^3*c^2*n + (a*b^2*c^2*n - 4*a^2*c^3*n)*x^(2*n) + (a*b^3*
c*n - 4*a^2*b*c^2*n)*x^n) + integrate((b^2*c*d^3*(n - 1) - (6*c*d*e^2 - b*
e^3)*a^2 - (2*c^2*d^3*(2*n - 1) - 3*b*c*d^2*e)*a - (2*a^2*c*e^3*(n + 1) -
b*c^2*d^3*(n - 1) + (6*c^2*d^2*e*(n - 1) - 3*b*c*d*e^2*(n - 1) - b^2*e^3)*
a)*x^n)/(a^2*b^2*c*n - 4*a^3*c^2*n + (a*b^2*c^2*n - 4*a^2*c^3*n)*x^(2*n) +
 (a*b^3*c*n - 4*a^2*b*c^2*n)*x^n), x)
```

3.75.8 Giac [F]

$$\int \frac{(d+ex^n)^3}{(a+bx^n+cx^{2n})^2}\,dx = \int \frac{(ex^n+d)^3}{(cx^{2n}+bx^n+a)^2}\,dx$$

input
```
integrate((d+e*x^n)^3/(a+b*x^n+c*x^(2*n))^2,x, algorithm="giac")
```

output
```
integrate((e*x^n + d)^3/(c*x^(2*n) + b*x^n + a)^2, x)
```

3.75.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^n)^3}{(a+bx^n+cx^{2n})^2}\,dx = \int \frac{(d+e\,x^n)^3}{(a+b\,x^n+c\,x^{2n})^2}\,dx$$

input
```
int((d + e*x^n)^3/(a + b*x^n + c*x^(2*n))^2,x)
```

output
```
int((d + e*x^n)^3/(a + b*x^n + c*x^(2*n))^2, x)
```

3.76 $\int \frac{(d+ex^n)^2}{(a+bx^n+cx^{2n})^2} dx$

3.76.1 Optimal result

Integrand size = 26, antiderivative size = 543

$$\int \frac{(d+ex^n)^2}{(a+bx^n+cx^{2n})^2} dx = \frac{x(b^2d^2-2abde-2a(cd^2-ae^2)+(bcd^2-4acde+abe^2)x^n)}{a(b^2-4ac)n(a+bx^n+cx^{2n})}$$
$$-\frac{2e^2x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{b^2-4ac-b\sqrt{b^2-4ac}}$$
$$-\frac{\left((bcd^2-4acde+abe^2)(1-n)-\frac{b^2(ae^2(1-3n)-cd^2(1-n))+4ac(cd^2-ae^2)(1-2n)+4abcden}{\sqrt{b^2-4ac}}\right)x\,\text{Hypergeometric2F1}}{a(b^2-4ac)\left(b-\sqrt{b^2-4ac}\right)n}$$
$$-\frac{2e^2x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{b^2-4ac+b\sqrt{b^2-4ac}}$$
$$-\frac{\left((bcd^2-4acde+abe^2)(1-n)+\frac{b^2(ae^2(1-3n)-cd^2(1-n))+4ac(cd^2-ae^2)(1-2n)+4abcden}{\sqrt{b^2-4ac}}\right)x\,\text{Hypergeometric2F1}}{a(b^2-4ac)\left(b+\sqrt{b^2-4ac}\right)n}$$

output
```
x*(b^2*d^2-2*a*b*d*e-2*a*(-a*e^2+c*d^2)+(a*b*e^2-4*a*c*d*e+b*c*d^2)*x^n)/a
/(-4*a*c+b^2)/n/(a+b*x^n+c*x^(2*n))-x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/
(b-(-4*a*c+b^2)^(1/2)))*((a*b*e^2-4*a*c*d*e+b*c*d^2)*(1-n)+(-b^2*(a*e^2*(1
-3*n)-c*d^2*(1-n))-4*a*c*(-a*e^2+c*d^2)*(1-2*n)-4*a*b*c*d*e*n)/(-4*a*c+b^2
)^(1/2))/a/(-4*a*c+b^2)/n/(b-(-4*a*c+b^2)^(1/2))-x*hypergeom([1, 1/n],[1+1
/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*((a*b*e^2-4*a*c*d*e+b*c*d^2)*(1-n)+(b
^2*(a*e^2*(1-3*n)-c*d^2*(1-n))+4*a*c*(-a*e^2+c*d^2)*(1-2*n)+4*a*b*c*d*e*n)
/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)/n/(b+(-4*a*c+b^2)^(1/2))-2*e^2*x*hyper
geom([1, 1/n],[1+1/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))/(b^2-4*a*c-b*(-4*a*
c+b^2)^(1/2))-2*e^2*x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b+(-4*a*c+b^2)^
(1/2)))/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))
```

3.76.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2980 vs. $2(543) = 1086$.

Time = 3.84 (sec) , antiderivative size = 2980, normalized size of antiderivative = 5.49

$$\int \frac{(d+ex^n)^2}{(a+bx^n+cx^{2n})^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[(d + e*x^n)^2/(a + b*x^n + c*x^(2*n))^2,x]
```

output
```
-((x*(-(a*Sqrt[b^2 - 4*a*c]*(b^2*d^2 + 2*a^2*e^2 + b*c*d^2*x^n + a*b*e*(-2
*d + e*x^n) - 2*a*c*d*(d + 2*e*x^n))) + (a*b*c*d^2*(a + x^n*(b + c*x^n))*(
Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, (b - Sqrt[b^2 - 4*a*c])/(b
 - Sqrt[b^2 - 4*a*c] + 2*c*x^n)]/((c*x^n)/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n
))^n^(-1) - Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, (b + Sqrt[b^2
- 4*a*c])/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)]/((c*x^n)/(b + Sqrt[b^2 - 4*a*
c] + 2*c*x^n))^n^(-1)))/2^n^(-1) - 2^(2 - n^(-1))*a^2*c*d*e*(a + x^n*(b +
c*x^n))*(Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, (b - Sqrt[b^2 - 4
*a*c])/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)]/((c*x^n)/(b - Sqrt[b^2 - 4*a*c]
+ 2*c*x^n))^n^(-1) - Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, (b +
Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)]/((c*x^n)/(b + Sqrt[b
^2 - 4*a*c] + 2*c*x^n))^n^(-1)) + (a^2*b*e^2*(a + x^n*(b + c*x^n))*(Hyperg
eometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, (b - Sqrt[b^2 - 4*a*c])/(b - Sqr
t[b^2 - 4*a*c] + 2*c*x^n)]/((c*x^n)/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n))^n^(
-1) - Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, (b + Sqrt[b^2 - 4*a*
c])/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)]/((c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2
*c*x^n))^n^(-1)))/2^n^(-1) - (a*b*c*d^2*n*(a + x^n*(b + c*x^n))*(Hypergeom
etric2F1[-n^(-1), -n^(-1), (-1 + n)/n, (b - Sqrt[b^2 - 4*a*c])/(b - Sqrt[b
^2 - 4*a*c] + 2*c*x^n)]/((c*x^n)/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n))^n^(-1)
 - Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, (b + Sqrt[b^2 - 4*a*...
```

3.76.3 Rubi [A] (verified)

Time = 1.55 (sec) , antiderivative size = 543, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1766, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^n)^2}{(a+bx^n+cx^{2n})^2}\,dx$$

$$\Big\downarrow \text{1766}$$

$$\int \left(\frac{-ae^2+x^n(2cde-be^2)+cd^2}{c(a+bx^n+cx^{2n})^2}+\frac{e^2}{c(a+bx^n+cx^{2n})}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{x\left((1-n)\left(abe^2-4acde+bcd^2\right)-\frac{b^2\left(ae^2(1-3n)-cd^2(1-n)\right)+4abcden+4ac(1-2n)\left(cd^2-ae^2\right)}{\sqrt{b^2-4ac}}\right)\text{Hypergeometric2F1}\left(1,\frac{1}{n},\right.}{an\left(b^2-4ac\right)\left(b-\sqrt{b^2-4ac}\right)}$$
$$\frac{x\left(\frac{b^2\left(ae^2(1-3n)-cd^2(1-n)\right)+4abcden+4ac(1-2n)\left(cd^2-ae^2\right)}{\sqrt{b^2-4ac}}+(1-n)\left(abe^2-4acde+bcd^2\right)\right)\text{Hypergeometric2F1}\left(1,\frac{1}{n},1\right.}{an\left(b^2-4ac\right)\left(\sqrt{b^2-4ac}+b\right)}$$
$$\frac{x\left(x^n\left(abe^2-4acde+bcd^2\right)-2abde-2a\left(cd^2-ae^2\right)+b^2d^2\right)}{an\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)}-$$
$$\frac{2e^2x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{-b\sqrt{b^2-4ac}-4ac+b^2}-$$
$$\frac{2e^2x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{b\sqrt{b^2-4ac}-4ac+b^2}$$

input
```
Int[(d + e*x^n)^2/(a + b*x^n + c*x^(2*n))^2,x]
```

output
```
(x*(b^2*d^2 - 2*a*b*d*e - 2*a*(c*d^2 - a*e^2) + (b*c*d^2 - 4*a*c*d*e + a*b
*e^2)*x^n))/(a*(b^2 - 4*a*c)*n*(a + b*x^n + c*x^(2*n))) - (2*e^2*x*Hyperge
ometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(b^
2 - 4*a*c - b*Sqrt[b^2 - 4*a*c]) - (((b*c*d^2 - 4*a*c*d*e + a*b*e^2)*(1 -
n) - (b^2*(a*e^2*(1 - 3*n) - c*d^2*(1 - n)) + 4*a*c*(c*d^2 - a*e^2)*(1 - 2
*n) + 4*a*b*c*d*e*n)/Sqrt[b^2 - 4*a*c])*x*Hypergeometric2F1[1, n^(-1), 1 +
 n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*(b - Sqrt[b
^2 - 4*a*c])*n) - (2*e^2*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*
x^n)/(b + Sqrt[b^2 - 4*a*c])])/(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c]) - (((b*
c*d^2 - 4*a*c*d*e + a*b*e^2)*(1 - n) + (b^2*(a*e^2*(1 - 3*n) - c*d^2*(1 -
n)) + 4*a*c*(c*d^2 - a*e^2)*(1 - 2*n) + 4*a*b*c*d*e*n)/Sqrt[b^2 - 4*a*c])*
x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*
c])])/(a*(b^2 - 4*a*c)*(b + Sqrt[b^2 - 4*a*c])*n)
```

3.76.3.1 Defintions of rubi rules used

rule 1766
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
))^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2
*n))^p, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ((IntegersQ[p, q] &&
!IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] &&  !IntegerQ[n]))
```

3.76. $\int \frac{(d+ex^n)^2}{(a+bx^n+cx^{2n})^2}\,dx$

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.76.4 Maple [F]

$$\int \frac{(d+e\,x^n)^2}{(a+b\,x^n+c\,x^{2n})^2}dx$$

input
```
int((d+e*x^n)^2/(a+b*x^n+c*x^(2*n))^2,x)
```

output
```
int((d+e*x^n)^2/(a+b*x^n+c*x^(2*n))^2,x)
```

3.76.5 Fricas [F]

$$\int \frac{(d+ex^n)^2}{(a+bx^n+cx^{2n})^2}\,dx = \int \frac{(ex^n+d)^2}{(cx^{2\,n}+bx^n+a)^2}\,dx$$

input
```
integrate((d+e*x^n)^2/(a+b*x^n+c*x^(2*n))^2,x, algorithm="fricas")
```

output
```
integral((e^2*x^(2*n) + 2*d*e*x^n + d^2)/(c^2*x^(4*n) + b^2*x^(2*n) + 2*a*
b*x^n + a^2 + 2*(b*c*x^n + a*c)*x^(2*n)), x)
```

3.76.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^n)^2}{(a+bx^n+cx^{2n})^2}\,dx = \text{Timed out}$$

input
```
integrate((d+e*x**n)**2/(a+b*x**n+c*x**(2*n))**2,x)
```

output
```
Timed out
```

3.76.7 Maxima [F]

$$\int \frac{(d+ex^n)^2}{(a+bx^n+cx^{2n})^2}\,dx = \int \frac{(ex^n+d)^2}{(cx^{2n}+bx^n+a)^2}\,dx$$

input
```
integrate((d+e*x^n)^2/(a+b*x^n+c*x^(2*n))^2,x, algorithm="maxima")
```

output
```
((b*c*d^2 - (4*c*d*e - b*e^2)*a)*x*x^n + (b^2*d^2 + 2*a^2*e^2 - 2*(c*d^2 +
 b*d*e)*a)*x)/(a^2*b^2*n - 4*a^3*c*n + (a*b^2*c*n - 4*a^2*c^2*n)*x^(2*n) +
 (a*b^3*n - 4*a^2*b*c*n)*x^n) - integrate(-(b^2*d^2*(n - 1) - 2*a^2*e^2 -
2*(c*d^2*(2*n - 1) - b*d*e)*a + (b*c*d^2*(n - 1) - (4*c*d*e*(n - 1) - b*e^
2*(n - 1))*a)*x^n)/(a^2*b^2*n - 4*a^3*c*n + (a*b^2*c*n - 4*a^2*c^2*n)*x^(2
*n) + (a*b^3*n - 4*a^2*b*c*n)*x^n), x)
```

3.76.8 Giac [F]

$$\int \frac{(d+ex^n)^2}{(a+bx^n+cx^{2n})^2}\,dx = \int \frac{(ex^n+d)^2}{(cx^{2n}+bx^n+a)^2}\,dx$$

input
```
integrate((d+e*x^n)^2/(a+b*x^n+c*x^(2*n))^2,x, algorithm="giac")
```

output
```
integrate((e*x^n + d)^2/(c*x^(2*n) + b*x^n + a)^2, x)
```

3.76.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^n)^2}{(a+bx^n+cx^{2n})^2}\,dx = \int \frac{(d+e\,x^n)^2}{(a+b\,x^n+c\,x^{2\,n})^2}\,dx$$

input
```
int((d + e*x^n)^2/(a + b*x^n + c*x^(2*n))^2,x)
```

output
```
int((d + e*x^n)^2/(a + b*x^n + c*x^(2*n))^2, x)
```

3.77 $\int \frac{d+ex^n}{\left(a+bx^n+cx^{2n}\right)^2}\,dx$

3.77.1 Optimal result

Integrand size = 24, antiderivative size = 362

$$\int \frac{d+ex^n}{\left(a+bx^n+cx^{2n}\right)^2}\,dx = \frac{x(b^2d-2acd-abe+c(bd-2ae)x^n)}{a\left(b^2-4ac\right)n\left(a+bx^n+cx^{2n}\right)}$$
$$-\frac{c\left(2a\left(2cd(1-2n)+\sqrt{b^2-4ac}e(1-n)\right)-b^2(d-dn)-b\left(\sqrt{b^2-4ac}d(1-n)-2aen\right)\right)x\,\text{Hypergeome}}{a\left(b^2-4ac\right)\left(b^2-4ac-b\sqrt{b^2-4ac}\right)n}$$
$$-\frac{c\left(2a\left(cd(2-4n)-\sqrt{b^2-4ac}e(1-n)\right)-b^2d(1-n)+b\left(\sqrt{b^2-4ac}d(1-n)+2aen\right)\right)x\,\text{Hypergeomet}}{a\left(b^2-4ac\right)\left(b^2-4ac+b\sqrt{b^2-4ac}\right)n}$$

output
```
x*(b^2*d-2*a*c*d-a*b*e+c*(-2*a*e+b*d)*x^n)/a/(-4*a*c+b^2)/n/(a+b*x^n+c*x^(
2*n))-c*x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(-b^
2*d*(1-n)+b*(2*a*e*n+d*(1-n)*(-4*a*c+b^2)^(1/2))+2*a*(c*d*(2-4*n)-e*(1-n)*
(-4*a*c+b^2)^(1/2)))/a/(-4*a*c+b^2)/n/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))-c*x
*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))*(-b^2*(-d*n+d
)-b*(-2*a*e*n+d*(1-n)*(-4*a*c+b^2)^(1/2))+2*a*(2*c*d*(1-2*n)+e*(1-n)*(-4*a
*c+b^2)^(1/2)))/a/(-4*a*c+b^2)/n/(b^2-4*a*c-b*(-4*a*c+b^2)^(1/2))
```

3.77.2 Mathematica [A] (verified)

Time = 4.18 (sec) , antiderivative size = 603, normalized size of antiderivative = 1.67

$$\int \frac{d+ex^n}{(a+bx^n+cx^{2n})^2}\,dx$$
$$= \frac{cx\left(\frac{4(b^2-4ac)\left(b^2d(-1+n)x^n(b+cx^n)-2a^2c(2dn+ex^n)+a\left(-2c^2d(-1+2n)x^{2n}+bcx^n(3d-4dn+ex^n)+b^2(dn+ex^n)\right)\right)}{\left(b^2-4ac-b\sqrt{b^2-4ac}\right)\left(b^2-4ac+b\sqrt{b^2-4ac}\right)(a+x^n(b+cx^n))}+\frac{2^{-1/n}\left(4ac\left(\sqrt{b^2-4\right.\right.}{}}{}\right.}{}$$

input
```
Integrate[(d + e*x^n)/(a + b*x^n + c*x^(2*n))^2,x]
```

output
```
(c*x*((4*(b^2 - 4*a*c)*(b^2*d*(-1 + n)*x^n*(b + c*x^n) - 2*a^2*c*(2*d*n +
e*x^n) + a*(-2*c^2*d*(-1 + 2*n)*x^(2*n) + b*c*x^n*(3*d - 4*d*n + e*x^n) +
b^2*(d*n + e*x^n))))/((b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*(b^2 - 4*a*c + b
*Sqrt[b^2 - 4*a*c])*(a + x^n*(b + c*x^n))) + ((4*a*c*(Sqrt[b^2 - 4*a*c]*d*
(1 - 2*n) + 2*a*e*(-1 + n)) + b^3*d*(-1 + n) + b^2*(Sqrt[b^2 - 4*a*c]*d -
2*a*e)*(-1 + n) + 2*a*b*(-2*c*d*(-1 + n) + Sqrt[b^2 - 4*a*c]*e*n))*Hyperge
ometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, (b - Sqrt[b^2 - 4*a*c])/(b - Sqrt
[b^2 - 4*a*c] + 2*c*x^n)])/(2^n^(-1)*Sqrt[b^2 - 4*a*c]*(-b^2 + 4*a*c + b*S
qrt[b^2 - 4*a*c])*((c*x^n)/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n))^n^(-1)) + ((
b*Sqrt[b^2 - 4*a*c]*d*(-1 + n) - 2*a*Sqrt[b^2 - 4*a*c]*e*(-1 + n) - 2*a*b*
e*n + 4*a*c*d*(-1 + 2*n) + b^2*(d - d*n))*Hypergeometric2F1[-n^(-1), -n^(-
1), (-1 + n)/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)]
)/(2^n^(-1)*Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*((c*x^n)/(b + Sqrt[b
^2 - 4*a*c] + 2*c*x^n))^n^(-1))))/(a*(-b^2 + 4*a*c)*n)
```

3.77.3 Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 312, normalized size of antiderivative = 0.86, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1760, 25, 1752, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^n}{(a+bx^n+cx^{2n})^2}\,dx$$
$$\Big\downarrow \; 1760$$

$$\frac{x\left(cx^n(bd-2ae)-abe-2acd+b^2d\right)}{an\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)}-\frac{\int -\frac{-c(bd-2ae)(1-n)x^n+abe+2acd(1-2n)-b^2(d-dn)}{bx^n+cx^{2n}+a}dx}{an\left(b^2-4ac\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{-c(bd-2ae)(1-n)x^n+abe+2acd(1-2n)-b^2d(1-n)}{bx^n+cx^{2n}+a}dx}{an\left(b^2-4ac\right)}+\frac{x\left(cx^n(bd-2ae)-abe-2acd+b^2d\right)}{an\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)}$$

$$\Big\downarrow \; 1752$$

$$\frac{\frac{c\left(-(1-n)\sqrt{b^2-4ac}(bd-2ae)+2aben+2acd(2-4n)+b^2(-d)(1-n)\right)\int \frac{1}{cx^n+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx}{2\sqrt{b^2-4ac}}-\frac{1}{2}c\left(\frac{2aben+4acd(1-2n)+b^2(-d)(1-n)}{\sqrt{b^2-4ac}}+(1 -\right.}{an\left(b^2-4ac\right)}$$

$$\frac{x\left(cx^n(bd-2ae)-abe-2acd+b^2d\right)}{an\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)}$$

$$\Big\downarrow \; 778$$

$$\frac{\frac{cx\left(-(1-n)\sqrt{b^2-4ac}(bd-2ae)+2aben+2acd(2-4n)+b^2(-d)(1-n)\right)\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{cx\left(\frac{2aben+4acd(1-2n)+b^2}{\sqrt{b^2-4ac}}\right.}{}}{an\left(b^2-4ac\right)}$$

$$\frac{x\left(cx^n(bd-2ae)-abe-2acd+b^2d\right)}{an\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)}$$

input
```
Int[(d + e*x^n)/(a + b*x^n + c*x^(2*n))^2,x]
```

output
```
(x*(b^2*d - 2*a*c*d - a*b*e + c*(b*d - 2*a*e)*x^n))/(a*(b^2 - 4*a*c)*n*(a
+ b*x^n + c*x^(2*n))) + ((c*(2*a*c*d*(2 - 4*n) - b^2*d*(1 - n) - Sqrt[b^2
- 4*a*c]*(b*d - 2*a*e)*(1 - n) + 2*a*b*e*n)*x*Hypergeometric2F1[1, n^(-1),
 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b -
Sqrt[b^2 - 4*a*c])) - (c*((b*d - 2*a*e)*(1 - n) + (4*a*c*d*(1 - 2*n) - b^2
*d*(1 - n) + 2*a*b*e*n)/Sqrt[b^2 - 4*a*c])*x*Hypergeometric2F1[1, n^(-1),
1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(b + Sqrt[b^2 - 4*a*c]))/
(a*(b^2 - 4*a*c)*n)
```

3.77. $\int \frac{d+ex^n}{\left(a+bx^n+cx^{2n}\right)^2}\,dx$

3.77.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 1752
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2
 - 4*a*c] ||  !IGtQ[n/2, 0])
```

rule 1760
```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p
_), x_Symbol] :> Simp[(-x)*(d*b^2 - a*b*e - 2*a*c*d + (b*d - 2*a*e)*c*x^n)*
((a + b*x^n + c*x^(2*n))^(p + 1)/(a*n*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/
(a*n*(p + 1)*(b^2 - 4*a*c))   Int[Simp[(n*p + n + 1)*d*b^2 - a*b*e - 2*a*c*
d*(2*n*p + 2*n + 1) + (2*n*p + 3*n + 1)*(d*b - 2*a*e)*c*x^n, x]*(a + b*x^n
+ c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n
] && NeQ[b^2 - 4*a*c, 0] && ILtQ[p, -1]
```

3.77.4 Maple [F]

$$\int \frac{d + e\,x^n}{\left(a + b\,x^n + c\,x^{2n}\right)^2}dx$$

input
```
int((d+e*x^n)/(a+b*x^n+c*x^(2*n))^2,x)
```

output
```
int((d+e*x^n)/(a+b*x^n+c*x^(2*n))^2,x)
```

3.77.5 Fricas [F]

$$\int \frac{d+ex^n}{(a+bx^n+cx^{2n})^2}\,dx=\int \frac{ex^n+d}{(cx^{2n}+bx^n+a)^2}\,dx$$

input
```
integrate((d+e*x^n)/(a+b*x^n+c*x^(2*n))^2,x, algorithm="fricas")
```

output
```
integral((e*x^n + d)/(c^2*x^(4*n) + b^2*x^(2*n) + 2*a*b*x^n + a^2 + 2*(b*c
*x^n + a*c)*x^(2*n)), x)
```

3.77.6 Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^n}{(a+bx^n+cx^{2n})^2}\,dx=\text{Timed out}$$

input
```
integrate((d+e*x**n)/(a+b*x**n+c*x**(2*n))**2,x)
```

output
```
Timed out
```

3.77.7 Maxima [F]

$$\int \frac{d+ex^n}{(a+bx^n+cx^{2n})^2}\,dx=\int \frac{ex^n+d}{(cx^{2n}+bx^n+a)^2}\,dx$$

input
```
integrate((d+e*x^n)/(a+b*x^n+c*x^(2*n))^2,x, algorithm="maxima")
```

output
```
((b*c*d - 2*a*c*e)*x*x^n + (b^2*d - (2*c*d + b*e)*a)*x)/(a^2*b^2*n - 4*a^3
*c*n + (a*b^2*c*n - 4*a^2*c^2*n)*x^(2*n) + (a*b^3*n - 4*a^2*b*c*n)*x^n) +
integrate((b^2*d*(n - 1) - (2*c*d*(2*n - 1) - b*e)*a + (b*c*d*(n - 1) - 2*
a*c*e*(n - 1))*x^n)/(a^2*b^2*n - 4*a^3*c*n + (a*b^2*c*n - 4*a^2*c^2*n)*x^(
2*n) + (a*b^3*n - 4*a^2*b*c*n)*x^n), x)
```

3.77.8 Giac [F]

$$\int \frac{d+ex^n}{(a+bx^n+cx^{2n})^2}\,dx = \int \frac{ex^n+d}{(cx^{2n}+bx^n+a)^2}\,dx$$

input
```
integrate((d+e*x^n)/(a+b*x^n+c*x^(2*n))^2,x, algorithm="giac")
```

output
```
integrate((e*x^n + d)/(c*x^(2*n) + b*x^n + a)^2, x)
```

3.77.9 Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^n}{(a+bx^n+cx^{2n})^2}\,dx = \int \frac{d+e\,x^n}{(a+b\,x^n+c\,x^{2\,n})^2}\,dx$$

input
```
int((d + e*x^n)/(a + b*x^n + c*x^(2*n))^2,x)
```

output
```
int((d + e*x^n)/(a + b*x^n + c*x^(2*n))^2, x)
```

3.78 $\int \frac{1}{(d+ex^n)\left(a+bx^n+cx^{2n}\right)^2}\,dx$

3.78.1 Optimal result

Integrand size = 26, antiderivative size = 726

$$\int \frac{1}{(d+ex^n)\left(a+bx^n+cx^{2n}\right)^2}\,dx = \frac{x(b^2cd-2ac^2d-b^3e+3abce+c(bcd-b^2e+2ace)\,x^n)}{a\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)n\left(a+bx^n+cx^{2n}\right)}$$

$$-\frac{ce^2\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac-b\sqrt{b^2-4ac}\right)\left(cd^2-bde+ae^2\right)^2}$$

$$-\frac{c\left(\frac{2abce(2-3n)-4ac^2d(1-2n)+b^2cd(1-n)-b^3e(1-n)}{\sqrt{b^2-4ac}}+(bcd-b^2e+2ace)\,(1-n)\right)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1-\right.}{a\left(b^2-4ac\right)\left(b-\sqrt{b^2-4ac}\right)\left(cd^2-bde+ae^2\right)n}$$

$$-\frac{ce^2\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac+b\sqrt{b^2-4ac}\right)\left(cd^2-bde+ae^2\right)^2}$$

$$+\frac{c\left(bc\left(2ae(2-3n)-\sqrt{b^2-4ac}d(1-n)\right)-2ac\left(2cd(1-2n)+\sqrt{b^2-4ac}e(1-n)\right)-b^3e(1-n)+b^2\left(c\right.\right.}{a\left(b^2-4ac\right)\left(b^2-4ac+b\sqrt{b^2-4ac}\right)\left(cd^2-\right.}$$

$$+\frac{e^4x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)}{d\left(cd^2-bde+ae^2\right)^2}$$

output
```
x*(b^2*c*d-2*a*c^2*d-b^3*e+3*a*b*c*e+c*(2*a*c*e-b^2*e+b*c*d)*x^n)/a/(-4*a*
c+b^2)/(a*e^2-b*d*e+c*d^2)/n/(a+b*x^n+c*x^(2*n))+e^4*x*hypergeom([1, 1/n],
[1+1/n],-e*x^n/d)/d/(a*e^2-b*d*e+c*d^2)^2-c*x*hypergeom([1, 1/n],[1+1/n],-
2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))*((2*a*c*e-b^2*e+b*c*d)*(1-n)+(2*a*b*c*e*(2
-3*n)-4*a*c^2*d*(1-2*n)+b^2*c*d*(1-n)-b^3*e*(1-n))/(-4*a*c+b^2)^(1/2))/a/(
-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)/n/(b-(-4*a*c+b^2)^(1/2))-c*e^2*x*hypergeom
([1, 1/n],[1+1/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(2*c*d-e*(b-(-4*a*c+b^2
)^(1/2)))/(a*e^2-b*d*e+c*d^2)^2/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))-c*e^2*x*h
ypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))*(2*c*d-e*(b+(-4
*a*c+b^2)^(1/2)))/(a*e^2-b*d*e+c*d^2)^2/(b^2-4*a*c-b*(-4*a*c+b^2)^(1/2))+c
*x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(-b^3*e*(1-
n)+b^2*(1-n)*(c*d+e*(-4*a*c+b^2)^(1/2))+b*c*(2*a*e*(2-3*n)-d*(1-n)*(-4*a*c
+b^2)^(1/2))-2*a*c*(2*c*d*(1-2*n)+e*(1-n)*(-4*a*c+b^2)^(1/2)))/a/(-4*a*c+b
^2)/(a*e^2-b*d*e+c*d^2)/n/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))
```

3.78.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 11767 vs. $2(726) = 1452$.

Time = 6.91 (sec) , antiderivative size = 11767, normalized size of antiderivative = 16.21

$$\int \frac{1}{(d+ex^n)(a+bx^n+cx^{2n})^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[1/((d + e*x^n)*(a + b*x^n + c*x^(2*n))^2),x]
```

output
```
Result too large to show
```

3.78.3 Rubi [A] (verified)

Time = 1.66 (sec) , antiderivative size = 726, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1766, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^n)(a+bx^n+cx^{2n})^2}\,dx$$

$$\Big\downarrow \text{1766}$$

$$\int \left(-\frac{e^2(be-cd+cex^n)}{(ae^2-bde+cd^2)^2(a+bx^n+cx^{2n})}+\frac{-be+cd-cex^n}{(ae^2-bde+cd^2)(a+bx^n+cx^{2n})^2}+\frac{e^4}{(d+ex^n)(ae^2-bde+cd^2)^2}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ce^2x\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{\left(-b\sqrt{b^2-4ac}-4ac+b^2\right)(ae^2-bde+cd^2)^2}-$$

$$\frac{ce^2x\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\left(b\sqrt{b^2-4ac}-4ac+b^2\right)(ae^2-bde+cd^2)^2}-$$

$$\frac{cx\left((1-n)(2ace+b^2(-e)+bcd)+\frac{2abce(2-3n)-4ac^2d(1-2n)+b^3(-e)(1-n)+b^2cd(1-n)}{\sqrt{b^2-4ac}}\right)\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\right.}{an(b^2-4ac)\left(b-\sqrt{b^2-4ac}\right)(ae^2-bde+cd^2)}$$

$$\frac{x(cx^n(2ace+b^2(-e)+bcd)+3abce-2ac^2d-b^3e+b^2cd)}{an(b^2-4ac)(ae^2-bde+cd^2)(a+bx^n+cx^{2n})}+$$

$$\frac{cx\left(b^2(1-n)\left(e\sqrt{b^2-4ac}+cd\right)+bc\left(2ae(2-3n)-d(1-n)\sqrt{b^2-4ac}\right)-2ac\left(e(1-n)\sqrt{b^2-4ac}+2cd(1-2\right.\right.}{an(b^2-4ac)\left(b\sqrt{b^2-4ac}-4ac+b^2\right)(ae^2-bde}$$

$$\frac{e^4x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)}{d(ae^2-bde+cd^2)^2}$$

input
```
Int[1/((d + e*x^n)*(a + b*x^n + c*x^(2*n))^2),x]
```

3.78. $\int \frac{1}{(d+ex^n)(a+bx^n+cx^{2n})^2}\,dx$

output
```
(x*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e + c*(b*c*d - b^2*e + 2*a*c*e)*
x^n))/(a*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*n*(a + b*x^n + c*x^(2*n)))
- (c*e^2*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x*Hypergeometric2F1[1, n^(-1)
, 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/((b^2 - 4*a*c - b*Sqrt[
b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)^2) - (c*((2*a*b*c*e*(2 - 3*n) - 4*a*
c^2*d*(1 - 2*n) + b^2*c*d*(1 - n) - b^3*e*(1 - n))/Sqrt[b^2 - 4*a*c] + (b*
c*d - b^2*e + 2*a*c*e)*(1 - n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1),
 (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*(b - Sqrt[b^2 - 4*a
*c])*(c*d^2 - b*d*e + a*e^2)*n) - (c*e^2*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*
e)*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4
*a*c])])/((b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)^2) +
 (c*(b*c*(2*a*e*(2 - 3*n) - Sqrt[b^2 - 4*a*c]*d*(1 - n)) - 2*a*c*(2*c*d*(1
 - 2*n) + Sqrt[b^2 - 4*a*c]*e*(1 - n)) - b^3*e*(1 - n) + b^2*(c*d + Sqrt[b
^2 - 4*a*c]*e)*(1 - n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x
^n)/(b + Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*(b^2 - 4*a*c + b*Sqrt[b^2 -
 4*a*c])*(c*d^2 - b*d*e + a*e^2)*n) + (e^4*x*Hypergeometric2F1[1, n^(-1),
1 + n^(-1), -((e*x^n)/d)])/(d*(c*d^2 - b*d*e + a*e^2)^2)
```

3.78.3.1 Defintions of rubi rules used

rule 1766
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
))^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2
*n))^p, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ((IntegersQ[p, q] &&
!IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] &&  !IntegerQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.78.4 Maple [F]

$$\int \frac{1}{(d+e\,x^n)\,(a+b\,x^n+c\,x^{2n})^2}dx$$

input
```
int(1/(d+e*x^n)/(a+b*x^n+c*x^(2*n))^2,x)
```

output
```
int(1/(d+e*x^n)/(a+b*x^n+c*x^(2*n))^2,x)
```

3.78.5 Fricas [F]

$$\int \frac{1}{(d+ex^n)\left(a+bx^n+cx^{2n}\right)^2}\,dx = \int \frac{1}{(cx^{2n}+bx^n+a)^2(ex^n+d)}\,dx$$

input
```
integrate(1/(d+e*x^n)/(a+b*x^n+c*x^(2*n))^2,x, algorithm="fricas")
```

output
```
integral(1/(b^2*e*x^(3*n) + a^2*d + (c^2*e*x^n + c^2*d)*x^(4*n) + 2*(b*c*e
*x^(2*n) + a*c*d + (b*c*d + a*c*e)*x^n)*x^(2*n) + (b^2*d + 2*a*b*e)*x^(2*n
) + (2*a*b*d + a^2*e)*x^n), x)
```

3.78.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^n)\left(a+bx^n+cx^{2n}\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(d+e*x**n)/(a+b*x**n+c*x**(2*n))**2,x)
```

output
```
Timed out
```

3.78.7 Maxima [F]

$$\int \frac{1}{(d+ex^n)\left(a+bx^n+cx^{2n}\right)^2}\,dx = \int \frac{1}{(cx^{2n}+bx^n+a)^2(ex^n+d)}\,dx$$

input
```
integrate(1/(d+e*x^n)/(a+b*x^n+c*x^(2*n))^2,x, algorithm="maxima")
```

output
```
e^4*integrate(1/(c^2*d^5 - 2*b*c*d^4*e + b^2*d^3*e^2 + a^2*d*e^4 + 2*(c*d^
3*e^2 - b*d^2*e^3)*a + (c^2*d^4*e - 2*b*c*d^3*e^2 + b^2*d^2*e^3 + a^2*e^5
+ 2*(c*d^2*e^3 - b*d*e^4)*a)*x^n), x) - ((b*c^2*d - b^2*c*e + 2*a*c^2*e)*x
*x^n + (b^2*c*d - b^3*e - (2*c^2*d - 3*b*c*e)*a)*x)/(4*a^4*c*e^2*n + (4*c^
2*d^2*n - 4*b*c*d*e*n - b^2*e^2*n)*a^3 - (b^2*c*d^2*n - b^3*d*e*n)*a^2 + (
4*a^3*c^2*e^2*n + (4*c^3*d^2*n - 4*b*c^2*d*e*n - b^2*c*e^2*n)*a^2 - (b^2*c
^2*d^2*n - b^3*c*d*e*n)*a)*x^(2*n) + (4*a^3*b*c*e^2*n + (4*b*c^2*d^2*n - 4
*b^2*c*d*e*n - b^3*e^2*n)*a^2 - (b^3*c*d^2*n - b^4*d*e*n)*a)*x^n) - integr
ate((b^2*c^2*d^3*(n - 1) - 2*b^3*c*d^2*e*(n - 1) + b^4*d*e^2*(n - 1) + (b*
c*e^3*(8*n - 3) - 2*c^2*d*e^2*(4*n - 1))*a^2 + (b*c^2*d^2*e*(8*n - 5) - 2*
c^3*d^3*(2*n - 1) - b^3*e^3*(2*n - 1) - 2*b^2*c*d*e^2*(n - 1))*a + (2*a^2*
c^2*e^3*(3*n - 1) + b*c^3*d^3*(n - 1) - 2*b^2*c^2*d^2*e*(n - 1) + b^3*c*d*
e^2*(n - 1) - (b^2*c*e^3*(2*n - 1) - 2*c^3*d^2*e*(n - 1) + b*c^2*d*e^2*(n
- 1))*a)*x^n)/(4*a^5*c*e^4*n + (8*c^2*d^2*e^2*n - 8*b*c*d*e^3*n - b^2*e^4*
n)*a^4 + 2*(2*c^3*d^4*n - 4*b*c^2*d^3*e*n + b^2*c*d^2*e^2*n + b^3*d*e^3*n)
*a^3 - (b^2*c^2*d^4*n - 2*b^3*c*d^3*e*n + b^4*d^2*e^2*n)*a^2 + (4*a^4*c^2*
e^4*n + (8*c^3*d^2*e^2*n - 8*b*c^2*d*e^3*n - b^2*c*e^4*n)*a^3 + 2*(2*c^4*d
^4*n - 4*b*c^3*d^3*e*n + b^2*c^2*d^2*e^2*n + b^3*c*d*e^3*n)*a^2 - (b^2*c^3
*d^4*n - 2*b^3*c^2*d^3*e*n + b^4*c*d^2*e^2*n)*a)*x^(2*n) + (4*a^4*b*c*e^4*
n + (8*b*c^2*d^2*e^2*n - 8*b^2*c*d*e^3*n - b^3*e^4*n)*a^3 + 2*(2*b*c^3*...
```

3.78.8 Giac [F]

$$\int \frac{1}{(d+ex^n)\left(a+bx^n+cx^{2n}\right)^2}\,dx = \int \frac{1}{\left(cx^{2\,n}+bx^n+a\right)^2(ex^n+d)}\,dx$$

input
```
integrate(1/(d+e*x^n)/(a+b*x^n+c*x^(2*n))^2,x, algorithm="giac")
```

output
```
integrate(1/((c*x^(2*n) + b*x^n + a)^2*(e*x^n + d)), x)
```

3.78.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^n)\left(a+bx^n+cx^{2n}\right)^2}\,dx=\int \frac{1}{(d+e\,x^n)\,\left(a+b\,x^n+c\,x^{2\,n}\right)^2}\,dx$$

input
```
int(1/((d + e*x^n)*(a + b*x^n + c*x^(2*n))^2),x)
```

output
```
int(1/((d + e*x^n)*(a + b*x^n + c*x^(2*n))^2), x)
```

3.79 $\int \frac{1}{(d+ex^n)^2(a+bx^n+cx^{2n})^2}\,dx$

3.79.1 Optimal result

Integrand size = 26, antiderivative size = 1129

$$\int \frac{1}{(d+ex^n)^2(a+bx^n+cx^{2n})^2}\,dx =$$

$$-\frac{x(2b^3cde-6abc^2de-b^4e^2-b^2c(cd^2-4ae^2)+2ac^2(cd^2-ae^2)+c(2b^2cde-4ac^2de-b^3e^2-bc(cd^2-}{a(b^2-4ac)(cd^2-bde+ae^2)^2n(a+bx^n+cx^{2n})}$$

$$-\frac{2ce^2\left(3c^2d^2+b\left(b+\sqrt{b^2-4ac}\right)e^2-ce\left(3bd+2\sqrt{b^2-4ac}d+ae\right)\right)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\right.}{\left(b^2-4ac-b\sqrt{b^2-4ac}\right)(cd^2-bde+ae^2)^3}$$

$$+\frac{c\left(4ac^2\left(e\left(ae(1-2n)+\sqrt{b^2-4ac}d(1-n)\right)-cd^2(1-2n)\right)-b^2c\left(e\left(ae(5-7n)+2\sqrt{b^2-4ac}d(1-n)\right)\right.\right.}{}$$

$$-\frac{2ce^2\left(3c^2d^2+b\left(b-\sqrt{b^2-4ac}\right)e^2-ce\left(3bd-2\sqrt{b^2-4ac}d+ae\right)\right)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\right.}{\left(b^2-4ac+b\sqrt{b^2-4ac}\right)(cd^2-bde+ae^2)^3}$$

$$+\frac{c\left(4ac^2\left(e\left(ae(1-2n)-\sqrt{b^2-4ac}d(1-n)\right)-cd^2(1-2n)\right)-b^2c\left(e\left(ae(5-7n)-2\sqrt{b^2-4ac}d(1-n)\right)\right.\right.}{}$$

$$+\frac{2e^4(2cd-be)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)}{d(cd^2-bde+ae^2)^3}$$

$$+\frac{e^4x\,\text{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)}{d^2(cd^2-bde+ae^2)^2}$$

output
```
-x*(2*b^3*c*d*e-6*a*b*c^2*d*e-b^4*e^2-b^2*c*(-4*a*e^2+c*d^2)+2*a*c^2*(-a*e
^2+c*d^2)+c*(2*b^2*c*d*e-4*a*c^2*d*e-b^3*e^2-b*c*(-3*a*e^2+c*d^2))*x^n)/a/
(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)^2/n/(a+b*x^n+c*x^(2*n))+2*e^4*(-b*e+2*c*d
)*x*hypergeom([1, 1/n],[1+1/n],-e*x^n/d)/d/(a*e^2-b*d*e+c*d^2)^3+e^4*x*hyp
ergeom([2, 1/n],[1+1/n],-e*x^n/d)/d^2/(a*e^2-b*d*e+c*d^2)^2-2*c*e^2*x*hype
rgeom([1, 1/n],[1+1/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(3*c^2*d^2+b*e^2*(
b-(-4*a*c+b^2)^(1/2))-c*e*(3*b*d+a*e-2*d*(-4*a*c+b^2)^(1/2)))/(a*e^2-b*d*e
+c*d^2)^3/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))-2*c*e^2*x*hypergeom([1, 1/n],[1
+1/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))*(3*c^2*d^2+b*e^2*(b+(-4*a*c+b^2)^(1
/2))-c*e*(3*b*d+a*e+2*d*(-4*a*c+b^2)^(1/2)))/(a*e^2-b*d*e+c*d^2)^3/(b^2-4*
a*c-b*(-4*a*c+b^2)^(1/2))+c*x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b+(-4*a
*c+b^2)^(1/2)))*(b^4*e^2*(1-n)-b^3*e*(1-n)*(2*c*d+e*(-4*a*c+b^2)^(1/2))-b^
2*c*(-c*d^2*(1-n)+e*(a*e*(5-7*n)-2*d*(1-n)*(-4*a*c+b^2)^(1/2)))+b*c*(3*a*e
^2*(1-n)*(-4*a*c+b^2)^(1/2)+c*d*(4*a*e*(2-3*n)-d*(1-n)*(-4*a*c+b^2)^(1/2))
)+4*a*c^2*(-c*d^2*(1-2*n)+e*(a*e*(1-2*n)-d*(1-n)*(-4*a*c+b^2)^(1/2))))/a/(
-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)^2/n/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))+c*x*h
ypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))*(b^4*e^2*(1-n)-
b^3*e*(1-n)*(2*c*d-e*(-4*a*c+b^2)^(1/2))+b*c*(-3*a*e^2*(1-n)*(-4*a*c+b^2)^
(1/2)+c*d*(4*a*e*(2-3*n)+d*(1-n)*(-4*a*c+b^2)^(1/2)))+4*a*c^2*(-c*d^2*(1-2
*n)+e*(a*e*(1-2*n)+d*(1-n)*(-4*a*c+b^2)^(1/2)))-b^2*c*(-c*d^2*(1-n)+e*(...
```

3.79.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 16855 vs. 2(1129) = 2258.

Time = 7.87 (sec) , antiderivative size = 16855, normalized size of antiderivative = 14.93

$$\int \frac{1}{(d+ex^n)^2 (a+bx^n+cx^{2n})^2} dx = \text{Result too large to show}$$

input
```
Integrate[1/((d + e*x^n)^2*(a + b*x^n + c*x^(2*n))^2),x]
```

output
```
Result too large to show
```

3.79.3 Rubi [A] (verified)

Time = 2.84 (sec) , antiderivative size = 1129, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1766, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^n)^2 (a+bx^n+cx^{2n})^2}dx$$

$$\Big\downarrow \text{1766}$$

$$\int \left(\frac{e^2(-ace^2+2b^2e^2+x^n(2bce^2-4c^2de)-5bcde+3c^2d^2)}{(ae^2-bde+cd^2)^3(a+bx^n+cx^{2n})}+\frac{-ace^2+b^2e^2-(x^n(2c^2de-bce^2))-2bcde+c^2\right.$$
$$\left.\overline{(ae^2-bde+cd^2)^2(a+bx^n+cx^{2n})^2}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{2(2cd-be)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)e^4}{d(cd^2-bed+ae^2)^3}+$$
$$\frac{x\,\text{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)e^4}{d^2(cd^2-bed+ae^2)^2}-$$
$$\frac{2c\left(3c^2d^2+b\left(b+\sqrt{b^2-4ac}\right)e^2-ce\left(3bd+2\sqrt{b^2-4ac}d+ae\right)\right)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right.}{\left(b^2-\sqrt{b^2-4ac}b-4ac\right)(cd^2-bed+ae^2)^3}$$
$$\frac{2c\left(3c^2d^2+b\left(b-\sqrt{b^2-4ac}\right)e^2-ce\left(3bd-2\sqrt{b^2-4ac}d+ae\right)\right)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right.}{\left(b^2+\sqrt{b^2-4ac}b-4ac\right)(cd^2-bed+ae^2)^3}$$
$$\frac{c\left(e^2(1-n)b^4-e\left(2cd-\sqrt{b^2-4ac}e\right)(1-n)b^3-c\left(e\left(ae(5-7n)+2\sqrt{b^2-4ac}d(1-n)\right)-cd^2(1-n)\right)b^2+c\left(\right.}{}$$
$$\frac{c\left(e^2(1-n)b^4-e\left(2cd+\sqrt{b^2-4ac}e\right)(1-n)b^3-c\left(e\left(ae(5-7n)-2\sqrt{b^2-4ac}d(1-n)\right)-cd^2(1-n)\right)b^2+c\left(\right.}{}$$
$$\frac{x(c(-e^2b^3+2cdeb^2-c(cd^2-3ae^2)b-4ac^2de)x^n-b^4e^2-6abc^2de+2b^3cde-b^2c(cd^2-4ae^2)+2ac^2(cd^2-a}{a(b^2-4ac)(cd^2-bed+ae^2)^2n(bx^n+cx^{2n}+a)}$$

input
```
Int[1/((d + e*x^n)^2*(a + b*x^n + c*x^(2*n))^2),x]
```

output
```
-((x*(2*b^3*c*d*e - 6*a*b*c^2*d*e - b^4*e^2 - b^2*c*(c*d^2 - 4*a*e^2) + 2*
a*c^2*(c*d^2 - a*e^2) + c*(2*b^2*c*d*e - 4*a*c^2*d*e - b^3*e^2 - b*c*(c*d^
2 - 3*a*e^2))*x^n))/(a*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)^2*n*(a + b*x^
n + c*x^(2*n)))) - (2*c*e^2*(3*c^2*d^2 + b*(b + Sqrt[b^2 - 4*a*c])*e^2 - c
*e*(3*b*d + 2*Sqrt[b^2 - 4*a*c]*d + a*e))*x*Hypergeometric2F1[1, n^(-1), 1
 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/((b^2 - 4*a*c - b*Sqrt[b^2
 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)^3) + (c*(4*a*c^2*(e*(a*e*(1 - 2*n) + Sq
rt[b^2 - 4*a*c]*d*(1 - n)) - c*d^2*(1 - 2*n)) - b^2*c*(e*(a*e*(5 - 7*n) +
2*Sqrt[b^2 - 4*a*c]*d*(1 - n)) - c*d^2*(1 - n)) + b*c*(c*d*(4*a*e*(2 - 3*n
) + Sqrt[b^2 - 4*a*c]*d*(1 - n)) - 3*a*Sqrt[b^2 - 4*a*c]*e^2*(1 - n)) + b^
4*e^2*(1 - n) - b^3*e*(2*c*d - Sqrt[b^2 - 4*a*c]*e)*(1 - n))*x*Hypergeomet
ric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(a*(b^2
 - 4*a*c)*(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)^2*n)
 - (2*c*e^2*(3*c^2*d^2 + b*(b - Sqrt[b^2 - 4*a*c])*e^2 - c*e*(3*b*d - 2*Sq
rt[b^2 - 4*a*c]*d + a*e))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c
*x^n)/(b + Sqrt[b^2 - 4*a*c])])/((b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*(c*d^
2 - b*d*e + a*e^2)^3) + (c*(4*a*c^2*(e*(a*e*(1 - 2*n) - Sqrt[b^2 - 4*a*c]*
d*(1 - n)) - c*d^2*(1 - 2*n)) - b^2*c*(e*(a*e*(5 - 7*n) - 2*Sqrt[b^2 - 4*a
*c]*d*(1 - n)) - c*d^2*(1 - n)) + b*c*(c*d*(4*a*e*(2 - 3*n) - Sqrt[b^2 - 4
*a*c]*d*(1 - n)) + 3*a*Sqrt[b^2 - 4*a*c]*e^2*(1 - n)) + b^4*e^2*(1 - n)...
```

3.79.3.1 Defintions of rubi rules used

rule 1766
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
))^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2
*n))^p, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ((IntegersQ[p, q] &&
!IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] &&  !IntegerQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.79.4 Maple [F]

$$\int \frac{1}{\left(d+e\,x^{n}\right)^{2}\left(a+b\,x^{n}+c\,x^{2 n}\right)^{2}}d x$$

input
```
int(1/(d+e*x^n)^2/(a+b*x^n+c*x^(2*n))^2,x)
```

output
```
int(1/(d+e*x^n)^2/(a+b*x^n+c*x^(2*n))^2,x)
```

3.79.5 Fricas [F]

$$\int \frac{1}{(d+ex^n)^2\left(a+bx^n+cx^{2n}\right)^2}\,dx=\int \frac{1}{(cx^{2\,n}+bx^n+a)^2(ex^n+d)^2}\,dx$$

input
```
integrate(1/(d+e*x^n)^2/(a+b*x^n+c*x^(2*n))^2,x, algorithm="fricas")
```

output
```
integral(1/(b^2*e^2*x^(4*n) + a^2*d^2 + (c^2*e^2*x^(2*n) + 2*c^2*d*e*x^n +
 c^2*d^2)*x^(4*n) + 2*(b^2*d*e + a*b*e^2)*x^(3*n) + 2*(b*c*e^2*x^(3*n) + a
*c*d^2 + (2*b*c*d*e + a*c*e^2)*x^(2*n) + (b*c*d^2 + 2*a*c*d*e)*x^n)*x^(2*n
) + (b^2*d^2 + 4*a*b*d*e + a^2*e^2)*x^(2*n) + 2*(a*b*d^2 + a^2*d*e)*x^n),
x)
```

3.79.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^n)^2\left(a+bx^n+cx^{2n}\right)^2}\,dx=\text{Timed out}$$

input
```
integrate(1/(d+e*x**n)**2/(a+b*x**n+c*x**(2*n))**2,x)
```

output
```
Timed out
```

3.79.7 Maxima [F]

$$\int \frac{1}{(d+ex^n)^2 (a+bx^n+cx^{2n})^2} dx = \int \frac{1}{(cx^{2n}+bx^n+a)^2 (ex^n+d)^2} dx$$

input
```
integrate(1/(d+e*x^n)^2/(a+b*x^n+c*x^(2*n))^2,x, algorithm="maxima")
```

output
```
(c*d^2*e^4*(5*n - 1) - b*d*e^5*(3*n - 1) + a*e^6*(n - 1))*integrate(1/(c^3
*d^8*n - 3*b*c^2*d^7*e*n + 3*b^2*c*d^6*e^2*n - b^3*d^5*e^3*n + a^3*d^2*e^6
*n + 3*(c*d^4*e^4*n - b*d^3*e^5*n)*a^2 + 3*(c^2*d^6*e^2*n - 2*b*c*d^5*e^3*
n + b^2*d^4*e^4*n)*a + (c^3*d^7*e*n - 3*b*c^2*d^6*e^2*n + 3*b^2*c*d^5*e^3*
n - b^3*d^4*e^4*n + a^3*d*e^7*n + 3*(c*d^3*e^5*n - b*d^2*e^6*n)*a^2 + 3*(c
^2*d^5*e^3*n - 2*b*c*d^4*e^4*n + b^2*d^3*e^5*n)*a)*x^n), x) - ((b*c^3*d^3*
e - 2*b^2*c^2*d^2*e^2 + b^3*c*d*e^3 - 4*a^2*c^2*e^4 + (4*c^3*d^2*e^2 - 3*b
*c^2*d*e^3 + b^2*c*e^4)*a)*x*x^(2*n) + (b*c^3*d^4 - b^2*c^2*d^3*e - b^3*c*
d^2*e^2 + b^4*d*e^3 + 2*(c^2*d*e^3 - 2*b*c*e^4)*a^2 + (2*c^3*d^3*e + 3*b*c
^2*d^2*e^2 - 4*b^2*c*d*e^3 + b^3*e^4)*a)*x*x^n + (b^2*c^2*d^4 - 2*b^3*c*d^
3*e + b^4*d^2*e^2 - 4*a^3*c*e^4 + (2*c^2*d^2*e^2 + b^2*e^4)*a^2 - 2*(c^3*d
^4 - 3*b*c^2*d^3*e + 2*b^2*c*d^2*e^2)*a)*x)/(4*a^5*c*d^2*e^4*n + (8*c^2*d^
4*e^2*n - 8*b*c*d^3*e^3*n - b^2*d^2*e^4*n)*a^4 + 2*(2*c^3*d^6*n - 4*b*c^2*
d^5*e*n + b^2*c*d^4*e^2*n + b^3*d^3*e^3*n)*a^3 - (b^2*c^2*d^6*n - 2*b^3*c*
d^5*e*n + b^4*d^4*e^2*n)*a^2 + (4*a^4*c^2*d*e^5*n + (8*c^3*d^3*e^3*n - 8*b
*c^2*d^2*e^4*n - b^2*c*d*e^5*n)*a^3 + 2*(2*c^4*d^5*e*n - 4*b*c^3*d^4*e^2*n
 + b^2*c^2*d^3*e^3*n + b^3*c*d^2*e^4*n)*a^2 - (b^2*c^3*d^5*e*n - 2*b^3*c^2
*d^4*e^2*n + b^4*c*d^3*e^3*n)*a)*x^(3*n) + (4*(c^2*d^2*e^4*n + b*c*d*e^5*n
)*a^4 + (8*c^3*d^4*e^2*n - 9*b^2*c*d^2*e^4*n - b^3*d*e^5*n)*a^3 + 2*(2*c^4
*d^6*n - 2*b*c^3*d^5*e*n - 3*b^2*c^2*d^4*e^2*n + 2*b^3*c*d^3*e^3*n + b^...
```

3.79.8 Giac [F]

$$\int \frac{1}{(d+ex^n)^2 (a+bx^n+cx^{2n})^2} dx = \int \frac{1}{(cx^{2n}+bx^n+a)^2 (ex^n+d)^2} dx$$

input
```
integrate(1/(d+e*x^n)^2/(a+b*x^n+c*x^(2*n))^2,x, algorithm="giac")
```

output
```
integrate(1/((c*x^(2*n) + b*x^n + a)^2*(e*x^n + d)^2), x)
```

3.79.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^n)^2 (a+bx^n+cx^{2n})^2}\,dx = \int \frac{1}{(d+e\,x^n)^2 \left(a+b\,x^n+c\,x^{2\,n}\right)^2}\,dx$$

input
```
int(1/((d + e*x^n)^2*(a + b*x^n + c*x^(2*n))^2),x)
```

output
```
int(1/((d + e*x^n)^2*(a + b*x^n + c*x^(2*n))^2), x)
```

3.80 $\int \frac{(d+ex^n)^3}{(a+bx^n+cx^{2n})^3}\,dx$

3.80.1 Optimal result

Integrand size = 26, antiderivative size = 1707

$$\int \frac{(d+ex^n)^3}{(a+bx^n+cx^{2n})^3}\,dx = \text{Too large to display}$$

output
```
1/2*x*(b^2*c*d^3-2*a*c*d*(-3*a*e^2+c*d^2)-a*b*e*(a*e^2+3*c*d^2)-(a*b^2*e^3
+2*a*c*e*(-a*e^2+3*c*d^2)-b*c*d*(3*a*e^2+c*d^2))*x^n)/a/c/(-4*a*c+b^2)/n/(
a+b*x^n+c*x^(2*n))^2+e^2*x*(3*b^2*c*d-6*a*c^2*d-b^3*e+a*b*c*e+c*(-2*a*c*e-
b^2*e+3*b*c*d)*x^n)/a/c^2/(-4*a*c+b^2)/n/(a+b*x^n+c*x^(2*n))-1/2*x*(a*b^2*
c^2*d*(3*a*e^2*(1-9*n)-5*c*d^2*(1-3*n))+4*a^2*c^3*d*(-3*a*e^2+c*d^2)*(1-4*
n)-2*a*b^5*e^3*n+2*a^2*b*c^2*e*(3*c*d^2*(2-3*n)-5*a*e^2*n)-3*a*b^3*c*e*(-3
*a*e^2*n+c*d^2)+b^4*c*d*(c*d^2*(1-2*n)+6*a*e^2*n)+c*(4*a^2*c^2*e*(-a*e^2+3
*c*d^2)*(1-3*n)-2*a*b^4*e^3*n-2*a*b*c^2*d*(c*d^2*(2-7*n)+3*a*e^2*n)+b^3*c*
d*(c*d^2*(1-2*n)+6*a*e^2*n)-a*b^2*c*e*(3*c*d^2-a*e^2*(1+2*n)))*x^n)/a^2/c^
2/(-4*a*c+b^2)^2/n^2/(a+b*x^n+c*x^(2*n))+1/2*x*hypergeom([1, 1/n],[1+1/n],
-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))*((1-n)*(4*a^2*c^2*e*(-a*e^2+3*c*d^2)*(1-3
*n)-2*a*b^4*e^3*n-2*a*b*c^2*d*(c*d^2*(2-7*n)+3*a*e^2*n)+b^3*c*d*(c*d^2*(1-
2*n)+6*a*e^2*n)-a*b^2*c*e*(3*c*d^2-a*e^2*(1+2*n)))+(-2*a*b^5*e^3*(1-n)*n+b
^4*c*d*(1-n)*(c*d^2*(1-2*n)+6*a*e^2*n)+8*a^2*c^3*d*(-3*a*e^2+c*d^2)*(8*n^2
-6*n+1)-6*a*b^2*c^2*d*(c*d^2*(3*n^2-4*n+1)-a*e^2*(15*n^2-10*n+1))+4*a^2*b*
c^2*e*(3*c*d^2*(-3*n^2-n+1)+a*e^2*(19*n^2-11*n+1))-a*b^3*c*e*(3*c*d^2*(1-n
)+a*e^2*(30*n^2-19*n+1)))/(-4*a*c+b^2)^(1/2))/a^2/c/(-4*a*c+b^2)^2/n^2/(b-
(-4*a*c+b^2)^(1/2))+1/2*x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b+(-4*a*c+b
^2)^(1/2)))*((1-n)*(4*a^2*c^2*e*(-a*e^2+3*c*d^2)*(1-3*n)-2*a*b^4*e^3*n-2*a
*b*c^2*d*(c*d^2*(2-7*n)+3*a*e^2*n)+b^3*c*d*(c*d^2*(1-2*n)+6*a*e^2*n)-a*...
```

3.80.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 13018 vs. $2(1707) = 3414$.

Time = 7.82 (sec) , antiderivative size = 13018, normalized size of antiderivative = 7.63

$$\int \frac{(d+ex^n)^3}{(a+bx^n+cx^{2n})^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[(d + e*x^n)^3/(a + b*x^n + c*x^(2*n))^3,x]
```

output
```
Result too large to show
```

3.80.3 Rubi [A] (verified)

Time = 4.33 (sec) , antiderivative size = 1707, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1766, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^n)^3}{(a+bx^n+cx^{2n})^3}\,dx$$

$$\Big\downarrow \text{1766}$$

$$\int \left(\frac{x^n\left(-ace^3+b^2e^3-3bcde^2+3c^2d^2e\right)+abe^3-3acde^2+c^2d^3}{c^2\left(a+bx^n+cx^{2n}\right)^3}+\frac{e^2(-be+3cd+cex^n)}{c^2\left(a+bx^n+cx^{2n}\right)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(-e(1-n)b^3+\left(3cd-\sqrt{b^2-4ac}e\right)(1-n)b^2+c\left(2ae(2-5n)+3\sqrt{b^2-4ac}d(1-n)\right)b-2ac\left(6cd(1-2n)+\sqrt{\right.\right.}{ac\left(b^2-4ac\right)\left(b^2-\sqrt{b^2-4ac}b-4ac\right)n}$$

$$\frac{\left(-e(1-n)b^3+\left(3cd+\sqrt{b^2-4ac}e\right)(1-n)b^2+c\left(2ae(2-5n)-3\sqrt{b^2-4ac}d(1-n)\right)b-2ac\left(6cd(1-2n)-\sqrt{\right.\right.}{ac\left(b^2-4ac\right)\left(b^2+\sqrt{b^2-4ac}b-4ac\right)n}$$

$$\frac{x\left(c\left(-eb^2+3cdb-2ace\right)x^n-6ac^2d+3b^2cd-b^3e+abce\right)e^2}{ac^2\left(b^2-4ac\right)n\left(bx^n+cx^{2n}+a\right)}+$$

$$\left((1-n)\left(-2ae^3nb^4+cd\left(c(1-2n)d^2+6ae^2n\right)b^3-ace\left(3cd^2-ae^2(2n+1)\right)b^2-2ac^2d\left(c(2-7n)d^2+3ae^2n\right)b\right.\right.$$

$$\left((1-n)\left(-2ae^3nb^4+cd\left(c(1-2n)d^2+6ae^2n\right)b^3-ace\left(3cd^2-ae^2(2n+1)\right)b^2-2ac^2d\left(c(2-7n)d^2+3ae^2n\right)b\right.\right.$$

$$x\left(c\left(-2ae^3nb^4+cd\left(c(1-2n)d^2+6ae^2n\right)b^3-ace\left(3cd^2-ae^2(2n+1)\right)b^2-2ac^2d\left(c(2-7n)d^2+3ae^2n\right)b+4a^2\right.\right.$$

$$\frac{x\left(-\left(\left(ab^2e^3+2ac\left(3cd^2-ae^2\right)e-bcd\left(cd^2+3ae^2\right)\right)x^n\right)+b^2cd^3-2acd\left(cd^2-3ae^2\right)-abe\left(3cd^2+ae^2\right)\right)}{2ac\left(b^2-4ac\right)n\left(bx^n+cx^{2n}+a\right)^2}$$

input
```
Int[(d + e*x^n)^3/(a + b*x^n + c*x^(2*n))^3,x]
```

3.80. $\int \frac{(d+ex^n)^3}{(a+bx^n+cx^{2n})^3}\,dx$

output
```
(x*(b^2*c*d^3 - 2*a*c*d*(c*d^2 - 3*a*e^2) - a*b*e*(3*c*d^2 + a*e^2) - (a*b
^2*e^3 + 2*a*c*e*(3*c*d^2 - a*e^2) - b*c*d*(c*d^2 + 3*a*e^2))*x^n))/(2*a*c
*(b^2 - 4*a*c)*n*(a + b*x^n + c*x^(2*n))^2) + (e^2*x*(3*b^2*c*d - 6*a*c^2*
d - b^3*e + a*b*c*e + c*(3*b*c*d - b^2*e - 2*a*c*e)*x^n))/(a*c^2*(b^2 - 4*
a*c)*n*(a + b*x^n + c*x^(2*n))) - (x*(a*b^2*c^2*d*(3*a*e^2*(1 - 9*n) - 5*c
*d^2*(1 - 3*n)) + 4*a^2*c^3*d*(c*d^2 - 3*a*e^2)*(1 - 4*n) - 2*a*b^5*e^3*n
+ 2*a^2*b*c^2*e*(3*c*d^2*(2 - 3*n) - 5*a*e^2*n) - 3*a*b^3*c*e*(c*d^2 - 3*a
*e^2*n) + b^4*c*d*(c*d^2*(1 - 2*n) + 6*a*e^2*n) + c*(4*a^2*c^2*e*(3*c*d^2
- a*e^2)*(1 - 3*n) - 2*a*b^4*e^3*n - 2*a*b*c^2*d*(c*d^2*(2 - 7*n) + 3*a*e^
2*n) + b^3*c*d*(c*d^2*(1 - 2*n) + 6*a*e^2*n) - a*b^2*c*e*(3*c*d^2 - a*e^2*
(1 + 2*n)))*x^n))/(2*a^2*c^2*(b^2 - 4*a*c)^2*n^2*(a + b*x^n + c*x^(2*n)))
+ (e^2*(b*c*(2*a*e*(2 - 5*n) + 3*Sqrt[b^2 - 4*a*c]*d*(1 - n)) - 2*a*c*(6*c
*d*(1 - 2*n) + Sqrt[b^2 - 4*a*c]*e*(1 - n)) - b^3*e*(1 - n) + b^2*(3*c*d -
 Sqrt[b^2 - 4*a*c]*e)*(1 - n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1),
(-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(a*c*(b^2 - 4*a*c)*(b^2 - 4*a*c - b*S
qrt[b^2 - 4*a*c])*n) + (((1 - n)*(4*a^2*c^2*e*(3*c*d^2 - a*e^2)*(1 - 3*n)
- 2*a*b^4*e^3*n - 2*a*b*c^2*d*(c*d^2*(2 - 7*n) + 3*a*e^2*n) + b^3*c*d*(c*d
^2*(1 - 2*n) + 6*a*e^2*n) - a*b^2*c*e*(3*c*d^2 - a*e^2*(1 + 2*n))) - (2*a*
b^5*e^3*(1 - n)*n - b^4*c*d*(1 - n)*(c*d^2*(1 - 2*n) + 6*a*e^2*n) - 8*a^2*
c^3*d*(c*d^2 - 3*a*e^2)*(1 - 6*n + 8*n^2) + 6*a*b^2*c^2*d*(c*d^2*(1 - 4...
```

3.80.3.1 Defintions of rubi rules used

rule 1766
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
))^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2
*n))^p, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ((IntegersQ[p, q] &&
!IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] &&  !IntegerQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.80.4 Maple [F]

$$\int \frac{(d+e\,x^{n})^{3}}{(a+b\,x^{n}+c\,x^{2n})^{3}}dx$$

input
```
int((d+e*x^n)^3/(a+b*x^n+c*x^(2*n))^3,x)
```

output
```
int((d+e*x^n)^3/(a+b*x^n+c*x^(2*n))^3,x)
```

3.80.5 Fricas [F]

$$\int \frac{(d+ex^{n})^{3}}{(a+bx^{n}+cx^{2n})^{3}}\,dx = \int \frac{(ex^{n}+d)^{3}}{(cx^{2\,n}+bx^{n}+a)^{3}}\,dx$$

input
```
integrate((d+e*x^n)^3/(a+b*x^n+c*x^(2*n))^3,x, algorithm="fricas")
```

output
```
integral((e^3*x^(3*n) + 3*d*e^2*x^(2*n) + 3*d^2*e*x^n + d^3)/(c^3*x^(6*n)
+ b^3*x^(3*n) + 3*a*b^2*x^(2*n) + 3*a^2*b*x^n + a^3 + 3*(b*c^2*x^n + a*c^2
)*x^(4*n) + 3*(b^2*c*x^(2*n) + 2*a*b*c*x^n + a^2*c)*x^(2*n)), x)
```

3.80.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^{n})^{3}}{(a+bx^{n}+cx^{2n})^{3}}\,dx = \text{Timed out}$$

input
```
integrate((d+e*x**n)**3/(a+b*x**n+c*x**(2*n))**3,x)
```

output
```
Timed out
```

3.80.7 Maxima [F]

$$\int \frac{(d+ex^n)^3}{(a+bx^n+cx^{2n})^3}\,dx = \int \frac{(ex^n+d)^3}{(cx^{2n}+bx^n+a)^3}\,dx$$

input
```
integrate((d+e*x^n)^3/(a+b*x^n+c*x^(2*n))^3,x, algorithm="maxima")
```

output
```
1/2*((b^3*c^2*d^3*(2*n - 1) + 4*a^3*c^2*e^3*(n + 1) + (12*c^3*d^2*e*(3*n -
 1) + b^2*c*e^3*(2*n - 1) - 18*b*c^2*d*e^2*n)*a^2 - (2*b*c^3*d^3*(7*n - 2)
 - 3*b^2*c^2*d^2*e)*a)*x*x^(3*n) + (2*b^4*c*d^3*(2*n - 1) + 2*(b*c*e^3*(3*
n + 2) + 6*c^2*d*e^2)*a^3 - (3*b^2*c*d*e^2*(9*n + 1) - 6*b*c^2*d^2*e*(9*n
- 4) - 4*c^3*d^3*(4*n - 1) - b^3*e^3*(3*n - 1))*a^2 - (b^2*c^2*d^3*(29*n -
 9) - 6*b^3*c*d^2*e)*a)*x*x^(2*n) + (b^5*d^3*(2*n - 1) - 4*a^4*c*e^3*(n -
1) + (b^2*e^3*(10*n - 1) + 12*c^2*d^2*e*(5*n - 1) - 6*b*c*d*e^2*(5*n - 2))
*a^3 + (3*b^2*c*d^2*e*(4*n - 3) - 3*b^3*d*e^2*(2*n + 1) - 2*b*c^2*d^3*n)*a
^2 - (4*b^3*c*d^3*(3*n - 1) - 3*b^4*d^2*e)*a)*x*x^n + (a*b^4*d^3*(3*n - 1)
 - 6*(2*c*d*e^2*(2*n - 1) - b*e^3*n)*a^4 + (4*c^2*d^3*(6*n - 1) + 6*b*c*d^
2*e*(5*n - 2) - 3*b^2*d*e^2*(n + 1))*a^3 - (b^2*c*d^3*(21*n - 5) + 3*b^3*d
^2*e*(n - 1))*a^2)*x)/(a^4*b^4*n^2 - 8*a^5*b^2*c*n^2 + 16*a^6*c^2*n^2 + (a
^2*b^4*c^2*n^2 - 8*a^3*b^2*c^3*n^2 + 16*a^4*c^4*n^2)*x^(4*n) + 2*(a^2*b^5*
c*n^2 - 8*a^3*b^3*c^2*n^2 + 16*a^4*b*c^3*n^2)*x^(3*n) + (a^2*b^6*n^2 - 6*a
^3*b^4*c*n^2 + 32*a^5*c^3*n^2)*x^(2*n) + 2*(a^3*b^5*n^2 - 8*a^4*b^3*c*n^2
+ 16*a^5*b*c^2*n^2)*x^n) + integrate(1/2*((2*n^2 - 3*n + 1)*b^4*d^3 + 6*(2
*c*d*e^2*(2*n - 1) - b*e^3*n)*a^3 + (4*(8*n^2 - 6*n + 1)*c^2*d^3 - 6*b*c*d
^2*e*(5*n - 2) + 3*b^2*d*e^2*(n + 1))*a^2 - ((16*n^2 - 21*n + 5)*b^2*c*d^3
 - 3*b^3*d^2*e*(n - 1))*a + ((2*n^2 - 3*n + 1)*b^3*c*d^3 + 4*(n^2 - 1)*a^3
*c*e^3 + (12*(3*n^2 - 4*n + 1)*c^2*d^2*e - 18*(n^2 - n)*b*c*d*e^2 + (2*...
```

3.80.8 Giac [F]

$$\int \frac{(d+ex^n)^3}{(a+bx^n+cx^{2n})^3}\,dx = \int \frac{(ex^n+d)^3}{(cx^{2n}+bx^n+a)^3}\,dx$$

input
```
integrate((d+e*x^n)^3/(a+b*x^n+c*x^(2*n))^3,x, algorithm="giac")
```

output
```
integrate((e*x^n + d)^3/(c*x^(2*n) + b*x^n + a)^3, x)
```

3.80.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^n)^3}{(a+bx^n+cx^{2n})^3}\,dx=\int \frac{{\left(d+e\,x^n\right)}^3}{{\left(a+b\,x^n+c\,x^{2\,n}\right)}^3} \,d x$$

input
```
int((d + e*x^n)^3/(a + b*x^n + c*x^(2*n))^3,x)
```

output
```
int((d + e*x^n)^3/(a + b*x^n + c*x^(2*n))^3, x)
```

3.81 $\int \frac{(d+ex^n)^2}{(a+bx^n+cx^{2n})^3}\,dx$

3.81.1 Optimal result

Integrand size = 26, antiderivative size = 1191

$$\int \frac{(d+ex^n)^2}{(a+bx^n+cx^{2n})^3}\,dx = \frac{x(b^2d^2-2abde-2a(cd^2-ae^2)+(bcd^2-4acde+abe^2)\,x^n)}{2a\,(b^2-4ac)\,n\,(a+bx^n+cx^{2n})^2}$$

$$+\frac{e^2x(b^2-2ac+bcx^n)}{ac\,(b^2-4ac)\,n\,(a+bx^n+cx^{2n})}$$

$$+\frac{x(2ab^3cde-ab^2c(ae^2(1-9n)-5cd^2(1-3n))-4a^2c^2(cd^2-ae^2)\,(1-4n)-4a^2bc^2de(2-3n)-b^4(c\ldots}{2a^2c\,(b^2-4a\ldots}$$

$$-\frac{e^2\left(4ac(1-2n)-b^2(1-n)-b\sqrt{b^2-4ac}(1-n)\right)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{a\,(b^2-4ac)\left(b^2-4ac-b\sqrt{b^2-4ac}\right)n}$$

$$-\frac{\Big((1-n)\,(2ab^2cde-8a^2c^2de(1-3n)+2abc(cd^2(2-7n)+ae^2n)-b^3(cd^2(1-2n)+2ae^2n))+\frac{2ab^3cde\ldots}{\ldots}}{\ldots}$$

$$-\frac{e^2\left(4ac(1-2n)-b^2(1-n)+b\sqrt{b^2-4ac}(1-n)\right)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{a\,(b^2-4ac)\left(b^2-4ac+b\sqrt{b^2-4ac}\right)n}$$

$$-\frac{\Big((1-n)\,(2ab^2cde-8a^2c^2de(1-3n)+2abc(cd^2(2-7n)+ae^2n)-b^3(cd^2(1-2n)+2ae^2n))-\frac{2ab^3cde\ldots}{\ldots}}{\ldots}$$

output
```
1/2*x*(b^2*d^2-2*a*b*d*e-2*a*(-a*e^2+c*d^2)+(a*b*e^2-4*a*c*d*e+b*c*d^2)*x^
n)/a/(-4*a*c+b^2)/n/(a+b*x^n+c*x^(2*n))^2+e^2*x*(b^2-2*a*c+b*c*x^n)/a/c/(-
4*a*c+b^2)/n/(a+b*x^n+c*x^(2*n))+1/2*x*(2*a*b^3*c*d*e-a*b^2*c*(a*e^2*(1-9*
n)-5*c*d^2*(1-3*n))-4*a^2*c^2*(-a*e^2+c*d^2)*(1-4*n)-4*a^2*b*c^2*d*e*(2-3*
n)-b^4*(c*d^2*(1-2*n)+2*a*e^2*n)+c*(2*a*b^2*c*d*e-8*a^2*c^2*d*e*(1-3*n)+2*
a*b*c*(c*d^2*(2-7*n)+a*e^2*n)-b^3*(c*d^2*(1-2*n)+2*a*e^2*n))*x^n)/a^2/c/(-
4*a*c+b^2)^2/n^2/(a+b*x^n+c*x^(2*n))-1/2*x*hypergeom([1, 1/n],[1+1/n],-2*c
*x^n/(b-(-4*a*c+b^2)^(1/2)))*((1-n)*(2*a*b^2*c*d*e-8*a^2*c^2*d*e*(1-3*n)+2
*a*b*c*(c*d^2*(2-7*n)+a*e^2*n)-b^3*(c*d^2*(1-2*n)+2*a*e^2*n))+(2*a*b^3*c*d
*e*(1-n)-b^4*(1-n)*(c*d^2*(1-2*n)+2*a*e^2*n)-8*a^2*b*c^2*d*e*(-3*n^2-n+1)-
8*a^2*c^2*(-a*e^2+c*d^2)*(8*n^2-6*n+1)+2*a*b^2*c*(3*c*d^2*(3*n^2-4*n+1)-a*
e^2*(15*n^2-10*n+1)))/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^2/n^2/(b-(-4*a*
c+b^2)^(1/2))-1/2*x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1
/2)))*((1-n)*(2*a*b^2*c*d*e-8*a^2*c^2*d*e*(1-3*n)+2*a*b*c*(c*d^2*(2-7*n)+a
*e^2*n)-b^3*(c*d^2*(1-2*n)+2*a*e^2*n))+(-2*a*b^3*c*d*e*(1-n)+b^4*(1-n)*(c*
d^2*(1-2*n)+2*a*e^2*n)+8*a^2*b*c^2*d*e*(-3*n^2-n+1)+8*a^2*c^2*(-a*e^2+c*d^
2)*(8*n^2-6*n+1)-2*a*b^2*c*(3*c*d^2*(3*n^2-4*n+1)-a*e^2*(15*n^2-10*n+1)))/
(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^2/n^2/(b+(-4*a*c+b^2)^(1/2))-e^2*x*hy
pergeom([1, 1/n],[1+1/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))*(4*a*c*(1-2*n)-b
^2*(1-n)-b*(1-n)*(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)/n/(b^2-4*a*c-b*(-4*...
```

3.81.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 10910 vs. 2(1191) = 2382.

Time = 7.52 (sec) , antiderivative size = 10910, normalized size of antiderivative = 9.16

$$\int \frac{(d+ex^n)^2}{(a+bx^n+cx^{2n})^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[(d + e*x^n)^2/(a + b*x^n + c*x^(2*n))^3,x]
```

output
```
Result too large to show
```

3.81.3 Rubi [A] (verified)

Time = 3.12 (sec) , antiderivative size = 1191, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1766, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^n)^2}{(a+bx^n+cx^{2n})^3}\,dx$$

$$\Big\downarrow \text{ 1766}$$

$$\int \left(\frac{-ae^2+x^n(2cde-be^2)+cd^2}{c(a+bx^n+cx^{2n})^3}+\frac{e^2}{c(a+bx^n+cx^{2n})^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\left(-((1-n)b^2)-\sqrt{b^2-4ac}(1-n)b+4ac(1-2n)\right)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)e^2}{a(b^2-4ac)\left(b^2-\sqrt{b^2-4ac}b-4ac\right)n}-$$
$$\frac{\left(-((1-n)b^2)+\sqrt{b^2-4ac}(1-n)b+4ac(1-2n)\right)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)e^2}{a(b^2-4ac)\left(b^2+\sqrt{b^2-4ac}b-4ac\right)n}+$$
$$\frac{x(bcx^n+b^2-2ac)e^2}{ac(b^2-4ac)n(bx^n+cx^{2n}+a)}-$$
$$\left((1-n)\left(-\left(\left(c(1-2n)d^2+2ae^2n\right)b^3\right)+2acdeb^2+2ac\left(c(2-7n)d^2+ae^2n\right)b-8a^2c^2de(1-3n)\right)+\frac{-((1-n)(c(1-\cdots}{\cdots}\right.$$
$$\left((1-n)\left(-\left(\left(c(1-2n)d^2+2ae^2n\right)b^3\right)+2acdeb^2+2ac\left(c(2-7n)d^2+ae^2n\right)b-8a^2c^2de(1-3n)\right)-\frac{-((1-n)(c(1-\cdots}{\cdots}\right.$$
$$\frac{x\left(c\left(-\left(\left(c(1-2n)d^2+2ae^2n\right)b^3\right)+2acdeb^2+2ac\left(c(2-7n)d^2+ae^2n\right)b-8a^2c^2de(1-3n)\right)x^n+2ab^3cde-ab^2\cdots\right.}{2a^2c\left(b^2-4ac\right)^2n^2(\cdots}$$
$$\frac{x\left(\left(bcd^2-4aced+abe^2\right)x^n+b^2d^2-2abde-2a\left(cd^2-ae^2\right)\right)}{2a(b^2-4ac)n(bx^n+cx^{2n}+a)^2}$$

input
```
Int[(d + e*x^n)^2/(a + b*x^n + c*x^(2*n))^3,x]
```

output
```
(x*(b^2*d^2 - 2*a*b*d*e - 2*a*(c*d^2 - a*e^2) + (b*c*d^2 - 4*a*c*d*e + a*b
*e^2)*x^n))/(2*a*(b^2 - 4*a*c)*n*(a + b*x^n + c*x^(2*n))^2) + (e^2*x*(b^2
- 2*a*c + b*c*x^n))/(a*c*(b^2 - 4*a*c)*n*(a + b*x^n + c*x^(2*n))) + (x*(2*
a*b^3*c*d*e - a*b^2*c*(a*e^2*(1 - 9*n) - 5*c*d^2*(1 - 3*n)) - 4*a^2*c^2*(c
*d^2 - a*e^2)*(1 - 4*n) - 4*a^2*b*c^2*d*e*(2 - 3*n) - b^4*(c*d^2*(1 - 2*n)
 + 2*a*e^2*n) + c*(2*a*b^2*c*d*e - 8*a^2*c^2*d*e*(1 - 3*n) + 2*a*b*c*(c*d^
2*(2 - 7*n) + a*e^2*n) - b^3*(c*d^2*(1 - 2*n) + 2*a*e^2*n))*x^n))/(2*a^2*c
*(b^2 - 4*a*c)^2*n^2*(a + b*x^n + c*x^(2*n))) - (e^2*(4*a*c*(1 - 2*n) - b^
2*(1 - n) - b*Sqrt[b^2 - 4*a*c]*(1 - n))*x*Hypergeometric2F1[1, n^(-1), 1
+ n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*(b^2 - 4*a
*c - b*Sqrt[b^2 - 4*a*c])*n) - (((1 - n)*(2*a*b^2*c*d*e - 8*a^2*c^2*d*e*(1
 - 3*n) + 2*a*b*c*(c*d^2*(2 - 7*n) + a*e^2*n) - b^3*(c*d^2*(1 - 2*n) + 2*a
*e^2*n)) + (2*a*b^3*c*d*e*(1 - n) - b^4*(1 - n)*(c*d^2*(1 - 2*n) + 2*a*e^2
*n) - 8*a^2*b*c^2*d*e*(1 - n - 3*n^2) - 8*a^2*c^2*(c*d^2 - a*e^2)*(1 - 6*n
 + 8*n^2) + 2*a*b^2*c*(3*c*d^2*(1 - 4*n + 3*n^2) - a*e^2*(1 - 10*n + 15*n^
2)))/Sqrt[b^2 - 4*a*c])*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x
^n)/(b - Sqrt[b^2 - 4*a*c])])/(2*a^2*(b^2 - 4*a*c)^2*(b - Sqrt[b^2 - 4*a*c
])*n^2) - (e^2*(4*a*c*(1 - 2*n) - b^2*(1 - n) + b*Sqrt[b^2 - 4*a*c]*(1 - n
))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4
*a*c])])/(a*(b^2 - 4*a*c)*(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*n) - (((1...
```

3.81.3.1 Defintions of rubi rules used

rule 1766
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
))^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2
*n))^p, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ((IntegersQ[p, q] &&
!IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] &&  !IntegerQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.81.4 Maple [F]

$$\int \frac{(d + e\,x^n)^2}{(a + b\,x^n + c\,x^{2n})^3}dx$$

input
```
int((d+e*x^n)^2/(a+b*x^n+c*x^(2*n))^3,x)
```

output
```
int((d+e*x^n)^2/(a+b*x^n+c*x^(2*n))^3,x)
```

3.81.5 Fricas [F]

$$\int \frac{(d + ex^n)^2}{(a + bx^n + cx^{2n})^3}\,dx = \int \frac{(ex^n + d)^2}{(cx^{2n} + bx^n + a)^3}\,dx$$

input
```
integrate((d+e*x^n)^2/(a+b*x^n+c*x^(2*n))^3,x, algorithm="fricas")
```

output
```
integral((e^2*x^(2*n) + 2*d*e*x^n + d^2)/(c^3*x^(6*n) + b^3*x^(3*n) + 3*a*
b^2*x^(2*n) + 3*a^2*b*x^n + a^3 + 3*(b*c^2*x^n + a*c^2)*x^(4*n) + 3*(b^2*c
*x^(2*n) + 2*a*b*c*x^n + a^2*c)*x^(2*n)), x)
```

3.81.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d + ex^n)^2}{(a + bx^n + cx^{2n})^3}\,dx = \text{Timed out}$$

input
```
integrate((d+e*x**n)**2/(a+b*x**n+c*x**(2*n))**3,x)
```

output
```
Timed out
```

3.81.7 Maxima [F]

$$\int \frac{(d+ex^n)^2}{(a+bx^n+cx^{2n})^3}\,dx = \int \frac{(ex^n+d)^2}{(cx^{2n}+bx^n+a)^3}\,dx$$

input
```
integrate((d+e*x^n)^2/(a+b*x^n+c*x^(2*n))^3,x, algorithm="maxima")
```

output
```
1/2*((b^3*c^2*d^2*(2*n - 1) + 2*(4*c^3*d*e*(3*n - 1) - 3*b*c^2*e^2*n)*a^2
- 2*(b*c^3*d^2*(7*n - 2) - b^2*c^2*d*e)*a)*x*x^(3*n) + (2*b^4*c*d^2*(2*n -
 1) + 4*a^3*c^2*e^2 - (b^2*c*e^2*(9*n + 1) - 4*b*c^2*d*e*(9*n - 4) - 4*c^3
*d^2*(4*n - 1))*a^2 - (b^2*c^2*d^2*(29*n - 9) - 4*b^3*c*d*e)*a)*x*x^(2*n)
+ (b^5*d^2*(2*n - 1) + 2*(4*c^2*d*e*(5*n - 1) - b*c*e^2*(5*n - 2))*a^3 + (
2*b^2*c*d*e*(4*n - 3) - b^3*e^2*(2*n + 1) - 2*b*c^2*d^2*n)*a^2 - 2*(2*b^3*
c*d^2*(3*n - 1) - b^4*d*e)*a)*x*x^n + (a*b^4*d^2*(3*n - 1) - 4*a^4*c*e^2*(
2*n - 1) + (4*c^2*d^2*(6*n - 1) + 4*b*c*d*e*(5*n - 2) - b^2*e^2*(n + 1))*a
^3 - (b^2*c*d^2*(21*n - 5) + 2*b^3*d*e*(n - 1))*a^2)*x)/(a^4*b^4*n^2 - 8*a
^5*b^2*c*n^2 + 16*a^6*c^2*n^2 + (a^2*b^4*c^2*n^2 - 8*a^3*b^2*c^3*n^2 + 16*
a^4*c^4*n^2)*x^(4*n) + 2*(a^2*b^5*c*n^2 - 8*a^3*b^3*c^2*n^2 + 16*a^4*b*c^3
*n^2)*x^(3*n) + (a^2*b^6*n^2 - 6*a^3*b^4*c*n^2 + 32*a^5*c^3*n^2)*x^(2*n) +
 2*(a^3*b^5*n^2 - 8*a^4*b^3*c*n^2 + 16*a^5*b*c^2*n^2)*x^n) - integrate(-1/
2*((2*n^2 - 3*n + 1)*b^4*d^2 + 4*a^3*c*e^2*(2*n - 1) + (4*(8*n^2 - 6*n + 1
)*c^2*d^2 - 4*b*c*d*e*(5*n - 2) + b^2*e^2*(n + 1))*a^2 - ((16*n^2 - 21*n +
 5)*b^2*c*d^2 - 2*b^3*d*e*(n - 1))*a + ((2*n^2 - 3*n + 1)*b^3*c*d^2 + 2*(4
*(3*n^2 - 4*n + 1)*c^2*d*e - 3*(n^2 - n)*b*c*e^2)*a^2 - 2*((7*n^2 - 9*n +
2)*b*c^2*d^2 - b^2*c*d*e*(n - 1))*a)*x^n)/(a^3*b^4*n^2 - 8*a^4*b^2*c*n^2 +
 16*a^5*c^2*n^2 + (a^2*b^4*c*n^2 - 8*a^3*b^2*c^2*n^2 + 16*a^4*c^3*n^2)*x^(
2*n) + (a^2*b^5*n^2 - 8*a^3*b^3*c*n^2 + 16*a^4*b*c^2*n^2)*x^n), x)
```

3.81.8 Giac [F]

$$\int \frac{(d+ex^n)^2}{(a+bx^n+cx^{2n})^3}\,dx = \int \frac{(ex^n+d)^2}{(cx^{2n}+bx^n+a)^3}\,dx$$

input
```
integrate((d+e*x^n)^2/(a+b*x^n+c*x^(2*n))^3,x, algorithm="giac")
```

output
```
integrate((e*x^n + d)^2/(c*x^(2*n) + b*x^n + a)^3, x)
```

3.81.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^n)^2}{(a+bx^n+cx^{2n})^3}\,dx=\int \frac{{\left(d+e\,x^n\right)}^2}{{\left(a+b\,x^n+c\,x^{2\,n}\right)}^3}\,dx$$

input
```
int((d + e*x^n)^2/(a + b*x^n + c*x^(2*n))^3,x)
```

output
```
int((d + e*x^n)^2/(a + b*x^n + c*x^(2*n))^3, x)
```

3.82 $\int \frac{d+ex^n}{(a+bx^n+cx^{2n})^3}\,dx$

3.82.1 Optimal result

Integrand size = 24, antiderivative size = 713

$$\int \frac{d+ex^n}{(a+bx^n+cx^{2n})^3}\,dx = \frac{x(b^2d-2acd-abe+c(bd-2ae)x^n)}{2a(b^2-4ac)\,n\,(a+bx^n+cx^{2n})^2}$$
$$+\frac{x(ab^3e-4a^2c^2d(1-4n)+5ab^2cd(1-3n)-2a^2bce(2-3n)-b^4d(1-2n)+c(ab^2e+2abcd(2-7n)\ldots}{2a^2(b^2-4ac)^2n^2(a+bx^n+cx^{2n})}$$
$$+\frac{c\left(ab^2\left(\sqrt{b^2-4ac}e+6cd(1-3n)\right)(1-n)+b^3\left(ae-\sqrt{b^2-4ac}d(1-2n)\right)(1-n)-b^4d(1-3n+2n^2)\ldots}{\ldots}$$
$$-\frac{c\left(ab^2\left(\sqrt{b^2-4ac}e-6cd(1-3n)\right)(1-n)-b^3\left(ae+\sqrt{b^2-4ac}d(1-2n)\right)(1-n)+b^4d(1-3n+2n^2)\ldots}{\ldots}$$

output
```
1/2*x*(b^2*d-2*a*c*d-a*b*e+c*(-2*a*e+b*d)*x^n)/a/(-4*a*c+b^2)/n/(a+b*x^n+c
*x^(2*n))^2+1/2*x*(a*b^3*e-4*a^2*c^2*d*(1-4*n)+5*a*b^2*c*d*(1-3*n)-2*a^2*b
*c*e*(2-3*n)-b^4*d*(1-2*n)+c*(a*b^2*e+2*a*b*c*d*(2-7*n)-4*a^2*c*e*(1-3*n)-
b^3*d*(1-2*n))*x^n)/a^2/(-4*a*c+b^2)^2/n^2/(a+b*x^n+c*x^(2*n))+1/2*c*x*hyp
ergeom([1, 1/n],[1+1/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))*(-b^4*d*(2*n^2-3*
n+1)+a*b^2*(1-n)*(6*c*d*(1-3*n)+e*(-4*a*c+b^2)^(1/2))+b^3*(1-n)*(a*e-d*(1-
2*n)*(-4*a*c+b^2)^(1/2))-4*a^2*c*(2*c*d*(8*n^2-6*n+1)+e*(3*n^2-4*n+1)*(-4*
a*c+b^2)^(1/2))-2*a*b*c*(2*a*e*(-3*n^2-n+1)-d*(7*n^2-9*n+2)*(-4*a*c+b^2)^(
1/2)))/a^2/(-4*a*c+b^2)^2/n^2/(b^2-4*a*c-b*(-4*a*c+b^2)^(1/2))-1/2*c*x*hyp
ergeom([1, 1/n],[1+1/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(b^4*d*(2*n^2-3*n
+1)+a*b^2*(1-n)*(-6*c*d*(1-3*n)+e*(-4*a*c+b^2)^(1/2))-b^3*(1-n)*(a*e+d*(1-
2*n)*(-4*a*c+b^2)^(1/2))-4*a^2*c*(-2*c*d*(8*n^2-6*n+1)+e*(3*n^2-4*n+1)*(-4
*a*c+b^2)^(1/2))+2*a*b*c*(2*a*e*(-3*n^2-n+1)+d*(7*n^2-9*n+2)*(-4*a*c+b^2)^
(1/2)))/a^2/(-4*a*c+b^2)^2/n^2/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))
```

3.82.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 8593 vs. $2(713) = 1426$.

Time = 6.73 (sec) , antiderivative size = 8593, normalized size of antiderivative = 12.05

$$\int \frac{d+ex^n}{(a+bx^n+cx^{2n})^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[(d + e*x^n)/(a + b*x^n + c*x^(2*n))^3,x]
```

output
```
Result too large to show
```

3.82.3 Rubi [A] (verified)

Time = 2.87 (sec) , antiderivative size = 722, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1760, 25, 1760, 25, 1752, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^n}{(a+bx^n+cx^{2n})^3}\,dx$$

$$\Big\downarrow \ 1760$$

$$\frac{x\left(cx^n(bd-2ae)-abe-2acd+b^2d\right)}{2an\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)^2}-\frac{\int -\frac{-c(bd-2ae)(1-3n)x^n+abe+2acd(1-4n)-b^2(d-2dn)}{\left(bx^n+cx^{2n}+a\right)^2}dx}{2an\left(b^2-4ac\right)}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{-c(bd-2ae)(1-3n)x^n+abe+2acd(1-4n)-b^2d(1-2n)}{\left(bx^n+cx^{2n}+a\right)^2}dx}{2an\left(b^2-4ac\right)}+\frac{x\left(cx^n(bd-2ae)-abe-2acd+b^2d\right)}{2an\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)^2}$$

$$\Big\downarrow \ 1760$$

$$\frac{\frac{x\left(cx^n\left(-4a^2ce(1-3n)+ab^2e+2abcd(2-7n)+b^3(-d)(1-2n)\right)-2a^2bce(2-3n)-4a^2c^2d(1-4n)+ab^3e+5ab^2cd(1-3n)-b^4d(1-2n)\right)}{an\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)}-\frac{\int -\frac{-c\left(-d(1\cdots\right.}{\cdots}}{\cdots}}{2an\left(b^2-4\cdots\right.}+$$

$$\frac{x\left(cx^n(bd-2ae)-abe-2acd+b^2d\right)}{2an\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{-c\left(-d(1-2n)b^3+aeb^2+2acd(2-7n)b-4a^2ce(1-3n)\right)(1-n)x^n+2a^2bce(2-5n)-ab^3e(1-n)+b^4d\left(2n^2-3n+1\right)+4a^2c^2d\left(8n^2-6n+1\right)-ab^2cd\left(16n^2-21n+5\right)}{bx^n+cx^{2n}+a}dx}{an\left(b^2-4ac\right)}\cdots}{2an\left(b^2-4a\cdots\right.}+$$

$$\frac{x\left(cx^n(bd-2ae)-abe-2acd+b^2d\right)}{2an\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)^2}$$

$$\Big\downarrow \ 1752$$

$$\frac{\frac{-\frac{c\left(-4a^2c\left(e\left(3n^2-4n+1\right)\sqrt{b^2-4ac}+2cd\left(8n^2-6n+1\right)\right)-2abc\left(2ae\left(-3n^2-n+1\right)-d\left(7n^2-9n+2\right)\sqrt{b^2-4ac}\right)+ab^2(1-n)\left(e\sqrt{b^2-4ac}+6cd(1-3n)\right)+b^3(1-n)\left(ae-\cdots\right.}{2\sqrt{b^2-4ac}}}{\cdots}}{\cdots}+$$

$$\frac{x\left(cx^n(bd-2ae)-abe-2acd+b^2d\right)}{2an\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)^2}$$

$$\Big\downarrow \ 778$$

$$\frac{\frac{x\left(cx^n\left(-4a^2ce(1-3n)+ab^2e+2abcd(2-7n)+b^3(-d)(1-2n)\right)-2a^2bce(2-3n)-4a^2c^2d(1-4n)+ab^3e+5ab^2cd(1-3n)-b^4d(1-2n)\right)}{an\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)}+\frac{-\frac{cx\left(-4a^2c\left(\cdots\right.\right.}{\cdots}}{\cdots}}{\cdots}+$$

$$\frac{x\left(cx^n(bd-2ae)-abe-2acd+b^2d\right)}{2an\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)^2}$$

3.82. $\int \frac{d+ex^n}{\left(a+bx^n+cx^{2n}\right)^3}\,dx$

input
```
Int[(d + e*x^n)/(a + b*x^n + c*x^(2*n))^3,x]
```

output
```
(x*(b^2*d - 2*a*c*d - a*b*e + c*(b*d - 2*a*e)*x^n))/(2*a*(b^2 - 4*a*c)*n*(
a + b*x^n + c*x^(2*n))^2) + ((x*(a*b^3*e - 4*a^2*c^2*d*(1 - 4*n) + 5*a*b^2
*c*d*(1 - 3*n) - 2*a^2*b*c*e*(2 - 3*n) - b^4*d*(1 - 2*n) + c*(a*b^2*e + 2*
a*b*c*d*(2 - 7*n) - 4*a^2*c*e*(1 - 3*n) - b^3*d*(1 - 2*n))*x^n))/(a*(b^2 -
 4*a*c)*n*(a + b*x^n + c*x^(2*n))) + (-((c*(a*b^2*(Sqrt[b^2 - 4*a*c]*e + 6
*c*d*(1 - 3*n))*(1 - n) + b^3*(a*e - Sqrt[b^2 - 4*a*c]*d*(1 - 2*n))*(1 - n
) - b^4*d*(1 - 3*n + 2*n^2) - 2*a*b*c*(2*a*e*(1 - n - 3*n^2) - Sqrt[b^2 -
4*a*c]*d*(2 - 9*n + 7*n^2)) - 4*a^2*c*(Sqrt[b^2 - 4*a*c]*e*(1 - 4*n + 3*n^
2) + 2*c*d*(1 - 6*n + 8*n^2)))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1),
(-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*
a*c]))) - (c*(a*b^2*(Sqrt[b^2 - 4*a*c]*e - 6*c*d*(1 - 3*n))*(1 - n) - b^3*
(a*e + Sqrt[b^2 - 4*a*c]*d*(1 - 2*n))*(1 - n) + b^4*d*(1 - 3*n + 2*n^2) +
2*a*b*c*(2*a*e*(1 - n - 3*n^2) + Sqrt[b^2 - 4*a*c]*d*(2 - 9*n + 7*n^2)) -
4*a^2*c*(Sqrt[b^2 - 4*a*c]*e*(1 - 4*n + 3*n^2) - 2*c*d*(1 - 6*n + 8*n^2)))
*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a
*c])])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])))/(a*(b^2 - 4*a*c)*n))/(
2*a*(b^2 - 4*a*c)*n)
```

3.82.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 1752
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2
 - 4*a*c] ||  !IGtQ[n/2, 0])
```

rule 1760
```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p
_), x_Symbol] :> Simp[(-x)*(d*b^2 - a*b*e - 2*a*c*d + (b*d - 2*a*e)*c*x^n)*
((a + b*x^n + c*x^(2*n))^(p + 1)/(a*n*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/
(a*n*(p + 1)*(b^2 - 4*a*c))   Int[Simp[(n*p + n + 1)*d*b^2 - a*b*e - 2*a*c*
d*(2*n*p + 2*n + 1) + (2*n*p + 3*n + 1)*(d*b - 2*a*e)*c*x^n, x]*(a + b*x^n
+ c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n
] && NeQ[b^2 - 4*a*c, 0] && ILtQ[p, -1]
```

3.82.4 Maple [F]

$$\int \frac{d+e\,x^{n}}{\left(a+b\,x^{n}+c\,x^{2n}\right)^{3}}dx$$

input
```
int((d+e*x^n)/(a+b*x^n+c*x^(2*n))^3,x)
```

output
```
int((d+e*x^n)/(a+b*x^n+c*x^(2*n))^3,x)
```

3.82.5 Fricas [F]

$$\int \frac{d+ex^n}{(a+bx^n+cx^{2n})^3}\,dx=\int \frac{ex^n+d}{(cx^{2n}+bx^n+a)^3}\,dx$$

input
```
integrate((d+e*x^n)/(a+b*x^n+c*x^(2*n))^3,x, algorithm="fricas")
```

output
```
integral((e*x^n + d)/(c^3*x^(6*n) + b^3*x^(3*n) + 3*a*b^2*x^(2*n) + 3*a^2*
b*x^n + a^3 + 3*(b*c^2*x^n + a*c^2)*x^(4*n) + 3*(b^2*c*x^(2*n) + 2*a*b*c*x
^n + a^2*c)*x^(2*n)), x)
```

3.82.6 Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^n}{(a+bx^n+cx^{2n})^3}\,dx = \text{Timed out}$$

input
```
integrate((d+e*x**n)/(a+b*x**n+c*x**(2*n))**3,x)
```

output
```
Timed out
```

3.82.7 Maxima [F]

$$\int \frac{d+ex^n}{(a+bx^n+cx^{2n})^3}\,dx = \int \frac{ex^n+d}{(cx^{2n}+bx^n+a)^3}\,dx$$

input
```
integrate((d+e*x^n)/(a+b*x^n+c*x^(2*n))^3,x, algorithm="maxima")
```

output
```
1/2*((4*a^2*c^3*e*(3*n - 1) + b^3*c^2*d*(2*n - 1) - (2*b*c^3*d*(7*n - 2) -
 b^2*c^2*e)*a)*x*x^(3*n) + (2*b^4*c*d*(2*n - 1) + 2*(b*c^2*e*(9*n - 4) + 2
*c^3*d*(4*n - 1))*a^2 - (b^2*c^2*d*(29*n - 9) - 2*b^3*c*e)*a)*x*x^(2*n) +
(4*a^3*c^2*e*(5*n - 1) + b^5*d*(2*n - 1) + (b^2*c*e*(4*n - 3) - 2*b*c^2*d*
n)*a^2 - (4*b^3*c*d*(3*n - 1) - b^4*e)*a)*x*x^n + (a*b^4*d*(3*n - 1) + 2*(
2*c^2*d*(6*n - 1) + b*c*e*(5*n - 2))*a^3 - (b^2*c*d*(21*n - 5) + b^3*e*(n
- 1))*a^2)*x)/(a^4*b^4*n^2 - 8*a^5*b^2*c*n^2 + 16*a^6*c^2*n^2 + (a^2*b^4*c
^2*n^2 - 8*a^3*b^2*c^3*n^2 + 16*a^4*c^4*n^2)*x^(4*n) + 2*(a^2*b^5*c*n^2 -
8*a^3*b^3*c^2*n^2 + 16*a^4*b*c^3*n^2)*x^(3*n) + (a^2*b^6*n^2 - 6*a^3*b^4*c
*n^2 + 32*a^5*c^3*n^2)*x^(2*n) + 2*(a^3*b^5*n^2 - 8*a^4*b^3*c*n^2 + 16*a^5
*b*c^2*n^2)*x^n) + integrate(1/2*((2*n^2 - 3*n + 1)*b^4*d + 2*(2*(8*n^2 -
6*n + 1)*c^2*d - b*c*e*(5*n - 2))*a^2 - ((16*n^2 - 21*n + 5)*b^2*c*d - b^3
*e*(n - 1))*a + ((2*n^2 - 3*n + 1)*b^3*c*d + 4*(3*n^2 - 4*n + 1)*a^2*c^2*e
 - (2*(7*n^2 - 9*n + 2)*b*c^2*d - b^2*c*e*(n - 1))*a)*x^n)/(a^3*b^4*n^2 -
8*a^4*b^2*c*n^2 + 16*a^5*c^2*n^2 + (a^2*b^4*c*n^2 - 8*a^3*b^2*c^2*n^2 + 16
*a^4*c^3*n^2)*x^(2*n) + (a^2*b^5*n^2 - 8*a^3*b^3*c*n^2 + 16*a^4*b*c^2*n^2)
*x^n), x)
```

3.82.8 Giac [F]

$$\int \frac{d+ex^n}{(a+bx^n+cx^{2n})^3}\,dx = \int \frac{ex^n+d}{(cx^{2n}+bx^n+a)^3}\,dx$$

input
```
integrate((d+e*x^n)/(a+b*x^n+c*x^(2*n))^3,x, algorithm="giac")
```

output
```
integrate((e*x^n + d)/(c*x^(2*n) + b*x^n + a)^3, x)
```

3.82.9 Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^n}{(a+bx^n+cx^{2n})^3}\,dx = \int \frac{d+e\,x^n}{(a+b\,x^n+c\,x^{2\,n})^3}\,dx$$

input
```
int((d + e*x^n)/(a + b*x^n + c*x^(2*n))^3,x)
```

output
```
int((d + e*x^n)/(a + b*x^n + c*x^(2*n))^3, x)
```

3.83 $\int \frac{1}{(d+ex^n)(a+bx^n+cx^{2n})^3} dx$

3.83.1 Optimal result

Integrand size = 26, antiderivative size = 1708

$$\int \frac{1}{(d+ex^n)(a+bx^n+cx^{2n})^3} dx = \text{Too large to display}$$

output
```
1/2*x*(b^2*c*d-2*a*c^2*d-b^3*e+3*a*b*c*e+c*(2*a*c*e-b^2*e+b*c*d)*x^n)/a/(-
4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)/n/(a+b*x^n+c*x^(2*n))^2+e^2*x*(b^2*c*d-2*a*
c^2*d-b^3*e+3*a*b*c*e+c*(2*a*c*e-b^2*e+b*c*d)*x^n)/a/(-4*a*c+b^2)/(a*e^2-b
*d*e+c*d^2)^2/n/(a+b*x^n+c*x^(2*n))+1/2*x*(2*a^2*b*c^2*e*(4-11*n)-3*a*b^3*
c*e*(2-5*n)-4*a^2*c^3*d*(1-4*n)+5*a*b^2*c^2*d*(1-3*n)-b^4*c*d*(1-2*n)+b^5*
(-2*e*n+e)-c*(a*b^2*c*e*(5-14*n)-2*a*b*c^2*d*(2-7*n)-4*a^2*c^2*e*(1-3*n)+b
^3*c*d*(1-2*n)-b^4*e*(1-2*n))*x^n)/a^2/(-4*a*c+b^2)^2/(a*e^2-b*d*e+c*d^2)/
n^2/(a+b*x^n+c*x^(2*n))+e^6*x*hypergeom([1, 1/n],[1+1/n],-e*x^n/d)/d/(a*e^
2-b*d*e+c*d^2)^3-c*e^4*x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b+(-4*a*c+b^
2)^(1/2)))*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))/(a*e^2-b*d*e+c*d^2)^3/(b^2-4*a
*c+b*(-4*a*c+b^2)^(1/2))-c*e^4*x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b-(-
4*a*c+b^2)^(1/2)))*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))/(a*e^2-b*d*e+c*d^2)^3/
(b^2-4*a*c-b*(-4*a*c+b^2)^(1/2))+c*e^2*x*hypergeom([1, 1/n],[1+1/n],-2*c*x
^n/(b-(-4*a*c+b^2)^(1/2)))*(-b^3*e*(1-n)+b^2*(1-n)*(c*d-e*(-4*a*c+b^2)^(1/
2))+b*c*(2*a*e*(2-3*n)+d*(1-n)*(-4*a*c+b^2)^(1/2))-2*a*c*(2*c*d*(1-2*n)-e*
(1-n)*(-4*a*c+b^2)^(1/2)))/a/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)^2/n/(b^2-4*a
*c-b*(-4*a*c+b^2)^(1/2))+c*e^2*x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b+(-
4*a*c+b^2)^(1/2)))*(-b^3*e*(1-n)+b^2*(1-n)*(c*d+e*(-4*a*c+b^2)^(1/2))+b*c*
(2*a*e*(2-3*n)-d*(1-n)*(-4*a*c+b^2)^(1/2))-2*a*c*(2*c*d*(1-2*n)+e*(1-n)*(-
4*a*c+b^2)^(1/2)))/a/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)^2/n/(b^2-4*a*c+b*...
```

3.83.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 43535 vs. 2(1708) = 3416.

Time = 7.87 (sec) , antiderivative size = 43535, normalized size of antiderivative = 25.49

$$\int \frac{1}{(d+ex^n)\left(a+bx^n+cx^{2n}\right)^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[1/((d + e*x^n)*(a + b*x^n + c*x^(2*n))^3),x]
```

output
```
Result too large to show
```

3.83.3 Rubi [A] (verified)

Time = 4.17 (sec) , antiderivative size = 1708, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1766, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^n)\left(a+bx^n+cx^{2n}\right)^3}\,dx$$

$$\Big\downarrow \; 1766$$

$$\int \left(-\frac{e^2(be-cd+cex^n)}{(ae^2-bde+cd^2)^2\left(a+bx^n+cx^{2n}\right)^2}+\frac{-be+cd-cex^n}{(ae^2-bde+cd^2)\left(a+bx^n+cx^{2n}\right)^3}+\frac{e^6}{(d+ex^n)(ae^2-bde+cd^2)\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)e^6}{d\left(cd^2-bed+ae^2\right)^3}-$$

$$\frac{c\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)e^4}{\left(b^2-\sqrt{b^2-4ac}b-4ac\right)\left(cd^2-bed+ae^2\right)^3}-$$

$$\frac{c\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)e^4}{\left(b^2+\sqrt{b^2-4ac}b-4ac\right)\left(cd^2-bed+ae^2\right)^3}+$$

$$\frac{c\left(-e(1-n)b^3+\left(cd-\sqrt{b^2-4ac}e\right)(1-n)b^2+c\left(2ae(2-3n)+\sqrt{b^2-4ac}d(1-n)\right)b-2ac\left(2cd(1-2n)-\sqrt{b\right.\right.}{a\left(b^2-4ac\right)\left(b^2-\sqrt{b^2-4ac}b-4ac\right)\left(cd^2-bed+\right.}$$

$$\frac{c\left(-e(1-n)b^3+\left(cd+\sqrt{b^2-4ac}e\right)(1-n)b^2+c\left(2ae(2-3n)-\sqrt{b^2-4ac}d(1-n)\right)b-2ac\left(2cd(1-2n)+\sqrt{b\right.\right.}{a\left(b^2-4ac\right)\left(b^2+\sqrt{b^2-4ac}b-4ac\right)\left(cd^2-bed+\right.}$$

$$\frac{x\left(c\left(-eb^2+cdb+2ace\right)x^n-2ac^2d+b^2cd-b^3e+3abce\right)e^2}{a\left(b^2-4ac\right)\left(cd^2-bed+ae^2\right)^2n\left(bx^n+cx^{2n}+a\right)}-$$

$$\frac{c\left(-e\left(2n^2-3n+1\right)b^5+\left(cd-\sqrt{b^2-4ac}e\right)\left(2n^2-3n+1\right)b^4+c\left(ae(7-18n)+\sqrt{b^2-4ac}d(1-2n)\right)(1-n)b\right.}{}$$

$$\frac{c\left(e\left(2n^2-3n+1\right)b^5-\left(cd+\sqrt{b^2-4ac}e\right)\left(2n^2-3n+1\right)b^4-c\left(ae(7-18n)-\sqrt{b^2-4ac}d(1-2n)\right)(1-n)b^3\right.}{}$$

$$\frac{x\left(-c\left(-e(1-2n)b^4+cd(1-2n)b^3+ace(5-14n)b^2-2ac^2d(2-7n)b-4a^2c^2e(1-3n)\right)x^n+2a^2bc^2e(4-11n)\right.}{2a^2\left(b^2-4ac\right)^2\left(cd^2-bed+ae^2\right)n^2\left(bx^{\cdots}\right.}$$

$$\frac{x\left(c\left(-eb^2+cdb+2ace\right)x^n-2ac^2d+b^2cd-b^3e+3abce\right)}{2a\left(b^2-4ac\right)\left(cd^2-bed+ae^2\right)n\left(bx^n+cx^{2n}+a\right)^2}$$

input
```
Int[1/((d + e*x^n)*(a + b*x^n + c*x^(2*n))^3),x]
```

3.83. $\int \frac{1}{(d+ex^n)(a+bx^n+cx^{2n})^3}\,dx$

output
```
(x*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e + c*(b*c*d - b^2*e + 2*a*c*e)*
x^n))/(2*a*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*n*(a + b*x^n + c*x^(2*n))
^2) + (e^2*x*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e + c*(b*c*d - b^2*e +
 2*a*c*e)*x^n))/(a*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)^2*n*(a + b*x^n +
c*x^(2*n))) + (x*(2*a^2*b*c^2*e*(4 - 11*n) - 3*a*b^3*c*e*(2 - 5*n) - 4*a^2
*c^3*d*(1 - 4*n) + 5*a*b^2*c^2*d*(1 - 3*n) - b^4*c*d*(1 - 2*n) + b^5*(e -
2*e*n) - c*(a*b^2*c*e*(5 - 14*n) - 2*a*b*c^2*d*(2 - 7*n) - 4*a^2*c^2*e*(1
- 3*n) + b^3*c*d*(1 - 2*n) - b^4*e*(1 - 2*n))*x^n))/(2*a^2*(b^2 - 4*a*c)^2
*(c*d^2 - b*d*e + a*e^2)*n^2*(a + b*x^n + c*x^(2*n))) - (c*e^4*(2*c*d - (b
 + Sqrt[b^2 - 4*a*c])*e)*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*
x^n)/(b - Sqrt[b^2 - 4*a*c])])/((b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*(c*d^2
 - b*d*e + a*e^2)^3) + (c*e^2*(b*c*(2*a*e*(2 - 3*n) + Sqrt[b^2 - 4*a*c]*d*
(1 - n)) - 2*a*c*(2*c*d*(1 - 2*n) - Sqrt[b^2 - 4*a*c]*e*(1 - n)) - b^3*e*(
1 - n) + b^2*(c*d - Sqrt[b^2 - 4*a*c]*e)*(1 - n))*x*Hypergeometric2F1[1, n
^(-1), 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*(
b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)^2*n) - (c*(a*b^
2*c*(Sqrt[b^2 - 4*a*c]*e*(5 - 14*n) - 6*c*d*(1 - 3*n))*(1 - n) + b^3*c*(a*
e*(7 - 18*n) + Sqrt[b^2 - 4*a*c]*d*(1 - 2*n))*(1 - n) - b^5*e*(1 - 3*n + 2
*n^2) + b^4*(c*d - Sqrt[b^2 - 4*a*c]*e)*(1 - 3*n + 2*n^2) - 4*a^2*c^2*(Sqr
t[b^2 - 4*a*c]*e*(1 - 4*n + 3*n^2) - 2*c*d*(1 - 6*n + 8*n^2)) - 2*a*b*c...
```

3.83.3.1 Defintions of rubi rules used

rule 1766
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
))^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2
*n))^p, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ((IntegersQ[p, q] &&
!IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] &&  !IntegerQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.83.4 Maple [F]

$$\int \frac{1}{(d+e\,x^n)\,(a+b\,x^n+c\,x^{2n})^3}dx$$

input
```
int(1/(d+e*x^n)/(a+b*x^n+c*x^(2*n))^3,x)
```

output
```
int(1/(d+e*x^n)/(a+b*x^n+c*x^(2*n))^3,x)
```

3.83.5 Fricas [F]

$$\int \frac{1}{(d+ex^n)\,(a+bx^n+cx^{2n})^3}\,dx = \int \frac{1}{(cx^{2\,n}+bx^n+a)^3(ex^n+d)}\,dx$$

input
```
integrate(1/(d+e*x^n)/(a+b*x^n+c*x^(2*n))^3,x, algorithm="fricas")
```

output
```
integral(1/(b^3*e*x^(4*n) + a^3*d + (c^3*e*x^n + c^3*d)*x^(6*n) + 3*(b*c^2
*e*x^(2*n) + a*c^2*d + (b*c^2*d + a*c^2*e)*x^n)*x^(4*n) + (b^3*d + 3*a*b^2
*e)*x^(3*n) + 3*(b^2*c*e*x^(3*n) + a^2*c*d + (b^2*c*d + 2*a*b*c*e)*x^(2*n)
 + (2*a*b*c*d + a^2*c*e)*x^n)*x^(2*n) + 3*(a*b^2*d + a^2*b*e)*x^(2*n) + (3
*a^2*b*d + a^3*e)*x^n), x)
```

3.83.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^n)\,(a+bx^n+cx^{2n})^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(d+e*x**n)/(a+b*x**n+c*x**(2*n))**3,x)
```

output
```
Timed out
```

3.83.7 Maxima [F]

$$\int \frac{1}{(d+ex^n)\left(a+bx^n+cx^{2n}\right)^3}\,dx = \int \frac{1}{(cx^{2n}+bx^n+a)^3(ex^n+d)}\,dx$$

input
```
integrate(1/(d+e*x^n)/(a+b*x^n+c*x^(2*n))^3,x, algorithm="maxima")
```

output
```
e^6*integrate(1/(c^3*d^7 - 3*b*c^2*d^6*e + 3*b^2*c*d^5*e^2 - b^3*d^4*e^3 +
 a^3*d*e^6 + 3*(c*d^3*e^4 - b*d^2*e^5)*a^2 + 3*(c^2*d^5*e^2 - 2*b*c*d^4*e^
3 + b^2*d^3*e^4)*a + (c^3*d^6*e - 3*b*c^2*d^5*e^2 + 3*b^2*c*d^4*e^3 - b^3*
d^3*e^4 + a^3*e^7 + 3*(c*d^2*e^5 - b*d*e^6)*a^2 + 3*(c^2*d^4*e^3 - 2*b*c*d
^3*e^4 + b^2*d^2*e^5)*a)*x^n), x) - 1/2*((4*a^3*c^4*e^3*(7*n - 1) - b^3*c^
4*d^3*(2*n - 1) + 2*b^4*c^3*d^2*e*(2*n - 1) - b^5*c^2*d*e^2*(2*n - 1) - (b
^2*c^3*e^3*(26*n - 5) - 4*c^5*d^2*e*(3*n - 1) - 10*b*c^4*d*e^2*n)*a^2 - (b
^2*c^4*d^2*e*(28*n - 9) - 2*b*c^5*d^3*(7*n - 2) - 2*b^3*c^3*d*e^2*(5*n - 2
) - b^4*c^2*e^3*(4*n - 1))*a)*x*x^(3*n) - (2*b^4*c^3*d^3*(2*n - 1) - 4*b^5
*c^2*d^2*e*(2*n - 1) + 2*b^6*c*d*e^2*(2*n - 1) - 2*(b*c^3*e^3*(37*n - 6) -
 2*c^4*d*e^2*(8*n - 1))*a^3 - (2*b*c^4*d^2*e*(25*n - 8) + 3*b^2*c^3*d*e^2*
(5*n + 1) - 11*b^3*c^2*e^3*(5*n - 1) - 4*c^5*d^3*(4*n - 1))*a^2 - (b^2*c^4
*d^3*(29*n - 9) - 2*b^3*c^3*d^2*e*(29*n - 10) + 3*b^4*c^2*d*e^2*(7*n - 3)
+ 2*b^5*c*e^3*(4*n - 1))*a)*x*x^(2*n) + (4*a^4*c^3*e^3*(9*n - 1) - b^5*c^2
*d^3*(2*n - 1) + 2*b^6*c*d^2*e*(2*n - 1) - b^7*d*e^2*(2*n - 1) + (b^2*c^2*
e^3*(14*n - 3) - 2*b*c^3*d*e^2*(13*n - 2) + 4*c^4*d^2*e*(5*n - 1))*a^3 - (
b^4*c*e^3*(24*n - 5) - b^3*c^2*d*e^2*(20*n - 1) - 2*b*c^4*d^3*n + 3*b^2*c^
3*d^2*e)*a^2 - (3*b^4*c^2*d^2*e*(8*n - 3) - b^6*e^3*(4*n - 1) - 4*b^3*c^3*
d^3*(3*n - 1) - 4*b^5*c*d*e^2*(2*n - 1))*a)*x*x^n + (2*(b*c^2*e^3*(29*n -
4) - 2*c^3*d*e^2*(10*n - 1))*a^4 + (2*b*c^3*d^2*e*(29*n - 6) - 4*c^4*d^...
```

3.83.8 Giac [F]

$$\int \frac{1}{(d+ex^n)\left(a+bx^n+cx^{2n}\right)^3}\,dx = \int \frac{1}{(cx^{2n}+bx^n+a)^3(ex^n+d)}\,dx$$

input
```
integrate(1/(d+e*x^n)/(a+b*x^n+c*x^(2*n))^3,x, algorithm="giac")
```

output
```
integrate(1/((c*x^(2*n) + b*x^n + a)^3*(e*x^n + d)), x)
```

3.83.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^n)(a+bx^n+cx^{2n})^3}\,dx=\int \frac{1}{(d+e\,x^n)\,\left(a+b\,x^n+c\,x^{2\,n}\right)^3}\,dx$$

input
```
int(1/((d + e*x^n)*(a + b*x^n + c*x^(2*n))^3),x)
```

output
```
int(1/((d + e*x^n)*(a + b*x^n + c*x^(2*n))^3), x)
```

3.84 $\int \frac{1}{(d+ex^n)^2 (a+bx^n+cx^{2n})^3} \, dx$

3.84.1 Optimal result

Integrand size = 26, antiderivative size = 2446

$$\int \frac{1}{(d+ex^n)^2 (a+bx^n+cx^{2n})^3} \, dx = \text{Too large to display}$$

output
```
-1/2*x*(2*b^3*c*d*e-6*a*b*c^2*d*e-b^4*e^2-b^2*c*(-4*a*e^2+c*d^2)+2*a*c^2*(
-a*e^2+c*d^2)+c*(2*b^2*c*d*e-4*a*c^2*d*e-b^3*e^2-b*c*(-3*a*e^2+c*d^2))*x^n
)/a/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)^2/n/(a+b*x^n+c*x^(2*n))^2-e^2*x*(5*b^
3*c*d*e-14*a*b*c^2*d*e-2*b^4*e^2-b^2*c*(-7*a*e^2+3*c*d^2)+2*a*c^2*(-a*e^2+
3*c*d^2)+c*(5*b^2*c*d*e-8*a*c^2*d*e-2*b^3*e^2-b*c*(-5*a*e^2+3*c*d^2))*x^n)
/a/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)^3/n/(a+b*x^n+c*x^(2*n))-1/2*x*(a*b^2*c
^2*(a*e^2*(13-37*n)-5*c*d^2*(1-3*n))-b^4*c*(a*e^2*(7-17*n)-c*d^2*(1-2*n))-
4*a^2*b*c^3*d*e*(4-11*n)+6*a*b^3*c^2*d*e*(2-5*n)+4*a^2*c^3*(-a*e^2+c*d^2)*
(1-4*n)-2*b^5*c*d*e*(1-2*n)+b^6*e^2*(1-2*n)+c*(2*a*b*c^2*(a*e^2*(4-13*n)-c
*d^2*(2-7*n))-b^3*c*(2*a*e^2*(3-8*n)-c*d^2*(1-2*n))+2*a*b^2*c^2*d*e*(5-14*
n)-8*a^2*c^3*d*e*(1-3*n)-2*b^4*c*d*e*(1-2*n)+b^5*e^2*(1-2*n))*x^n)/a^2/(-4
*a*c+b^2)^2/(a*e^2-b*d*e+c*d^2)^2/n^2/(a+b*x^n+c*x^(2*n))+3*e^6*(-b*e+2*c*
d)*x*hypergeom([1, 1/n],[1+1/n],-e*x^n/d)/d/(a*e^2-b*d*e+c*d^2)^4+e^6*x*hy
pergeom([2, 1/n],[1+1/n],-e*x^n/d)/d^2/(a*e^2-b*d*e+c*d^2)^3+1/2*c*x*hyper
geom([1, 1/n],[1+1/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))*((2*a*b*c^2*(a*e^2*
(4-13*n)-c*d^2*(2-7*n))-b^3*c*(2*a*e^2*(3-8*n)-c*d^2*(1-2*n))+2*a*b^2*c^2*
d*e*(5-14*n)-8*a^2*c^3*d*e*(1-3*n)-2*b^4*c*d*e*(1-2*n)+b^5*e^2*(1-2*n))*(1
-n)+(-b^4*c*(4*a*e^2*(2-5*n)-c*d^2*(1-2*n))*(1-n)-2*b^5*c*d*e*(2*n^2-3*n+1
)+b^6*e^2*(2*n^2-3*n+1)+8*a^2*c^3*(-a*e^2+c*d^2)*(8*n^2-6*n+1)-8*a^2*b*c^3
*d*e*(13*n^2-13*n+3)+2*a*b^3*c^2*d*e*(18*n^2-25*n+7)-2*a*b^2*c^2*(3*c*d...
```

3.84.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 56566 vs. 2(2446) = 4892.

Time = 9.30 (sec) , antiderivative size = 56566, normalized size of antiderivative = 23.13

$$\int \frac{1}{(d+ex^n)^2 (a+bx^n+cx^{2n})^3} \, dx = \text{Result too large to show}$$

input
```
Integrate[1/((d + e*x^n)^2*(a + b*x^n + c*x^(2*n))^3),x]
```

output
```
Result too large to show
```

3.84.3 Rubi [A] (verified)

Time = 6.31 (sec) , antiderivative size = 2446, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1766, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^n)^2 (a+bx^n+cx^{2n})^3} \, dx$$

$$\Big\downarrow \text{1766}$$

$$\int \left(\frac{e^2 \left(-ace^2+2b^2e^2+x^n \left(2bce^2-4c^2de\right)-5bcde+3c^2d^2\right)}{(ae^2-bde+cd^2)^3 (a+bx^n+cx^{2n})^2}+\frac{-ace^2+b^2e^2-\left(x^n \left(2c^2de-bce^2\right)\right)-2bcde+c^2\cdots}{(ae^2-bde+cd^2)^2 (a+bx^n+cx^{2n})^3}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{3(2cd-be)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)e^6}{d\left(cd^2-bed+ae^2\right)^4}+\frac{x\,\text{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{ex^n}{d}\right)e^6}{d^2\left(cd^2-bed+ae^2\right)^3}-$$

$$\frac{c\left(10c^2d^2+3b\left(b+\sqrt{b^2-4ac}\right)e^2-2ce\left(5bd+3\sqrt{b^2-4ac}d+ae\right)\right)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4}}\right.}{\left(b^2-\sqrt{b^2-4ac}b-4ac\right)\left(cd^2-bed+ae^2\right)^4}$$

$$\frac{c\left(10c^2d^2+3b\left(b-\sqrt{b^2-4ac}\right)e^2-2ce\left(5bd-3\sqrt{b^2-4ac}d+ae\right)\right)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b+\sqrt{b^2-4}}\right.}{\left(b^2+\sqrt{b^2-4ac}b-4ac\right)\left(cd^2-bed+ae^2\right)^4}$$

$$\frac{c\left(2e^2(1-n)b^4-e\left(5cd-2\sqrt{b^2-4ac}e\right)(1-n)b^3-c\left(e\left(ae(9-13n)+5\sqrt{b^2-4ac}d(1-n)\right)-3cd^2(1-n)\right)b^2}{}$$

$$\frac{c\left(2e^2(1-n)b^4-e\left(5cd+2\sqrt{b^2-4ac}e\right)(1-n)b^3-c\left(e\left(ae(9-13n)-5\sqrt{b^2-4ac}d(1-n)\right)-3cd^2(1-n)\right)b^2}{}$$

$$\frac{x\left(c\left(-2e^2b^3+5cdeb^2-c\left(3cd^2-5ae^2\right)b-8ac^2de\right)x^n-2b^4e^2-14abc^2de+5b^3cde-b^2c\left(3cd^2-7ae^2\right)+2ac^2\left(3c\right.\right.}{a\left(b^2-4ac\right)\left(cd^2-bed+ae^2\right)^3n\left(bx^n+cx^{2n}+a\right)}$$

$$\frac{c\left(\left(e^2(1-2n)b^5-2cde(1-2n)b^4-c\left(2ae^2(3-8n)-cd^2(1-2n)\right)b^3+2ac^2de(5-14n)b^2+2ac^2\left(ae^2(4-13n)\right.\right.\right.}{}$$

$$\frac{c\left(\left(e^2(1-2n)b^5-2cde(1-2n)b^4-c\left(2ae^2(3-8n)-cd^2(1-2n)\right)b^3+2ac^2de(5-14n)b^2+2ac^2\left(ae^2(4-13n)\right.\right.\right.}{}$$

$$\frac{x\left(c\left(e^2(1-2n)b^5-2cde(1-2n)b^4-c\left(2ae^2(3-8n)-cd^2(1-2n)\right)b^3+2ac^2de(5-14n)b^2+2ac^2\left(ae^2(4-13n\right.\right.\right.}{}$$

$$\frac{x\left(c\left(-e^2b^3+2cdeb^2-c\left(cd^2-3ae^2\right)b-4ac^2de\right)x^n-b^4e^2-6abc^2de+2b^3cde-b^2c\left(cd^2-4ae^2\right)+2ac^2\left(cd^2-a\right.\right.}{2a\left(b^2-4ac\right)\left(cd^2-bed+ae^2\right)^2n\left(bx^n+cx^{2n}+a\right)^2}$$

input
```
Int[1/((d + e*x^n)^2*(a + b*x^n + c*x^(2*n))^3),x]
```

3.84. $\int \frac{1}{(d+ex^n)^2(a+bx^n+cx^{2n})^3}\,dx$

output
```
-1/2*(x*(2*b^3*c*d*e - 6*a*b*c^2*d*e - b^4*e^2 - b^2*c*(c*d^2 - 4*a*e^2) +
 2*a*c^2*(c*d^2 - a*e^2) + c*(2*b^2*c*d*e - 4*a*c^2*d*e - b^3*e^2 - b*c*(c
*d^2 - 3*a*e^2))*x^n))/(a*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)^2*n*(a + b
*x^n + c*x^(2*n))^2) - (e^2*x*(5*b^3*c*d*e - 14*a*b*c^2*d*e - 2*b^4*e^2 -
b^2*c*(3*c*d^2 - 7*a*e^2) + 2*a*c^2*(3*c*d^2 - a*e^2) + c*(5*b^2*c*d*e - 8
*a*c^2*d*e - 2*b^3*e^2 - b*c*(3*c*d^2 - 5*a*e^2))*x^n))/(a*(b^2 - 4*a*c)*(
c*d^2 - b*d*e + a*e^2)^3*n*(a + b*x^n + c*x^(2*n))) - (x*(a*b^2*c^2*(a*e^2
*(13 - 37*n) - 5*c*d^2*(1 - 3*n)) - b^4*c*(a*e^2*(7 - 17*n) - c*d^2*(1 - 2
*n)) - 4*a^2*b*c^3*d*e*(4 - 11*n) + 6*a*b^3*c^2*d*e*(2 - 5*n) + 4*a^2*c^3*
(c*d^2 - a*e^2)*(1 - 4*n) - 2*b^5*c*d*e*(1 - 2*n) + b^6*e^2*(1 - 2*n) + c*
(2*a*b*c^2*(a*e^2*(4 - 13*n) - c*d^2*(2 - 7*n)) - b^3*c*(2*a*e^2*(3 - 8*n)
 - c*d^2*(1 - 2*n)) + 2*a*b^2*c^2*d*e*(5 - 14*n) - 8*a^2*c^3*d*e*(1 - 3*n)
 - 2*b^4*c*d*e*(1 - 2*n) + b^5*e^2*(1 - 2*n))*x^n))/(2*a^2*(b^2 - 4*a*c)^2
*(c*d^2 - b*d*e + a*e^2)^2*n^2*(a + b*x^n + c*x^(2*n))) - (c*e^4*(10*c^2*d
^2 + 3*b*(b + Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(5*b*d + 3*Sqrt[b^2 - 4*a*c]*
d + a*e))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[
b^2 - 4*a*c])])/((b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^
2)^4) + (c*e^2*(4*a*c^2*(e*(a*e*(1 - 2*n) + 2*Sqrt[b^2 - 4*a*c]*d*(1 - n))
 - 3*c*d^2*(1 - 2*n)) - b^2*c*(e*(a*e*(9 - 13*n) + 5*Sqrt[b^2 - 4*a*c]*d*(
1 - n)) - 3*c*d^2*(1 - n)) + b*c*(c*d*(4*a*e*(5 - 8*n) + 3*Sqrt[b^2 - 4...
```

3.84.3.1 Defintions of rubi rules used

rule 1766
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
))^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2
*n))^p, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ((IntegersQ[p, q] &&
!IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] &&  !IntegerQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.84.4 Maple [F]

$$\int \frac{1}{(d+e\,x^n)^2\,(a+b\,x^n+c\,x^{2n})^3}dx$$

input
```
int(1/(d+e*x^n)^2/(a+b*x^n+c*x^(2*n))^3,x)
```

output
```
int(1/(d+e*x^n)^2/(a+b*x^n+c*x^(2*n))^3,x)
```

3.84.5 Fricas [F]

$$\int \frac{1}{(d+ex^n)^2\,(a+bx^n+cx^{2n})^3}\,dx = \int \frac{1}{(cx^{2n}+bx^n+a)^3(ex^n+d)^2}\,dx$$

input
```
integrate(1/(d+e*x^n)^2/(a+b*x^n+c*x^(2*n))^3,x, algorithm="fricas")
```

output
```
integral(1/(b^3*e^2*x^(5*n) + a^3*d^2 + (c^3*e^2*x^(2*n) + 2*c^3*d*e*x^n +
 c^3*d^2)*x^(6*n) + 3*(b*c^2*e^2*x^(3*n) + a*c^2*d^2 + (2*b*c^2*d*e + a*c^
2*e^2)*x^(2*n) + (b*c^2*d^2 + 2*a*c^2*d*e)*x^n)*x^(4*n) + (2*b^3*d*e + 3*a
*b^2*e^2)*x^(4*n) + (b^3*d^2 + 6*a*b^2*d*e + 3*a^2*b*e^2)*x^(3*n) + 3*(b^2
*c*e^2*x^(4*n) + a^2*c*d^2 + 2*(b^2*c*d*e + a*b*c*e^2)*x^(3*n) + (b^2*c*d^
2 + 4*a*b*c*d*e + a^2*c*e^2)*x^(2*n) + 2*(a*b*c*d^2 + a^2*c*d*e)*x^n)*x^(2
*n) + (3*a*b^2*d^2 + 6*a^2*b*d*e + a^3*e^2)*x^(2*n) + (3*a^2*b*d^2 + 2*a^3
*d*e)*x^n), x)
```

3.84.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^n)^2\,(a+bx^n+cx^{2n})^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(d+e*x**n)**2/(a+b*x**n+c*x**(2*n))**3,x)
```

output
```
Timed out
```

3.84.7 Maxima [F]

$$\int \frac{1}{(d+ex^n)^2 (a+bx^n+cx^{2n})^3}\,dx = \int \frac{1}{(cx^{2n}+bx^n+a)^3(ex^n+d)^2}\,dx$$

input
```
integrate(1/(d+e*x^n)^2/(a+b*x^n+c*x^(2*n))^3,x, algorithm="maxima")
```

output
```
(c*d^2*e^6*(7*n - 1) - b*d*e^7*(4*n - 1) + a*e^8*(n - 1))*integrate(1/(c^4
*d^10*n - 4*b*c^3*d^9*e*n + 6*b^2*c^2*d^8*e^2*n - 4*b^3*c*d^7*e^3*n + b^4*
d^6*e^4*n + a^4*d^2*e^8*n + 4*(c*d^4*e^6*n - b*d^3*e^7*n)*a^3 + 6*(c^2*d^6
*e^4*n - 2*b*c*d^5*e^5*n + b^2*d^4*e^6*n)*a^2 + 4*(c^3*d^8*e^2*n - 3*b*c^2
*d^7*e^3*n + 3*b^2*c*d^6*e^4*n - b^3*d^5*e^5*n)*a + (c^4*d^9*e*n - 4*b*c^3
*d^8*e^2*n + 6*b^2*c^2*d^7*e^3*n - 4*b^3*c*d^6*e^4*n + b^4*d^5*e^5*n + a^4
*d*e^9*n + 4*(c*d^3*e^7*n - b*d^2*e^8*n)*a^3 + 6*(c^2*d^5*e^5*n - 2*b*c*d^
4*e^6*n + b^2*d^3*e^7*n)*a^2 + 4*(c^3*d^7*e^3*n - 3*b*c^2*d^6*e^4*n + 3*b^
2*c*d^5*e^5*n - b^3*d^4*e^6*n)*a)*x^n), x) + 1/2*((b^3*c^5*d^5*e*(2*n - 1)
 - 3*b^4*c^4*d^4*e^2*(2*n - 1) + 3*b^5*c^3*d^3*e^3*(2*n - 1) - b^6*c^2*d^2
*e^4*(2*n - 1) + 32*a^4*c^4*e^6*n + 2*(b*c^4*d*e^5*(33*n - 4) - 4*c^5*d^2*
e^4*(11*n - 1) - 8*b^2*c^3*e^6*n)*a^3 + 2*(b^2*c^4*d^2*e^4*(29*n - 1) - 3*
b^3*c^3*d*e^5*(7*n - 1) - 4*c^6*d^4*e^2*(3*n - 1) + 6*b*c^5*d^3*e^3*(n - 1
) + b^4*c^2*e^6*n)*a^2 - (3*b^3*c^4*d^3*e^3*(12*n - 5) + 2*b*c^6*d^5*e*(7*
n - 2) - b^5*c^2*d*e^5*(6*n - 1) - 14*b^2*c^5*d^4*e^2*(3*n - 1) - 2*b^4*c^
3*d^2*e^4*(n - 2))*a)*x*x^(4*n) + (b^3*c^5*d^6*(2*n - 1) - b^4*c^4*d^5*e*(
2*n - 1) - 3*b^5*c^3*d^4*e^2*(2*n - 1) + 5*b^6*c^2*d^3*e^3*(2*n - 1) - 2*b
^7*c*d^2*e^4*(2*n - 1) - 4*(c^4*d*e^5*(8*n - 1) - 16*b*c^3*e^6*n)*a^4 + (b
^2*c^3*d*e^5*(163*n - 21) - 6*b*c^4*d^2*e^4*(27*n - 2) - 8*c^5*d^3*e^3*(5*
n - 1) - 32*b^3*c^2*e^6*n)*a^3 - (b^4*c^2*d*e^5*(89*n - 13) - b^3*c^3*d...
```

3.84.8 Giac [F]

$$\int \frac{1}{(d+ex^n)^2 (a+bx^n+cx^{2n})^3}\,dx = \int \frac{1}{(cx^{2n}+bx^n+a)^3(ex^n+d)^2}\,dx$$

input
```
integrate(1/(d+e*x^n)^2/(a+b*x^n+c*x^(2*n))^3,x, algorithm="giac")
```

output
```
integrate(1/((c*x^(2*n) + b*x^n + a)^3*(e*x^n + d)^2), x)
```

3.84.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^n)^2\,(a+bx^n+cx^{2n})^3}\,dx = \int \frac{1}{(d+e\,x^n)^2\,(a+b\,x^n+c\,x^{2\,n})^3}\,dx$$

input
```
int(1/((d + e*x^n)^2*(a + b*x^n + c*x^(2*n))^3),x)
```

output
```
int(1/((d + e*x^n)^2*(a + b*x^n + c*x^(2*n))^3), x)
```

3.85 $\int (d+ex^n)\sqrt{a+bx^n+cx^{2n}}\,dx$

3.85.1 Optimal result

Integrand size = 26, antiderivative size = 292

$$\int (d+ex^n)\sqrt{a+bx^n+cx^{2n}}\,dx = \frac{ex^{1+n}\sqrt{a+bx^n+cx^{2n}}\operatorname{AppellF1}\left(1+\frac{1}{n},-\frac{1}{2},-\frac{1}{2},2+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{(1+n)\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}} + \frac{dx\sqrt{a+bx^n+cx^{2n}}\operatorname{AppellF1}\left(\frac{1}{n},-\frac{1}{2},-\frac{1}{2},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

output
```
e*x^(1+n)*AppellF1(1+1/n,-1/2,-1/2,2+1/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-
2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(a+b*x^n+c*x^(2*n))^(1/2)/(1+n)/(1+2*c*x^n
/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)+d*
x*AppellF1(1/n,-1/2,-1/2,1+1/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b
+(-4*a*c+b^2)^(1/2)))*(a+b*x^n+c*x^(2*n))^(1/2)/(1+2*c*x^n/(b-(-4*a*c+b^2)
^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

3.85.2 Mathematica [A] (verified)

Time = 1.22 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.45

$$\int (d+ex^n)\sqrt{a+bx^n+cx^{2n}}\,dx$$
$$= \frac{x\left(-n(-4ace(1+n)+b^2e(2+n)-2bcd(1+2n))\,x^n\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(1+\frac{1}{n},\right.\right.}{}$$

input
```
Integrate[(d + e*x^n)*Sqrt[a + b*x^n + c*x^(2*n)],x]
```

output
```
(x*(-(n*(-4*a*c*e*(1 + n) + b^2*e*(2 + n) - 2*b*c*d*(1 + 2*n))*x^n*Sqrt[(b
 - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^
2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1 + n^(-1), 1/2, 1
/2, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b
^2 - 4*a*c])]) + 2*(1 + n)*((a + x^n*(b + c*x^n))*(b*e*n + 2*c*(d + 2*d*n
+ e*(1 + n)*x^n)) + a*n*(-(b*e) + 2*c*(d + 2*d*n))*Sqrt[(b - Sqrt[b^2 - 4*
a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c
*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[n^(-1), 1/2, 1/2, 1 + n^(-1), (-2*
c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])))/(4*
(1 + n)^2*(c + 2*c*n)*Sqrt[a + x^n*(b + c*x^n)])
```

3.85.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1762, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex^n)\sqrt{a+bx^n+cx^{2n}}\,dx$$
$$\Big\downarrow \; 1762$$
$$\int \left(d\sqrt{a+bx^n+cx^{2n}}+ex^n\sqrt{a+bx^n+cx^{2n}}\right)dx$$
$$\Big\downarrow \; 2009$$

$$\frac{dx\sqrt{a+bx^n+cx^{2n}}\,\text{AppellF1}\left(\frac{1}{n},-\frac{1}{2},-\frac{1}{2},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}+\frac{ex^{n+1}\sqrt{a+bx^n+cx^{2n}}\,\text{AppellF1}\left(1+\frac{1}{n},-\frac{1}{2},-\frac{1}{2},2+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{(n+1)\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[(d + e*x^n)*Sqrt[a + b*x^n + c*x^(2*n)],x]
```

output
```
(e*x^(1 + n)*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[1 + n^(-1), -1/2, -1/2,
2 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 -
 4*a*c])])/((1 + n)*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (
2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]) + (d*x*Sqrt[a + b*x^n + c*x^(2*n)]*Appe
llF1[n^(-1), -1/2, -1/2, 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (
-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*
a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])
```

3.85.3.1 Defintions of rubi rules used

rule 1762
```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p
_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)*(a + b*x^n + c*x^(2*n))^p,
 x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.85.4 Maple [F]

$$\int (d+e\,x^n)\sqrt{a+b\,x^n+c\,x^{2n}}dx$$

input
```
int((d+e*x^n)*(a+b*x^n+c*x^(2*n))^(1/2),x)
```

output
```
int((d+e*x^n)*(a+b*x^n+c*x^(2*n))^(1/2),x)
```

3.85.5 Fricas [F(-2)]

Exception generated.

$$\int (d + ex^n) \sqrt{a + bx^n + cx^{2n}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((d+e*x^n)*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.85.6 Sympy [F]

$$\int (d + ex^n) \sqrt{a + bx^n + cx^{2n}} \, dx = \int (d + ex^n) \sqrt{a + bx^n + cx^{2n}} \, dx$$

input
```
integrate((d+e*x**n)*(a+b*x**n+c*x**(2*n))**(1/2),x)
```

output
```
Integral((d + e*x**n)*sqrt(a + b*x**n + c*x**(2*n)), x)
```

3.85.7 Maxima [F]

$$\int (d + ex^n) \sqrt{a + bx^n + cx^{2n}} \, dx = \int \sqrt{cx^{2n} + bx^n + a}(ex^n + d) \, dx$$

input
```
integrate((d+e*x^n)*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^(2*n) + b*x^n + a)*(e*x^n + d), x)
```

3.85.8 Giac [F]

$$\int (d+ex^n)\sqrt{a+bx^n+cx^{2n}}\,dx = \int \sqrt{cx^{2n}+bx^n+a}(ex^n+d)\,dx$$

input
```
integrate((d+e*x^n)*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^(2*n) + b*x^n + a)*(e*x^n + d), x)
```

3.85.9 Mupad [F(-1)]

Timed out.

$$\int (d+ex^n)\sqrt{a+bx^n+cx^{2n}}\,dx = \int (d+e\,x^n)\,\sqrt{a+b\,x^n+c\,x^{2\,n}}\,dx$$

input
```
int((d + e*x^n)*(a + b*x^n + c*x^(2*n))^(1/2),x)
```

output
```
int((d + e*x^n)*(a + b*x^n + c*x^(2*n))^(1/2), x)
```

3.86 $\int (d+ex^n) \left(a+bx^n+cx^{2n}\right)^{3/2} \, dx$

3.86.1 Optimal result

Integrand size = 26, antiderivative size = 294

$$\int (d+ex^n) \left(a+bx^n+cx^{2n}\right)^{3/2} \, dx = \frac{aex^{1+n}\sqrt{a+bx^n+cx^{2n}}\,\text{AppellF1}\left(1+\frac{1}{n},-\frac{3}{2},-\frac{3}{2},2+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{(1+n)\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}} + \frac{adx\sqrt{a+bx^n+cx^{2n}}\,\text{AppellF1}\left(\frac{1}{n},-\frac{3}{2},-\frac{3}{2},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

output
```
a*e*x^(1+n)*AppellF1(1+1/n,-3/2,-3/2,2+1/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2))
,-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(a+b*x^n+c*x^(2*n))^(1/2)/(1+n)/(1+2*c*x
^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)+
a*d*x*AppellF1(1/n,-3/2,-3/2,1+1/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^
n/(b+(-4*a*c+b^2)^(1/2)))*(a+b*x^n+c*x^(2*n))^(1/2)/(1+2*c*x^n/(b-(-4*a*c+
b^2)^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

3.86.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 690 vs. $2(294) = 588$.

Time = 3.26 (sec) , antiderivative size = 690, normalized size of antiderivative = 2.35

$$\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)^{3/2} dx = \frac{x\left(3n^2(16a^2c^2e(1+4n+3n^2)+b^4e(4+8n+3n^2)-2b^3cd(2+9n+4n^2)-4ab^2ce(5+14n\right.}{}$$

input
```
Integrate[(d + e*x^n)*(a + b*x^n + c*x^(2*n))^(3/2),x]
```

output
```
(x*(3*n^2*(16*a^2*c^2*e*(1 + 4*n + 3*n^2) + b^4*e*(4 + 8*n + 3*n^2) - 2*b^
3*c*d*(2 + 9*n + 4*n^2) - 4*a*b^2*c*e*(5 + 14*n + 6*n^2) + 8*a*b*c^2*d*(2
+ 11*n + 12*n^2))*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2
 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])
]*AppellF1[1 + n^(-1), 1/2, 1/2, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*
a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + 2*(1 + n)*((a + x^n*(b + c*x^
n))*(-3*b^3*e*n^2*(2 + 3*n) + 6*b^2*c*n^2*(d + 4*d*n + e*(1 + n)*x^n) + 8*
c^3*(1 + 3*n + 2*n^2)*x^(2*n)*(d + 4*d*n + e*(1 + 3*n)*x^n) + 4*b*c^2*(1 +
 n)*x^n*(d*(2 + 15*n + 28*n^2) + e*(2 + 13*n + 18*n^2)*x^n) + 4*a*c*(3*b*e
*n^2*(2 + 5*n) + 2*c*(d*(1 + 2*n)*(1 + 4*n)^2 + e*(1 + 9*n + 23*n^2 + 15*n
^3)*x^n))) + 3*a*n^2*(b^3*e*(2 + 3*n) - 2*b^2*c*d*(1 + 4*n) - 4*a*b*c*e*(2
 + 5*n) + 8*a*c^2*d*(1 + 6*n + 8*n^2))*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x
^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + S
qrt[b^2 - 4*a*c])]*AppellF1[n^(-1), 1/2, 1/2, 1 + n^(-1), (-2*c*x^n)/(b +
Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])))/(16*c^2*(1 + n)
^2*(1 + 2*n)*(1 + 3*n)*(1 + 4*n)*Sqrt[a + x^n*(b + c*x^n)])
```

3.86.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1762, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)^{3/2}\,dx$$

$$\Big\downarrow \text{1762}$$

$$\int \left(d\left(a+bx^n+cx^{2n}\right)^{3/2}+ex^n\left(a+bx^n+cx^{2n}\right)^{3/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{adx\sqrt{a+bx^n+cx^{2n}}\,\text{AppellF1}\left(\frac{1}{n},-\frac{3}{2},-\frac{3}{2},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}+\frac{aex^{n+1}\sqrt{a+bx^n+cx^{2n}}\,\text{AppellF1}\left(1+\frac{1}{n},-\frac{3}{2},-\frac{3}{2},2+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{(n+1)\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[(d + e*x^n)*(a + b*x^n + c*x^(2*n))^(3/2),x]
```

output
```
(a*e*x^(1 + n)*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[1 + n^(-1), -3/2, -3/2
, 2 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2
 - 4*a*c])])/((1 + n)*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 +
 (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]) + (a*d*x*Sqrt[a + b*x^n + c*x^(2*n)]*
AppellF1[n^(-1), -3/2, -3/2, 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]
), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2
- 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])
```

3.86.3.1 Defintions of rubi rules used

rule 1762
```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p
_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)*(a + b*x^n + c*x^(2*n))^p,
 x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.86.4 Maple [F]

$$\int (d+e\,x^{n})\left(a+b\,x^{n}+c\,x^{2n}\right)^{\frac{3}{2}}dx$$

input
```
int((d+e*x^n)*(a+b*x^n+c*x^(2*n))^(3/2),x)
```

output
```
int((d+e*x^n)*(a+b*x^n+c*x^(2*n))^(3/2),x)
```

3.86.5 Fricas [F(-2)]

Exception generated.

$$\int (d+ex^{n})\left(a+bx^{n}+cx^{2n}\right)^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d+e*x^n)*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.86.6 Sympy [F]

$$\int (d+ex^{n})\left(a+bx^{n}+cx^{2n}\right)^{3/2}\,dx = \int (d+ex^{n})\left(a+bx^{n}+cx^{2n}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((d+e*x**n)*(a+b*x**n+c*x**(2*n))**(3/2),x)
```

output
```
Integral((d + e*x**n)*(a + b*x**n + c*x**(2*n))**(3/2), x)
```

3.86.7 Maxima [F]

$$\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)^{3/2}\,dx=\int \left(cx^{2n}+bx^n+a\right)^{\frac{3}{2}}(ex^n+d)\,dx$$

input
```
integrate((d+e*x^n)*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^(3/2)*(e*x^n + d), x)
```

3.86.8 Giac [F]

$$\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)^{3/2}\,dx=\int \left(cx^{2n}+bx^n+a\right)^{\frac{3}{2}}(ex^n+d)\,dx$$

input
```
integrate((d+e*x^n)*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^(3/2)*(e*x^n + d), x)
```

3.86.9 Mupad [F(-1)]

Timed out.

$$\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)^{3/2}\,dx=\int (d+e\,x^n)\,\left(a+b\,x^n+c\,x^{2\,n}\right)^{3/2}\,dx$$

input
```
int((d + e*x^n)*(a + b*x^n + c*x^(2*n))^(3/2),x)
```

output
```
int((d + e*x^n)*(a + b*x^n + c*x^(2*n))^(3/2), x)
```

3.87 $\int \frac{d+ex^n}{\sqrt{a+bx^n+cx^{2n}}}\,dx$

3.87.1 Optimal result

Integrand size = 26, antiderivative size = 292

$$\int \frac{d+ex^n}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \frac{ex^{1+n}\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(1+\frac{1}{n},\frac{1}{2},\frac{1}{2},2+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{(1+n)\sqrt{a+bx^n+cx^{2n}}} + \frac{dx\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{1}{n},\frac{1}{2},\frac{1}{2},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{a+bx^n+cx^{2n}}}$$

output
```
e*x^(1+n)*AppellF1(1+1/n,1/2,1/2,2+1/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*
c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+
2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(1+n)/(a+b*x^n+c*x^(2*n))^(1/2)+d*x*
AppellF1(1/n,1/2,1/2,1+1/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-4
*a*c+b^2)^(1/2)))*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^n/(b+(
-4*a*c+b^2)^(1/2)))^(1/2)/(a+b*x^n+c*x^(2*n))^(1/2)
```

3.87.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.84

$$\int \frac{d+ex^n}{\sqrt{a+bx^n+cx^{2n}}}\,dx$$
$$= \frac{x\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\left(ex^n\,\text{AppellF1}\left(1+\frac{1}{n},\frac{1}{2},\frac{1}{2},2+\frac{1}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)+d(1+n)\right.}{(1+n)\sqrt{a+x^n(b+cx^n)}}$$

input
```
Integrate[(d + e*x^n)/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

output
```
(x*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b
 + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*(e*x^n*AppellF1[1
 + n^(-1), 1/2, 1/2, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*
x^n)/(-b + Sqrt[b^2 - 4*a*c])] + d*(1 + n)*AppellF1[n^(-1), 1/2, 1/2, 1 +
n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a
*c])]))/((1 + n)*Sqrt[a + x^n*(b + c*x^n)])
```

3.87.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1762, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^n}{\sqrt{a+bx^n+cx^{2n}}}\,dx$$
$$\Big\downarrow \text{1762}$$
$$\int \left(\frac{d}{\sqrt{a+bx^n+cx^{2n}}}+\frac{ex^n}{\sqrt{a+bx^n+cx^{2n}}}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{dx\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{1}{n},\frac{1}{2},\frac{1}{2},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{a+bx^n+cx^{2n}}}+$$
$$\frac{ex^{n+1}\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(1+\frac{1}{n},\frac{1}{2},\frac{1}{2},2+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{(n+1)\sqrt{a+bx^n+cx^{2n}}}$$

input
```
Int[(d + e*x^n)/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

output
```
(e*x^(1 + n)*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n
)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1 + n^(-1), 1/2, 1/2, 2 + n^(-1), (-2*
c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/((1 +
 n)*Sqrt[a + b*x^n + c*x^(2*n)]) + (d*x*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 -
 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[n^(-1), 1/2
, 1/2, 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqr
t[b^2 - 4*a*c])])/Sqrt[a + b*x^n + c*x^(2*n)]
```

3.87.3.1 Defintions of rubi rules used

rule 1762
```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p
_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)*(a + b*x^n + c*x^(2*n))^p,
 x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.87.4 Maple [F]

$$\int \frac{d+e\,x^n}{\sqrt{a+b\,x^n+c\,x^{2n}}}dx$$

input
```
int((d+e*x^n)/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

output
```
int((d+e*x^n)/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

3.87.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{d+ex^n}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d+e*x^n)/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.87.6 Sympy [F]

$$\int \frac{d+ex^n}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{d+ex^n}{\sqrt{a+bx^n+cx^{2n}}}\,dx$$

input
```
integrate((d+e*x**n)/(a+b*x**n+c*x**(2*n))**(1/2),x)
```

output
```
Integral((d + e*x**n)/sqrt(a + b*x**n + c*x**(2*n)), x)
```

3.87.7 Maxima [F]

$$\int \frac{d+ex^n}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{ex^n+d}{\sqrt{cx^{2n}+bx^n+a}}\,dx$$

input
```
integrate((d+e*x^n)/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x^n + d)/sqrt(c*x^(2*n) + b*x^n + a), x)
```

3.87.8 Giac [F]

$$\int \frac{d+ex^n}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{ex^n+d}{\sqrt{cx^{2n}+bx^n+a}}\,dx$$

input
```
integrate((d+e*x^n)/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x^n + d)/sqrt(c*x^(2*n) + b*x^n + a), x)
```

3.87.9 Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^n}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{d+e\,x^n}{\sqrt{a+b\,x^n+c\,x^{2\,n}}}\,dx$$

input
```
int((d + e*x^n)/(a + b*x^n + c*x^(2*n))^(1/2),x)
```

output
```
int((d + e*x^n)/(a + b*x^n + c*x^(2*n))^(1/2), x)
```

3.88 $\int \frac{d+ex^n}{(a+bx^n+cx^{2n})^{3/2}}\,dx$

3.88.1 Optimal result

Integrand size = 26, antiderivative size = 298

$$\int \frac{d+ex^n}{(a+bx^n+cx^{2n})^{3/2}}\,dx=\frac{ex^{1+n}\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(1+\frac{1}{n},\frac{3}{2},\frac{3}{2},2+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right.}{a(1+n)\sqrt{a+bx^n+cx^{2n}}}$$
$$+\frac{dx\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(\frac{1}{n},\frac{3}{2},\frac{3}{2},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{a\sqrt{a+bx^n+cx^{2n}}}$$

output
```
e*x^(1+n)*AppellF1(1+1/n,3/2,3/2,2+1/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*
c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+
2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/(1+n)/(a+b*x^n+c*x^(2*n))^(1/2)+d*
x*AppellF1(1/n,3/2,3/2,1+1/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+(
-4*a*c+b^2)^(1/2)))*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^n/(b
+(-4*a*c+b^2)^(1/2)))^(1/2)/a/(a+b*x^n+c*x^(2*n))^(1/2)
```

3.88.2 Mathematica [A] (verified)

Time = 1.56 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.39

$$\int \frac{d+ex^n}{(a+bx^n+cx^{2n})^{3/2}}\,dx=\frac{x\left(2c(bd-2ae)x^n\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(1+\frac{1}{n},\frac{1}{2},\frac{1}{2},2+\right.\right.}{}$$

input
```
Integrate[(d + e*x^n)/(a + b*x^n + c*x^(2*n))^(3/2),x]
```

output
```
(x*(2*c*(b*d - 2*a*e)*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt
[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a
*c])]*AppellF1[1 + n^(-1), 1/2, 1/2, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2
- 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - (1 + n)*(2*(b^2*d + b*(-(
a*e) + c*d*x^n) - 2*a*c*(d + e*x^n)) + (2*a*b*e + b^2*d*(-2 + n) - 4*a*c*d
*(-1 + n))*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]
*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[
n^(-1), 1/2, 1/2, 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n
)/(-b + Sqrt[b^2 - 4*a*c])])))/(a*(-b^2 + 4*a*c)*n*(1 + n)*Sqrt[a + x^n*(b
 + c*x^n)])
```

3.88.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1762, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^n}{(a+bx^n+cx^{2n})^{3/2}}\,dx$$

$$\Big\downarrow \text{1762}$$

$$\int \left(\frac{d}{(a+bx^n+cx^{2n})^{3/2}}+\frac{ex^n}{(a+bx^n+cx^{2n})^{3/2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{dx\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{1}{n},\frac{3}{2},\frac{3}{2},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{a\sqrt{a+bx^n+cx^{2n}}}+\frac{ex^{n+1}\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(1+\frac{1}{n},\frac{3}{2},\frac{3}{2},2+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{a(n+1)\sqrt{a+bx^n+cx^{2n}}}$$

input
```
Int[(d + e*x^n)/(a + b*x^n + c*x^(2*n))^(3/2),x]
```

output
```
(e*x^(1 + n)*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n
)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1 + n^(-1), 3/2, 3/2, 2 + n^(-1), (-2*
c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(a*(1
 + n)*Sqrt[a + b*x^n + c*x^(2*n)]) + (d*x*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2
 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[n^(-1), 3
/2, 3/2, 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + S
qrt[b^2 - 4*a*c])])/(a*Sqrt[a + b*x^n + c*x^(2*n)])
```

3.88.3.1 Defintions of rubi rules used

rule 1762
```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p
_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)*(a + b*x^n + c*x^(2*n))^p,
 x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.88.4 Maple [F]

$$\int \frac{d + e\,x^{n}}{\left(a + b\,x^{n} + c\,x^{2n}\right)^{\frac{3}{2}}}dx$$

input
```
int((d+e*x^n)/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

output
```
int((d+e*x^n)/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

3.88.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{d + ex^n}{(a + bx^n + cx^{2n})^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d+e*x^n)/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.88.6 Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^n}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((d+e*x**n)/(a+b*x**n+c*x**(2*n))**(3/2),x)
```

output
```
Timed out
```

3.88.7 Maxima [F]

$$\int \frac{d+ex^n}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \int \frac{ex^n+d}{(cx^{2n}+bx^n+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d+e*x^n)/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
```

output
```
integrate((e*x^n + d)/(c*x^(2*n) + b*x^n + a)^(3/2), x)
```

3.88.8 Giac [F]

$$\int \frac{d+ex^n}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \int \frac{ex^n+d}{(cx^{2n}+bx^n+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d+e*x^n)/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
```

output
```
integrate((e*x^n + d)/(c*x^(2*n) + b*x^n + a)^(3/2), x)
```

3.88.9 Mupad [F(-1)]

Timed out.

$$\int \frac{d + ex^n}{(a + bx^n + cx^{2n})^{3/2}}\, dx = \int \frac{d + e\, x^n}{(a + b\, x^n + c\, x^{2\, n})^{3/2}}\, dx$$

input
```
int((d + e*x^n)/(a + b*x^n + c*x^(2*n))^(3/2),x)
```

output
```
int((d + e*x^n)/(a + b*x^n + c*x^(2*n))^(3/2), x)
```

3.89 $\int \frac{d+ex^n}{(a+bx^n+cx^{2n})^{5/2}}\,dx$

3.89.1 Optimal result

Integrand size = 26, antiderivative size = 298

$$\int \frac{d+ex^n}{(a+bx^n+cx^{2n})^{5/2}}\,dx=\frac{ex^{1+n}\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(1+\frac{1}{n},\frac{5}{2},\frac{5}{2},2+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},\right.}{a^2(1+n)\sqrt{a+bx^n+cx^{2n}}}$$
$$+\frac{dx\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(\frac{1}{n},\frac{5}{2},\frac{5}{2},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{a^2\sqrt{a+bx^n+cx^{2n}}}$$

output
```
e*x^(1+n)*AppellF1(1+1/n,5/2,5/2,2+1/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*
c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+
2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a^2/(1+n)/(a+b*x^n+c*x^(2*n))^(1/2)+
d*x*AppellF1(1/n,5/2,5/2,1+1/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b
+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^n/
(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a^2/(a+b*x^n+c*x^(2*n))^(1/2)
```

3.89.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 701 vs. 2(298) = 596.

Time = 5.36 (sec) , antiderivative size = 701, normalized size of antiderivative = 2.35

$$\int \frac{d+ex^n}{(a+bx^n+cx^{2n})^{5/2}}\,dx=\frac{x\left(-2c(2ab^2e+4abcd(2-5n)+8a^2ce(-1+2n)+b^3d(-2+3n))\,x^n\sqrt{\frac{b-\sqrt{b^2-4}}{b-\sqrt{b}}}\right.}{}$$

input
```
Integrate[(d + e*x^n)/(a + b*x^n + c*x^(2*n))^(5/2),x]
```

output
```
(x*(-2*c*(2*a*b^2*e + 4*a*b*c*d*(2 - 5*n) + 8*a^2*c*e*(-1 + 2*n) + b^3*d*(
-2 + 3*n))*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*
c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*(a +
x^n*(b + c*x^n))*AppellF1[1 + n^(-1), 1/2, 1/2, 2 + n^(-1), (-2*c*x^n)/(b
+ Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + (1 + n)*(2*(b^
3*d*(-2 + 3*n)*x^n*(b + c*x^n)^2 + 4*a^3*c*(b*e*(-2 + 3*n) + c*d*(-2 + 8*n
) + 2*c*e*(-1 + 3*n)*x^n) + 2*a*b*(b + c*x^n)*(-2*c^2*d*(-2 + 5*n)*x^(2*n)
 + b*c*x^n*(d*(5 - 11*n) + e*x^n) + b^2*(d*(-1 + 2*n) + e*x^n)) + a^2*(-(b
^3*e*(-2 + n)) + 8*b*c^2*e*(-2 + 3*n)*x^(2*n) - 2*b^2*c*(d*(-5 + 14*n) - 3
*e*(-1 + n)*x^n) + 8*c^3*x^(2*n)*(d*(-1 + 3*n) + e*(-1 + 2*n)*x^n))) + (2*
a*b^3*e*(-2 + n) - 8*a^2*b*c*e*(-2 + 3*n) + b^4*d*(4 - 8*n + 3*n^2) + 16*a
^2*c^2*d*(1 - 4*n + 3*n^2) - 4*a*b^2*c*d*(5 - 14*n + 6*n^2))*Sqrt[(b - Sqr
t[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*
a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*(a + x^n*(b + c*x^n))*AppellF1[n^
(-1), 1/2, 1/2, 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/
(-b + Sqrt[b^2 - 4*a*c])])))/(3*a^2*(b^2 - 4*a*c)^2*n^2*(1 + n)*(a + x^n*(
b + c*x^n))^(3/2))
```

3.89.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1762, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^n}{(a+bx^n+cx^{2n})^{5/2}}\,dx$$

$$\Big\downarrow \text{1762}$$

$$\int \left(\frac{d}{(a+bx^n+cx^{2n})^{5/2}}+\frac{ex^n}{(a+bx^n+cx^{2n})^{5/2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{dx\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{1}{n},\frac{5}{2},\frac{5}{2},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{a^2\sqrt{a+bx^n+cx^{2n}}}+\frac{ex^{n+1}\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(1+\frac{1}{n},\frac{5}{2},\frac{5}{2},2+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{a^2(n+1)\sqrt{a+bx^n+cx^{2n}}}$$

input
```
Int[(d + e*x^n)/(a + b*x^n + c*x^(2*n))^(5/2),x]
```

output
```
(e*x^(1 + n)*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n
)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1 + n^(-1), 5/2, 5/2, 2 + n^(-1), (-2*
c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(a^2*
(1 + n)*Sqrt[a + b*x^n + c*x^(2*n)]) + (d*x*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b
^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[n^(-1),
 5/2, 5/2, 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b +
 Sqrt[b^2 - 4*a*c])])/(a^2*Sqrt[a + b*x^n + c*x^(2*n)])
```

3.89.3.1 Defintions of rubi rules used

rule 1762
```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p
_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)*(a + b*x^n + c*x^(2*n))^p,
 x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.89.4 Maple [F]

$$\int \frac{d+e\,x^n}{\left(a+b\,x^n+c\,x^{2n}\right)^{\frac{5}{2}}}dx$$

input
```
int((d+e*x^n)/(a+b*x^n+c*x^(2*n))^(5/2),x)
```

output
```
int((d+e*x^n)/(a+b*x^n+c*x^(2*n))^(5/2),x)
```

3.89.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{d + ex^n}{(a + bx^n + cx^{2n})^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d+e*x^n)/(a+b*x^n+c*x^(2*n))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.89.6 Sympy [F]

$$\int \frac{d + ex^n}{(a + bx^n + cx^{2n})^{5/2}}\,dx = \int \frac{d + ex^n}{(a + bx^n + cx^{2n})^{\frac{5}{2}}}\,dx$$

input
```
integrate((d+e*x**n)/(a+b*x**n+c*x**(2*n))**(5/2),x)
```

output
```
Integral((d + e*x**n)/(a + b*x**n + c*x**(2*n))**(5/2), x)
```

3.89.7 Maxima [F]

$$\int \frac{d + ex^n}{(a + bx^n + cx^{2n})^{5/2}}\,dx = \int \frac{ex^n + d}{(cx^{2n} + bx^n + a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d+e*x^n)/(a+b*x^n+c*x^(2*n))^(5/2),x, algorithm="maxima")
```

output
```
integrate((e*x^n + d)/(c*x^(2*n) + b*x^n + a)^(5/2), x)
```

3.89.8 Giac [F]

$$\int \frac{d+ex^n}{(a+bx^n+cx^{2n})^{5/2}}\,dx=\int \frac{ex^n+d}{(cx^{2n}+bx^n+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d+e*x^n)/(a+b*x^n+c*x^(2*n))^(5/2),x, algorithm="giac")
```

output
```
integrate((e*x^n + d)/(c*x^(2*n) + b*x^n + a)^(5/2), x)
```

3.89.9 Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^n}{(a+bx^n+cx^{2n})^{5/2}}\,dx=\int \frac{d+e\,x^n}{(a+b\,x^n+c\,x^{2\,n})^{5/2}}\,dx$$

input
```
int((d + e*x^n)/(a + b*x^n + c*x^(2*n))^(5/2),x)
```

output
```
int((d + e*x^n)/(a + b*x^n + c*x^(2*n))^(5/2), x)
```

3.90 $\int (d+ex^n)^q (a+bx^n+cx^{2n})^p \, dx$

3.90.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int (d+ex^n)^q \left(a+bx^n+cx^{2n}\right)^p \, dx = \text{Int}\left((d+ex^n)^q \left(a+bx^n+cx^{2n}\right)^p, x\right)$$

output
```
Unintegrable((d+e*x^n)^q*(a+b*x^n+c*x^(2*n))^p,x)
```

3.90.2 Mathematica [N/A]

Not integrable

Time = 1.33 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int (d+ex^n)^q \left(a+bx^n+cx^{2n}\right)^p \, dx = \int (d+ex^n)^q \left(a+bx^n+cx^{2n}\right)^p \, dx$$

input
```
Integrate[(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))^p,x]
```

output
```
Integrate[(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x]
```

3.90.3 Rubi [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {1769}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex^n)^q \left(a+bx^n+cx^{2n}\right)^p \, dx$$
$$\Big\downarrow \text{1769}$$
$$\int (d+ex^n)^q \left(a+bx^n+cx^{2n}\right)^p dx$$

input
```
Int[(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))^p,x]
```

output
```
$Aborted
```

3.90.3.1 Defintions of rubi rules used

rule 1769
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
))^(p_), x_Symbol] :> Unintegrable[(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))^p,
 x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && EqQ[n2, 2*n]
```

3.90.4 Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int (d+e\,x^n)^q \left(a+b\,x^n+c\,x^{2n}\right)^p dx$$

input
```
int((d+e*x^n)^q*(a+b*x^n+c*x^(2*n))^p,x)
```

output
```
int((d+e*x^n)^q*(a+b*x^n+c*x^(2*n))^p,x)
```

3.90.5 Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int (d+ex^n)^q \left(a+bx^n+cx^{2n}\right)^p \, dx = \int (cx^{2n}+bx^n+a)^p (ex^n+d)^q \, dx$$

input
```
integrate((d+e*x^n)^q*(a+b*x^n+c*x^(2*n))^p,x, algorithm="fricas")
```

output
```
integral((c*x^(2*n) + b*x^n + a)^p*(e*x^n + d)^q, x)
```

3.90.6 Sympy [F(-1)]

Timed out.

$$\int (d+ex^n)^q \left(a+bx^n+cx^{2n}\right)^p \, dx = \text{Timed out}$$

input
```
integrate((d+e*x**n)**q*(a+b*x**n+c*x**(2*n))**p,x)
```

output
```
Timed out
```

3.90.7 Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int (d+ex^n)^q \left(a+bx^n+cx^{2n}\right)^p \, dx = \int (cx^{2n}+bx^n+a)^p (ex^n+d)^q \, dx$$

input
```
integrate((d+e*x^n)^q*(a+b*x^n+c*x^(2*n))^p,x, algorithm="maxima")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^p*(e*x^n + d)^q, x)
```

3.90.8 Giac [N/A]

Not integrable

Time = 1.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int (d+ex^n)^q \left(a+bx^n+cx^{2n}\right)^p \,dx = \int \left(cx^{2n}+bx^n+a\right)^p (ex^n+d)^q \,dx$$

input
```
integrate((d+e*x^n)^q*(a+b*x^n+c*x^(2*n))^p,x, algorithm="giac")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^p*(e*x^n + d)^q, x)
```

3.90.9 Mupad [N/A]

Not integrable

Time = 10.63 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int (d+ex^n)^q \left(a+bx^n+cx^{2n}\right)^p \,dx = \int (d+e\,x^n)^q \left(a+b\,x^n+c\,x^{2\,n}\right)^p \,dx$$

input
```
int((d + e*x^n)^q*(a + b*x^n + c*x^(2*n))^p,x)
```

output
```
int((d + e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x)
```

3.91 $\int (d+ex^n)^3 (a+bx^n+cx^{2n})^p \, dx$

3.91.1 Optimal result

Integrand size = 26, antiderivative size = 606

$$\int (d+ex^n)^3 (a+bx^n+cx^{2n})^p \, dx$$
$$= \frac{3d^2ex^{1+n}\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^n+cx^{2n})^p \operatorname{AppellF1}\left(1+\frac{1}{n},-p,-p,2+\frac{1}{n},-\frac{2c}{b-\sqrt{b^2}}\right.}{1+n}$$
$$+\frac{3de^2x^{1+2n}\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^n+cx^{2n})^p \operatorname{AppellF1}\left(2+\frac{1}{n},-p,-p,3+\frac{1}{n},-\frac{}{b-}\right.}{1+2n}$$
$$+\frac{e^3x^{1+3n}\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^n+cx^{2n})^p \operatorname{AppellF1}\left(3+\frac{1}{n},-p,-p,4+\frac{1}{n},-\frac{2c}{b-\sqrt{}}\right.}{1+3n}$$
$$+d^3x\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^n+cx^{2n})^p \operatorname{AppellF1}\left(\frac{1}{n},\right.$$
$$\left.-p,-p,1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)$$

output
```
3*d^2*e*x^(1+n)*(a+b*x^n+c*x^(2*n))^p*AppellF1(1+1/n,-p,-p,2+1/n,-2*c*x^n/
(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/(1+n)/((1+2*c*x^n/
(b-(-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^p)+3*d*e^2*
x^(1+2*n)*(a+b*x^n+c*x^(2*n))^p*AppellF1(2+1/n,-p,-p,3+1/n,-2*c*x^n/(b-(-4
*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/(1+2*n)/((1+2*c*x^n/(b-(
-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^p)+e^3*x^(1+3*n
)*(a+b*x^n+c*x^(2*n))^p*AppellF1(3+1/n,-p,-p,4+1/n,-2*c*x^n/(b-(-4*a*c+b^2
)^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/(1+3*n)/((1+2*c*x^n/(b-(-4*a*c+b
^2)^(1/2)))^p)/((1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^p)+d^3*x*(a+b*x^n+c*x^(
2*n))^p*AppellF1(1/n,-p,-p,1+1/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/
(b+(-4*a*c+b^2)^(1/2)))/((1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x^n
/(b+(-4*a*c+b^2)^(1/2)))^p)
```

3.91.2 Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 438, normalized size of antiderivative = 0.72

$$\int (d+ex^n)^3 \left(a+bx^n+cx^{2n}\right)^p \, dx$$

$$= \frac{x\left(\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+x^n(b+cx^n))^p\left(3d^2e(1+5n+6n^2)\,x^n\,\text{AppellF1}\left(1+\frac{1}{n},-p\right.\right.}{}$$

input
```
Integrate[(d + e*x^n)^3*(a + b*x^n + c*x^(2*n))^p,x]
```

output
```
(x*(a + x^n*(b + c*x^n))^p*(3*d^2*e*(1 + 5*n + 6*n^2)*x^n*AppellF1[1 + n^(
-1), -p, -p, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b
 + Sqrt[b^2 - 4*a*c])] + (1 + n)*(3*d*e^2*(1 + 3*n)*x^(2*n)*AppellF1[2 + n
^(-1), -p, -p, 3 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(
-b + Sqrt[b^2 - 4*a*c])] + (1 + 2*n)*(e^3*x^(3*n)*AppellF1[3 + n^(-1), -p,
 -p, 4 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[
b^2 - 4*a*c])] + d^3*(1 + 3*n)*AppellF1[n^(-1), -p, -p, 1 + n^(-1), (-2*c*
x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))))/((1
+ n)*(1 + 2*n)*(1 + 3*n)*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2
- 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p
)
```

3.91.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 606, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1766, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex^n)^3 \left(a+bx^n+cx^{2n}\right)^p \, dx$$

$$\Big\downarrow \text{1766}$$

$$\int \left(d^3\left(a+bx^n+cx^{2n}\right)^p+3d^2ex^n\left(a+bx^n+cx^{2n}\right)^p+3de^2x^{2n}\left(a+bx^n+cx^{2n}\right)^p+e^3x^{3n}\left(a+bx^n+cx^{2n}\right)^p\right) dx$$

$$\Big\downarrow \text{2009}$$

$$d^3x\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p \text{AppellF1}\left(\frac{1}{n},-p,-p,1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-}}\right.$$
$$\frac{3d^2ex^{n+1}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p \text{AppellF1}\left(1+\frac{1}{n},-p,-p,2+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},\right.}{n+1}$$
$$\frac{3de^2x^{2n+1}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p \text{AppellF1}\left(2+\frac{1}{n},-p,-p,3+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right.}{2n+1}$$
$$\frac{e^3x^{3n+1}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p \text{AppellF1}\left(3+\frac{1}{n},-p,-p,4+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\right.}{3n+1}$$

input
```
Int[(d + e*x^n)^3*(a + b*x^n + c*x^(2*n))^p,x]
```

output
```
(3*d^2*e*x^(1 + n)*(a + b*x^n + c*x^(2*n))^p*AppellF1[1 + n^(-1), -p, -p,
2 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 -
 4*a*c])])/((1 + n)*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^
n)/(b + Sqrt[b^2 - 4*a*c]))^p) + (3*d*e^2*x^(1 + 2*n)*(a + b*x^n + c*x^(2*
n))^p*AppellF1[2 + n^(-1), -p, -p, 3 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 -
4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/((1 + 2*n)*(1 + (2*c*x^n)/(b
 - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p) + (e^3
*x^(1 + 3*n)*(a + b*x^n + c*x^(2*n))^p*AppellF1[3 + n^(-1), -p, -p, 4 + n^
(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
])])/((1 + 3*n)*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(
b + Sqrt[b^2 - 4*a*c]))^p) + (d^3*x*(a + b*x^n + c*x^(2*n))^p*AppellF1[n^(
-1), -p, -p, 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b
 + Sqrt[b^2 - 4*a*c])])/((1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2
*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

3.91.3.1 Defintions of rubi rules used

rule 1766
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
))^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2
*n))^p, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ((IntegersQ[p, q] &&
!IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] && !IntegerQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.91.4 Maple [F]

$$\int (d+e\,x^n)^3\left(a+b\,x^n+c\,x^{2n}\right)^p dx$$

input
```
int((d+e*x^n)^3*(a+b*x^n+c*x^(2*n))^p,x)
```

output
```
int((d+e*x^n)^3*(a+b*x^n+c*x^(2*n))^p,x)
```

3.91.5 Fricas [F]

$$\int (d+ex^n)^3 \left(a+bx^n+cx^{2n}\right)^p \, dx = \int (ex^n+d)^3 \left(cx^{2n}+bx^n+a\right)^p \, dx$$

input
```
integrate((d+e*x^n)^3*(a+b*x^n+c*x^(2*n))^p,x, algorithm="fricas")
```

output
```
integral((e^3*x^(3*n) + 3*d*e^2*x^(2*n) + 3*d^2*e*x^n + d^3)*(c*x^(2*n) +
b*x^n + a)^p, x)
```

3.91.6 Sympy [F(-1)]

Timed out.

$$\int (d+ex^n)^3 \left(a+bx^n+cx^{2n}\right)^p \, dx = \text{Timed out}$$

input
```
integrate((d+e*x**n)**3*(a+b*x**n+c*x**(2*n))**p,x)
```

output
```
Timed out
```

3.91.7 Maxima [F]

$$\int (d+ex^n)^3 \left(a+bx^n+cx^{2n}\right)^p \, dx = \int (ex^n+d)^3 \left(cx^{2n}+bx^n+a\right)^p \, dx$$

input
```
integrate((d+e*x^n)^3*(a+b*x^n+c*x^(2*n))^p,x, algorithm="maxima")
```

output
```
integrate((e*x^n + d)^3*(c*x^(2*n) + b*x^n + a)^p, x)
```

3.91.8 Giac [F(-2)]

Exception generated.

$$\int (d+ex^n)^3 \left(a+bx^n+cx^{2n}\right)^p \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((d+e*x^n)^3*(a+b*x^n+c*x^(2*n))^p,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{512,[1,0,7,4,9,5,1,8,0,3]%%%}+%%%{-3072,[1,0,7,4,9,5,0,9,1
,2]%%%}+%
```

3.91.9 Mupad [F(-1)]

Timed out.

$$\int (d+ex^n)^3 \left(a+bx^n+cx^{2n}\right)^p \, dx = \int (d+e\,x^n)^3 \left(a+b\,x^n+c\,x^{2\,n}\right)^p dx$$

input
```
int((d + e*x^n)^3*(a + b*x^n + c*x^(2*n))^p,x)
```

output
```
int((d + e*x^n)^3*(a + b*x^n + c*x^(2*n))^p, x)
```

3.92 $\int (d+ex^n)^2 (a+bx^n+cx^{2n})^p \, dx$

3.92.1 Optimal result

Integrand size = 26, antiderivative size = 447

$$\int (d+ex^n)^2 (a+bx^n+cx^{2n})^p \, dx = \frac{2dex^{1+n}\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^n+cx^{2n})^p \operatorname{AppellF1}\left(1+\frac{1}{n},-p,-p,2+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{1+n} + \frac{e^2x^{1+2n}\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^n+cx^{2n})^p \operatorname{AppellF1}\left(2+\frac{1}{n},-p,-p,3+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{1+2n} + d^2x\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^n+cx^{2n})^p \operatorname{AppellF1}\left(\frac{1}{n},-p,-p,1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)$$

output
```
2*d*e*x^(1+n)*(a+b*x^n+c*x^(2*n))^p*AppellF1(1+1/n,-p,-p,2+1/n,-2*c*x^n/(b
-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/(1+n)/((1+2*c*x^n/(b
-(-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^p)+e^2*x^(1+2
*n)*(a+b*x^n+c*x^(2*n))^p*AppellF1(2+1/n,-p,-p,3+1/n,-2*c*x^n/(b-(-4*a*c+b
^2)^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/(1+2*n)/((1+2*c*x^n/(b-(-4*a*c
+b^2)^(1/2)))^p)/((1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^p)+d^2*x*(a+b*x^n+c*x
^(2*n))^p*AppellF1(1/n,-p,-p,1+1/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^
n/(b+(-4*a*c+b^2)^(1/2)))/((1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x
^n/(b+(-4*a*c+b^2)^(1/2)))^p)
```

3.92.2 Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 338, normalized size of antiderivative = 0.76

$$\int (d+ex^n)^2 \left(a+bx^n+cx^{2n}\right)^p \, dx$$
$$= \frac{x\left(\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+x^n(b+cx^n))^p\left(2de(1+2n)x^n \operatorname{AppellF1}\left(1+\frac{1}{n},-p,-p,2+\right.\right.}{}$$

input
```
Integrate[(d + e*x^n)^2*(a + b*x^n + c*x^(2*n))^p,x]
```

output
```
(x*(a + x^n*(b + c*x^n))^p*(2*d*e*(1 + 2*n)*x^n*AppellF1[1 + n^(-1), -p, -
p, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^
2 - 4*a*c])] + (1 + n)*(e^2*x^(2*n)*AppellF1[2 + n^(-1), -p, -p, 3 + n^(-1
), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]
 + d^2*(1 + 2*n)*AppellF1[n^(-1), -p, -p, 1 + n^(-1), (-2*c*x^n)/(b + Sqrt
[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])))/((1 + n)*(1 + 2*n)*
((b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b
^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

3.92.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 447, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1766, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex^n)^2 \left(a+bx^n+cx^{2n}\right)^p \, dx$$
$$\Big\downarrow \ 1766$$
$$\int \left(d^2\left(a+bx^n+cx^{2n}\right)^p+2dex^n\left(a+bx^n+cx^{2n}\right)^p+e^2x^{2n}\left(a+bx^n+cx^{2n}\right)^p\right) dx$$
$$\Big\downarrow \ 2009$$

$$d^2x\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^n+cx^{2n})^p\,\text{AppellF1}\left(\frac{1}{n},-p,-p,1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-}}\right.$$
$$\frac{2dex^{n+1}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^n+cx^{2n})^p\,\text{AppellF1}\left(1+\frac{1}{n},-p,-p,2+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\right.}{n+1}$$
$$\frac{e^2x^{2n+1}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^n+cx^{2n})^p\,\text{AppellF1}\left(2+\frac{1}{n},-p,-p,3+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\right.}{2n+1}$$

input
```
Int[(d + e*x^n)^2*(a + b*x^n + c*x^(2*n))^p,x]
```

output
```
(2*d*e*x^(1 + n)*(a + b*x^n + c*x^(2*n))^p*AppellF1[1 + n^(-1), -p, -p, 2
+ n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4
*a*c])])/((1 + n)*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)
/(b + Sqrt[b^2 - 4*a*c]))^p) + (e^2*x^(1 + 2*n)*(a + b*x^n + c*x^(2*n))^p*
AppellF1[2 + n^(-1), -p, -p, 3 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]
), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/((1 + 2*n)*(1 + (2*c*x^n)/(b - Sqr
t[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p) + (d^2*x*(a
+ b*x^n + c*x^(2*n))^p*AppellF1[n^(-1), -p, -p, 1 + n^(-1), (-2*c*x^n)/(b
- Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/((1 + (2*c*x^n)
/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

3.92.3.1 Defintions of rubi rules used

rule 1766
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
))^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2
*n))^p, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ((IntegersQ[p, q] &&
!IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] &&  !IntegerQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.92.4 Maple [F]

$$\int \left(d+e\,x^{n}\right)^{2} \left(a+b\,x^{n}+c\,x^{2 n}\right)^{p}d x$$

input
```
int((d+e*x^n)^2*(a+b*x^n+c*x^(2*n))^p,x)
```

output
```
int((d+e*x^n)^2*(a+b*x^n+c*x^(2*n))^p,x)
```

3.92.5 Fricas [F]

$$\int (d+ex^n)^2 \left(a+bx^n+cx^{2n}\right)^p \, dx = \int (ex^n+d)^2 (cx^{2n}+bx^n+a)^p \, dx$$

input
```
integrate((d+e*x^n)^2*(a+b*x^n+c*x^(2*n))^p,x, algorithm="fricas")
```

output
```
integral((e^2*x^(2*n) + 2*d*e*x^n + d^2)*(c*x^(2*n) + b*x^n + a)^p, x)
```

3.92.6 Sympy [F(-1)]

Timed out.

$$\int (d+ex^n)^2 \left(a+bx^n+cx^{2n}\right)^p \, dx = \text{Timed out}$$

input
```
integrate((d+e*x**n)**2*(a+b*x**n+c*x**(2*n))**p,x)
```

output
```
Timed out
```

3.92.7 Maxima [F]

$$\int (d+ex^n)^2 \left(a+bx^n+cx^{2n}\right)^p dx = \int (ex^n+d)^2 (cx^{2n}+bx^n+a)^p dx$$

input
```
integrate((d+e*x^n)^2*(a+b*x^n+c*x^(2*n))^p,x, algorithm="maxima")
```

output
```
integrate((e*x^n + d)^2*(c*x^(2*n) + b*x^n + a)^p, x)
```

3.92.8 Giac [F(-2)]

Exception generated.

$$\int (d+ex^n)^2 \left(a+bx^n+cx^{2n}\right)^p dx = \text{Exception raised: TypeError}$$

input
```
integrate((d+e*x^n)^2*(a+b*x^n+c*x^(2*n))^p,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{-128,[1,0,5,3,6,4,1,6,0,2]%%%}+%%%{512,[1,0,5,3,6,4,0,7,1,
1]%%%}+%%
```

3.92.9 Mupad [F(-1)]

Timed out.

$$\int (d+ex^n)^2 \left(a+bx^n+cx^{2n}\right)^p dx = \int (d+e\,x^n)^2 \left(a+b\,x^n+c\,x^{2\,n}\right)^p dx$$

input
```
int((d + e*x^n)^2*(a + b*x^n + c*x^(2*n))^p,x)
```

output
```
int((d + e*x^n)^2*(a + b*x^n + c*x^(2*n))^p, x)
```

3.93 $\int (d+ex^n)(a+bx^n+cx^{2n})^p\,dx$

3.93.1 Optimal result

Integrand size = 24, antiderivative size = 288

$$\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)^p\,dx$$
$$=\frac{ex^{1+n}\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p\text{AppellF1}\left(1+\frac{1}{n},-p,-p,2+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4a}}\right.}{1+n}$$
$$+dx\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p\text{AppellF1}\left(\frac{1}{n},-p,-p,1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)$$

output
```
e*x^(1+n)*(a+b*x^n+c*x^(2*n))^p*AppellF1(1+1/n,-p,-p,2+1/n,-2*c*x^n/(b-(-4
*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/(1+n)/((1+2*c*x^n/(b-(-4
*a*c+b^2)^(1/2)))^p)/((1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^p)+d*x*(a+b*x^n+c
*x^(2*n))^p*AppellF1(1/n,-p,-p,1+1/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*
x^n/(b+(-4*a*c+b^2)^(1/2)))/((1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^p)/((1+2*c
*x^n/(b+(-4*a*c+b^2)^(1/2)))^p)
```

3.93.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.84

$$\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)^p\,dx$$
$$=\frac{x\left(\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+x^n(b+cx^n))^p\left(ex^n\,\text{AppellF1}\left(1+\frac{1}{n},-p,-p,2+\frac{1}{n},-\frac{2c}{b+\sqrt{b}}\right.\right.}{1+n}$$

input
```
Integrate[(d + e*x^n)*(a + b*x^n + c*x^(2*n))^p,x]
```

output
```
(x*(a + x^n*(b + c*x^n))^p*(e*x^n*AppellF1[1 + n^(-1), -p, -p, 2 + n^(-1),
 (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] +
 d*(1 + n)*AppellF1[n^(-1), -p, -p, 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 -
 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))/((1 + n)*((b - Sqrt[b^2 -
4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c
*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

3.93.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1762, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)^p\,dx$$
$$\Big\downarrow\ 1762$$
$$\int \left(d\left(a+bx^n+cx^{2n}\right)^p+ex^n\left(a+bx^n+cx^{2n}\right)^p\right)dx$$
$$\Big\downarrow\ 2009$$
$$dx\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p\text{AppellF1}\left(\frac{1}{n},-p,-p,1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-}}\right.$$
$$\frac{ex^{n+1}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p\text{AppellF1}\left(1+\frac{1}{n},-p,-p,2+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{}{b+}\right.}{n+1}$$

input
```
Int[(d + e*x^n)*(a + b*x^n + c*x^(2*n))^p,x]
```

output
```
(e*x^(1 + n)*(a + b*x^n + c*x^(2*n))^p*AppellF1[1 + n^(-1), -p, -p, 2 + n^
(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
])])/((1 + n)*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b
+ Sqrt[b^2 - 4*a*c]))^p) + (d*x*(a + b*x^n + c*x^(2*n))^p*AppellF1[n^(-1),
 -p, -p, 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + S
qrt[b^2 - 4*a*c])])/((1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x
^n)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

3.93.3.1 Defintions of rubi rules used

rule 1762
```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p
_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)*(a + b*x^n + c*x^(2*n))^p,
 x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.93.4 Maple [F]

$$\int (d + e\,x^{n}) \left(a + b\,x^{n} + c\,x^{2n}\right)^{p} dx$$

input
```
int((d+e*x^n)*(a+b*x^n+c*x^(2*n))^p,x)
```

output
```
int((d+e*x^n)*(a+b*x^n+c*x^(2*n))^p,x)
```

3.93.5 Fricas [F]

$$\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)^p\,dx = \int (ex^n+d)\left(cx^{2n}+bx^n+a\right)^p\,dx$$

input
```
integrate((d+e*x^n)*(a+b*x^n+c*x^(2*n))^p,x, algorithm="fricas")
```

output
```
integral((e*x^n + d)*(c*x^(2*n) + b*x^n + a)^p, x)
```

3.93.6 Sympy [F(-1)]

Timed out.

$$\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)^p\,dx = \text{Timed out}$$

input
```
integrate((d+e*x**n)*(a+b*x**n+c*x**(2*n))**p,x)
```

output
```
Timed out
```

3.93.7 Maxima [F]

$$\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)^p\,dx = \int (ex^n+d)\left(cx^{2n}+bx^n+a\right)^p\,dx$$

input
```
integrate((d+e*x^n)*(a+b*x^n+c*x^(2*n))^p,x, algorithm="maxima")
```

output
```
integrate((e*x^n + d)*(c*x^(2*n) + b*x^n + a)^p, x)
```

3.93.8 Giac [F]

$$\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)^p\,dx=\int (ex^n+d)\left(cx^{2\,n}+bx^n+a\right)^p\,dx$$

input
```
integrate((d+e*x^n)*(a+b*x^n+c*x^(2*n))^p,x, algorithm="giac")
```

output
```
integrate((e*x^n + d)*(c*x^(2*n) + b*x^n + a)^p, x)
```

3.93.9 Mupad [F(-1)]

Timed out.

$$\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)^p\,dx=\int (d+e\,x^n)\ \left(a+b\,x^n+c\,x^{2\,n}\right)^p\,dx$$

input
```
int((d + e*x^n)*(a + b*x^n + c*x^(2*n))^p,x)
```

output
```
int((d + e*x^n)*(a + b*x^n + c*x^(2*n))^p, x)
```

3.94 $\int \frac{(a+bx^n+cx^{2n})^p}{d+ex^n}\,dx$

3.94.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{(a+bx^n+cx^{2n})^p}{d+ex^n}\,dx=\text{Int}\left(\frac{(a+bx^n+cx^{2n})^p}{d+ex^n},x\right)$$

output
```
Unintegrable((a+b*x^n+c*x^(2*n))^p/(d+e*x^n),x)
```

3.94.2 Mathematica [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^n+cx^{2n})^p}{d+ex^n}\,dx=\int \frac{(a+bx^n+cx^{2n})^p}{d+ex^n}\,dx$$

input
```
Integrate[(a + b*x^n + c*x^(2*n))^p/(d + e*x^n),x]
```

output
```
Integrate[(a + b*x^n + c*x^(2*n))^p/(d + e*x^n), x]
```

3.94.3 Rubi [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {1769}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^n+cx^{2n})^p}{d+ex^n}\,dx$$

$$\Big\downarrow \text{ 1769}$$

$$\int \frac{(a+bx^n+cx^{2n})^p}{d+ex^n}dx$$

input
```
Int[(a + b*x^n + c*x^(2*n))^p/(d + e*x^n),x]
```

output
```
$Aborted
```

3.94.3.1 Defintions of rubi rules used

rule 1769
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
))^(p_), x_Symbol] :> Unintegrable[(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))^p,
 x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && EqQ[n2, 2*n]
```

3.94.4 Maple [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\,x^n+c\,x^{2n})^p}{d+e\,x^n}dx$$

input
```
int((a+b*x^n+c*x^(2*n))^p/(d+e*x^n),x)
```

output
```
int((a+b*x^n+c*x^(2*n))^p/(d+e*x^n),x)
```

3.94.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^n+cx^{2n})^p}{d+ex^n}\,dx=\int \frac{(cx^{2\,n}+bx^n+a)^p}{ex^n+d}\,dx$$

input
```
integrate((a+b*x^n+c*x^(2*n))^p/(d+e*x^n),x, algorithm="fricas")
```

output
```
integral((c*x^(2*n) + b*x^n + a)^p/(e*x^n + d), x)
```

3.94.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^n+cx^{2n})^p}{d+ex^n}\,dx=\text{Timed out}$$

input
```
integrate((a+b*x**n+c*x**(2*n))**p/(d+e*x**n),x)
```

output
```
Timed out
```

3.94.7 Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^n+cx^{2n})^p}{d+ex^n}\,dx=\int \frac{(cx^{2\,n}+bx^n+a)^p}{ex^n+d}\,dx$$

input
```
integrate((a+b*x^n+c*x^(2*n))^p/(d+e*x^n),x, algorithm="maxima")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^p/(e*x^n + d), x)
```

3.94.8 Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^n+cx^{2n})^p}{d+ex^n}\,dx=\int \frac{(cx^{2n}+bx^n+a)^p}{ex^n+d}\,dx$$

input
```
integrate((a+b*x^n+c*x^(2*n))^p/(d+e*x^n),x, algorithm="giac")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^p/(e*x^n + d), x)
```

3.94.9 Mupad [N/A]

Not integrable

Time = 10.36 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^n+cx^{2n})^p}{d+ex^n}\,dx=\int \frac{(a+b\,x^n+c\,x^{2\,n})^p}{d+e\,x^n}\,dx$$

input
```
int((a + b*x^n + c*x^(2*n))^p/(d + e*x^n),x)
```

output
```
int((a + b*x^n + c*x^(2*n))^p/(d + e*x^n), x)
```

3.95 $\int \frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^2} dx$

3.95.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^2} dx = \text{Int}\left(\frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^2}, x\right)$$

output
```
Unintegrable((a+b*x^n+c*x^(2*n))^p/(d+e*x^n)^2,x)
```

3.95.2 Mathematica [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^2} dx = \int \frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^2} dx$$

input
```
Integrate[(a + b*x^n + c*x^(2*n))^p/(d + e*x^n)^2,x]
```

output
```
Integrate[(a + b*x^n + c*x^(2*n))^p/(d + e*x^n)^2, x]
```

3.95.3 Rubi [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {1769}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^2} \, dx$$

$$\Big\downarrow \; 1769$$

$$\int \frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^2}dx$$

input
```
Int[(a + b*x^n + c*x^(2*n))^p/(d + e*x^n)^2,x]
```

output
```
$Aborted
```

3.95.3.1 Defintions of rubi rules used

rule 1769
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
))^(p_), x_Symbol] :> Unintegrable[(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))^p,
 x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && EqQ[n2, 2*n]
```

3.95.4 Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\,x^n+c\,x^{2n})^p}{(d+e\,x^n)^2}dx$$

input
```
int((a+b*x^n+c*x^(2*n))^p/(d+e*x^n)^2,x)
```

output
```
int((a+b*x^n+c*x^(2*n))^p/(d+e*x^n)^2,x)
```

3.95.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.58

$$\int \frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^2}\,dx = \int \frac{(cx^{2n}+bx^n+a)^p}{(ex^n+d)^2}\,dx$$

input
```
integrate((a+b*x^n+c*x^(2*n))^p/(d+e*x^n)^2,x, algorithm="fricas")
```

output
```
integral((c*x^(2*n) + b*x^n + a)^p/(e^2*x^(2*n) + 2*d*e*x^n + d^2), x)
```

3.95.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*x**n+c*x**(2*n))**p/(d+e*x**n)**2,x)
```

output
```
Timed out
```

3.95.7 Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^2}\,dx = \int \frac{(cx^{2n}+bx^n+a)^p}{(ex^n+d)^2}\,dx$$

input
```
integrate((a+b*x^n+c*x^(2*n))^p/(d+e*x^n)^2,x, algorithm="maxima")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^p/(e*x^n + d)^2, x)
```

3.95.8 Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^2}\,dx = \int \frac{(cx^{2n}+bx^n+a)^p}{(ex^n+d)^2}\,dx$$

input
```
integrate((a+b*x^n+c*x^(2*n))^p/(d+e*x^n)^2,x, algorithm="giac")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^p/(e*x^n + d)^2, x)
```

3.95.9 Mupad [N/A]

Not integrable

Time = 11.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^2}\,dx = \int \frac{(a+b\,x^n+c\,x^{2\,n})^p}{(d+e\,x^n)^2}\,dx$$

input
```
int((a + b*x^n + c*x^(2*n))^p/(d + e*x^n)^2,x)
```

output
```
int((a + b*x^n + c*x^(2*n))^p/(d + e*x^n)^2, x)
```

3.96 $\int \frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^3}\,dx$

3.96.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^3}\,dx = \text{Int}\left(\frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^3},x\right)$$

output
```
Unintegrable((a+b*x^n+c*x^(2*n))^p/(d+e*x^n)^3,x)
```

3.96.2 Mathematica [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^3}\,dx = \int \frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^3}\,dx$$

input
```
Integrate[(a + b*x^n + c*x^(2*n))^p/(d + e*x^n)^3,x]
```

output
```
Integrate[(a + b*x^n + c*x^(2*n))^p/(d + e*x^n)^3, x]
```

3.96.3 Rubi [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {1769}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^3} \, dx$$
$$\Big\downarrow \text{1769}$$
$$\int \frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^3} dx$$

input
```
Int[(a + b*x^n + c*x^(2*n))^p/(d + e*x^n)^3,x]
```

output
```
$Aborted
```

3.96.3.1 Defintions of rubi rules used

rule 1769
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
))^(p_), x_Symbol] :> Unintegrable[(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))^p,
 x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && EqQ[n2, 2*n]
```

3.96.4 Maple [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\,x^n+c\,x^{2n})^p}{(d+e\,x^n)^3}dx$$

input
```
int((a+b*x^n+c*x^(2*n))^p/(d+e*x^n)^3,x)
```

output
```
int((a+b*x^n+c*x^(2*n))^p/(d+e*x^n)^3,x)
```

3.96.5 Fricas [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.08

$$\int \frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^3}\,dx=\int \frac{(cx^{2n}+bx^n+a)^p}{(ex^n+d)^3}\,dx$$

input
```
integrate((a+b*x^n+c*x^(2*n))^p/(d+e*x^n)^3,x, algorithm="fricas")
```

output
```
integral((c*x^(2*n) + b*x^n + a)^p/(e^3*x^(3*n) + 3*d*e^2*x^(2*n) + 3*d^2*
e*x^n + d^3), x)
```

3.96.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^3}\,dx=\text{Timed out}$$

input
```
integrate((a+b*x**n+c*x**(2*n))**p/(d+e*x**n)**3,x)
```

output
```
Timed out
```

3.96.7 Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^3}\,dx=\int \frac{(cx^{2n}+bx^n+a)^p}{(ex^n+d)^3}\,dx$$

input
```
integrate((a+b*x^n+c*x^(2*n))^p/(d+e*x^n)^3,x, algorithm="maxima")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^p/(e*x^n + d)^3, x)
```

3.96.8 Giac [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^3}\,dx = \int \frac{(cx^{2n}+bx^n+a)^p}{(ex^n+d)^3}\,dx$$

input
```
integrate((a+b*x^n+c*x^(2*n))^p/(d+e*x^n)^3,x, algorithm="giac")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^p/(e*x^n + d)^3, x)
```

3.96.9 Mupad [N/A]

Not integrable

Time = 11.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^3}\,dx = \int \frac{(a+b\,x^n+c\,x^{2\,n})^p}{(d+e\,x^n)^3}\,dx$$

input
```
int((a + b*x^n + c*x^(2*n))^p/(d + e*x^n)^3,x)
```

output
```
int((a + b*x^n + c*x^(2*n))^p/(d + e*x^n)^3, x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
if ExpnType_result<=ExpnType_optimal then
    if debug then
         print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                    print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A"," ";
            else
               return "B",cat("Both result and optimal contain complex but leaf count of
                               convert(leaf_count_result,string)," vs. $2 (",
                               convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
            end if
        else #result contains complex but optimal is not
            if debug then
                    print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex
         # this assumes optimal do not as well. No check is needed here.
        if debug then
               print("result do not contain complex, this assumes optimal do not as well"
        fi;
        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A"," ";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
            fi;
            return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
        fi;
    fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))  #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):  #type(expn,'`^`')
        if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
            return expnType(expn.args[0])  #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):   #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))

leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation =" "
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
return grade, grade_annotation
```